An Improved Satellite Selection Method in Attitude Determination Using Global Positioning System (GPS)

Bo Wang*[,1,2], Shunting Wang[1], Lingjuan Miao[1] and Jun Shen[1]

[1]Department of Automatic Control, Beijing Institute of Technology, Beijing 100081, China

[2]School of Instrumentation Science and Opto-electronics Engineering, Beihang University, Beijing 100191, China

Abstract: In attitude determination using GPS, the accuracy of results is influenced directly by the satellite selection of the differenced observations. Based on the information of satellites, such as elevation, azimuth and signal to noise ratio, we apply a weighted selection method to obtain the satellite combination used in attitude determination. This method also considered the quality of satellite signal during building double difference observation equations. Experimental results show that this method reduces the noise influence of the double difference carrier phase measurements and increases the accuracy of the attitude determination. Compared to the traditional method, this is more suitable for the attitude determination.

Keywords: GPS, attitude determination, satellite selection.

1. INTRODUCTION

Using the subcentimeter precision of global positioning system (GPS) carrier phase measurements from multiple antennas mounted to a vehicle and line-of-sight (LOS) vectors from antennas to satellites the attitude of the vehicle can be determined in real time [1]. Compared to an inertial measurement unit, GPS has long-term stability and lower power consumption and cost. Therefore, research on attitude determination using GPS has been given considerable attention in the last decades [2-4].

In attitude determination using GPS, precise measurements of the double difference carrier phase observations between satellites and antennas are critical to the accurate solution. However, the double difference carrier phase measurements include many kinds of noise which cause errors in the attitude solution. The errors are dominated by the systematic errors caused by multipath, atmosphere, orbit effects and receiver noise [2]. Except for the receiver noise, other noise sources are related directly to the satellites used in the double difference. Therefore, elevation, azimuth and signal to noise (SNR) of different satellites have significant influence on the accuracy of the carrier phase measurements and the attitude determination [5]. The traditional satellite selection method chooses the satellite with the high elevation angle as the reference satellite to obtain the double difference measurements and all the satellites the same weight in the observation equation without considering the quality of the satellite signals. Therefore, in order to efficiently increase the accuracy of attitude determination, we should choose a group of satellites with minimum noise effect and their corresponding weight matrix.

*Address correspondence to this author at the Department of Automatic Control, Beijing Institute of Technology, Beijing 100081, China;
E-mail: wb1020@ bit.edu.cn

The functions of satellite selection methods can be considered as two types: positioning and differencing. Generally, minimum GDOP (Geometry Dilution of Precision) principle is used in positioning. We choose the satellites which form the largest volume as a group to calculate the position of the antenna. In attitude determination, the same as GDOP, we have ADOP (Attitude Dilution of Precision) which is also controlled by the geometry of the satellites [2]. In double difference, we always choose one as the main satellite, because the geometry would be better.

2. ATTITUDE DETERMINATION USING GPS

The attitude determination system using GPS includes at least two antennas. One is selected as the main antenna and the others are considered to be auxiliary antennas that form several baselines mounted to a vehicle. The carrier phase measurement equation is as follows:

$$\phi_i^j = \rho_i^j + c(\mathrm{d}t^j - \mathrm{d}T_i) + \lambda N_i^j - \delta_{i,I}^j + \delta_{i,T}^j + e_i^j \quad (1)$$

where i and j are the number of antenna and satellite respectively, ϕ_i^j is the carrier phase measurement, ρ_i^j is the geometric range from antenna to satellite, c is the speed of light, $\mathrm{d}t^j$ and $\mathrm{d}T_i$ are the offsets of the satellite and receiver clocks from GPS time, λ is the carrier wavelength, N_i^j is the integer ambiguity, $\delta_{i,I}^j$ and $\delta_{i,T}^j$ are the delays imparted by the ionosphere and troposphere respectively, and e_i^j is the effect of multipath and receiver noise. For the original observable vector of antenna 1 and 2,

$$O = \begin{bmatrix} O_1^1 & O_1^2 & \cdots & O_1^n & O_2^1 & O_2^2 & \cdots & O_2^n \end{bmatrix}^T \quad (2)$$

where $O = \phi, \rho, N, e$; n is the number of observed satellite. The difference between the simultaneous phase

1877-6116/09 2009 Bentham Open

measurements of a receiver of the signals transmitted by two different satellites, and simultaneous measurements at the same nominal time of a second receiver of the same signals follows from Eq. (1) as:

$$\mathrm{DD}\phi = \mathrm{DD}\rho + \lambda \cdot \mathrm{DD}N + \mathrm{DD}e \tag{3}$$

where DD is the double difference operator, and the dimension of Eq. (3) is *n*-1. The clock and atmospheric errors in the carrier phase measurement can be eliminated by the double difference. With *m*+1 satellites in view, the above model can be cast in the linearized observation equation. We use the least squares method to obtain the float solutions of double difference integer ambiguities and baseline vector components. After the integer ambiguity resolution and validation process, the baseline vector will be estimated independently for each auxiliary antenna by least squares method [6, 7].

Once the baseline vectors are solved, which can be considered as the relative positions of antennas in the Earth Centered Earth Fixed (ECEF) coordinate frame [8]. They must be transformed to the local level navigation frame by the transformation matrix. Then, the attitude angles yaw, pitch, and roll of the vehicle can be calculated by the direct computation method [8].

3. SELECTION OF THE SATELLITE COMBINATION AND THE WEIGHT MATRIX

The selection of the satellite combination for attitude estimation is actually the determination of the double difference operator DD. For the original observable vector in Eq. (1), the single difference can be formed by the operator SD as

$$\mathrm{SD} = \begin{bmatrix} 1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & -1 \end{bmatrix} = \begin{bmatrix} I_{n\times n} & -I_{n\times n} \end{bmatrix} \tag{4}$$

where $I_{n\times n}$ is an identity matrix that has the size of *n*. The double difference operator DD is formed between two single differences related to two observed satellites as

$$\mathrm{DD} = C_d \cdot \mathrm{SD} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \cdot \mathrm{SD} = \begin{bmatrix} \mathbf{1}_{m\times 1} & -I_{m\times m} \end{bmatrix} \cdot \mathrm{SD} \tag{5}$$

where **1** is a 1 vector (all elements of the vector are 1), *m* is the number of formed double differences, and *m*=*n*-1. The sequences of the row and column in the linear transformation matrix C_d in Eq. (5) are the same as the sequence of the satellite elevation angles. The measurements used in double difference depend on the satellite combination in Eq. (2). Not all the in-sight satellites should be included in the observation equation. In order to increase the measurement accuracy, some criterions are applied to choose the proper satellite combination. For attitude determination, the satellite with the highest elevation angle is selected as the reference satellite in the traditional double difference operator, namely the first column in C_d is the 1 vector. However, in the practical environment it is not always optimal to select the satellite with highest elevation angle as the reference one. We must take other error sources into account.

3.1. Expression of the Geometrical Dilution of Precision for Attitude

The variances of the position estimates depend upon two factors: variance of the user range error, and a term which depends entirely on the antenna-satellite geometry. The geometry dilution of precision (GDOP) parameter is defined on the basis of the positioning equations to characterize the contribution of the antenna-satellite geometry. As an analog to GDOP, ADOP, the geometrical dilution of precision for attitude, is defined by considering the geometrical component of the covariance matrix [2].

$$\mathrm{ADOP} = \sqrt{\mathrm{trace}\left[\left(nI - B^T B\right)^{-1}\right]} \tag{6}$$

where trace(•) is the trace of matrix. The error of the attitude determination increases with the increase of the ADOP value, as similar to what GDOP to positioning error.

As shown in Eq. (6), the ADOP value is determined by the LOS vector matrix *B*. The direction cosine vectors of antenna-satellite in matrix *B* can be expressed by the combination of elevation and azimuth angles of satellites. Therefore, in attitude determination using GPS, we should choose satellites used in double difference based on the angle information. The DOP value does not change in different coordinate frames. Therefore, the relationship of the LOS vector and the elevation and azimuth angles are expressed as follows

$$\begin{cases} \cos\alpha_i = \sin E_i \\ \cos\beta_i = \cos E_i \sin A_i \\ \cos\gamma_i = \cos E_i \cos A_i \end{cases} \tag{7}$$

where $\cos\alpha_i$, $\cos\beta_i$, and $\cos\gamma_i$ are direction cosine vectors of the *i*th satellite in WGS84 frame, E_i and A_i are the elevation and azimuth angles of the *i*th satellite respectively, $i = 1, 2, \cdots, n$. Thus, the LOS vector matrix *B* can be rewritten as

$$B = \begin{bmatrix} \sin E_1 & \cos E_1 \sin A_1 & \cos E_1 \cos A_1 \\ \sin E_2 & \cos E_2 \sin A_2 & \cos E_2 \cos A_2 \\ \vdots & \vdots & \vdots \\ \sin E_n & \cos E_n \sin A_n & \cos E_n \cos A_n \end{bmatrix} \tag{8}$$

Substituting Eq. (8) into Eq. (6), we obtain the expression of the ADOP with satellite angle information

$$\mathrm{ADOP} = \sqrt{\sigma_E + \sigma_A} \tag{9}$$

where σ_E and σ_A are errors with the elevation and azimuth angle of the satellites respectively. The ADOP value only depends on the satellite selection, which can be minimized by selecting proper satellites according to the elevation and azimuth angles.

3.2. Elevation Angle and SNR of Satellite

When the satellite combination is selected by the ADOP, we need to choose a reference satellite to form the double difference. The satellite with highest elevation angle is chosen in traditional method, because there will be low multipath effect in the received signal as shown in Fig. (**1**). In general, SNR increased with the increase of the satellite elevation angle and decreased with the increase of the noise of carrier phase measurement. However, in the practical application, atmosphere and orbit errors also impact the satellite signal and lower the SNR. Therefore, we should consider both of the elevation angle and the SNR of the satellite simultaneously during selection.

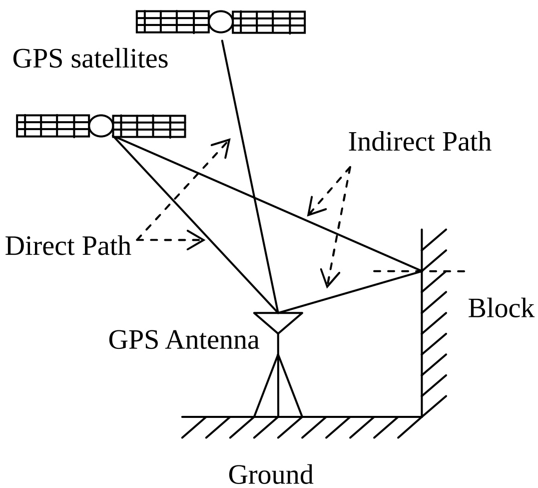

Fig. (1). Multipath effect.

Define the satellite selection function:

$$f_i = \frac{E_i}{E_{\max}} + \alpha \cdot \frac{SNR_i}{SNR_{\max}} \tag{10}$$

where SNR_i is the SNR of the *i*th satellite, $E_{\max}$ and $SNR_{\max}$ are the maximum elevation angle and SNR in this epoch, and α is the weight parameter. α is set to 1 in normal situation and greater in strong multipath environment. In Eq. (10), $E_i \propto Q_s$ where Q_s is the quality of satellites signal, if there is no block of signal, while $SNR_i \propto Q_s$ with or without block of signal. In the strong multipath environment, SNR gives better description of Q_s than elevation. Therefore, the weight parameter should be greater. Thus, f_i in Eq. (10) can be used to choose the main satellite according to the quality of satellites signal.

In practical application, we choose the satellite which maximizes Eq. (10) as the reference satellite to establish the double difference. The elevation angle and SNR of the satellite are considered simultaneously in the satellite selection function Eq. (10). Such selection can efficiently reduce the noise in the double difference carrier phase measurements. The corresponding linear transformation matrix $\tilde{C}_d$ can be rewritten as

$$\tilde{C}_d = \left[-I_{(k-1)\times(k-1)} \quad \mathbf{1}_{m\times 1} \quad -I_{(m-k+1)\times(m-k+1)} \right] \tag{11}$$

where k is the sequence of the reference satellite in the original C_d. For instance, if the satellite with the 2nd highest elevation angle is chosen as the reference satellite, k=2, and the linear transformation matrix is

$$\tilde{C}_d = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 1 & 0 & \cdots & -1 \end{bmatrix}$$

3.3. Covariance Matrix in the Observation Equation

After selection of satellite combinations and the reference satellite for double difference, the covariance matrix of the observation equation should be set properly according to the practical environment. In the common observation model, it is usually assumed that all the carrier phase measurements have the same variance and statistically independent. Therefore, the covariance matrix of the observation can be formulated as

$$Cov = \sigma^2 I \tag{12}$$

where σ^2 is the variance of the carrier phase. Through the error propagation law the time invariant covariance matrix of the double difference measurements can be obtained.

$$Q = \text{cov(DD)} = C_d \cdot \text{cov(SD)} \cdot C_d^T = 2\sigma^2 C_d C_d^T = \sigma^2 \begin{bmatrix} 4 & 2 & \cdots & 2 \\ 2 & 4 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \cdots & 4 \end{bmatrix} \tag{13}$$

As stated in section 3.2, the reference satellite in double difference is not the satellite with the highest elevation angle in some cases. The linear transformation matrices with different sequences have the following result

$$\tilde{C}_d \tilde{C}_d^T = C_d C_d^T \tag{14}$$

Therefore, based on the error propagation law the covariance matrices from Eq. (11) and (13) is the same. However, in the practical application, this simplified model may contain some misspecifications, and thus could result in unreliable GPS attitude results. GPS measurement errors are dominated by multipath, atmosphere, orbit effects and receiver noise, which are related to the elevation and azimuth angles of the satellite except for the last one. Therefore, the effects of these error sources are different for each satellite. In order to model the variances of carrier phase measurements from different satellites, a function of satellite elevation angle is used to describe the variance of the GPS measurements in practice [9].

$$\sigma_i^2 = \sigma^2 + \sigma_E^2 \cdot f^2\left(E_i\right) \tag{15}$$

Table 1. Angles of Observed Satellites

PRN	Elevation/Degree			Azimuth/Degree		
	Epoch			Epoch		
	1	2	3	1	2	3
14	71	71	71	78	81	83
30	57	56	56	69	68	67
31	56	57	57	279	281	282
32	36	37	37	306	305	305
5	35	34	34	51	50	50
22	32	31	30	181	180	180

where σ and σ_E are constant, and $f\left(E_i\right)$ is the function of satellite elevation angle. Because of noise in attitude determination, the relationship between the accuracy of carrier phase measurements and satellite elevation angles can only be approximately expressed. In practice, the trigonometric function of the elevation angle is usually used [10]

$$\tilde{\sigma}_i^2 = \sigma^2 + \sigma_E^2 / \sin^2(E_i) \qquad (16)$$

Substituting the variance form Eq. (16) into Eq. (13), we obtain the covariance matrix

$$\tilde{Q} = \tilde{\sigma}^2 \begin{bmatrix} 4 & 2 & \cdots & 2 \\ 2 & 4 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \cdots & 4 \end{bmatrix} \qquad (17)$$

where $\tilde{\sigma}$ is obtained form Eq. (16). The obtained covariance matrix $\tilde{Q}$, which includes the influence of the satellite elevation angles, can be substituted into the least square equations while solving the observation equation, and thus the satellite signals are properly weighted during solution to lower the noise error in the results.

4. EXPERIMENTAL RESULTS

Attitude determination using two GPS antennas forming a single baseline is a basic building block of the attitude determination system. Therefore, the proposed satellite selection method in static attitude determination is evaluated by two NovAtel 12 channels single frequency C/A code GPS receivers with 1Hz output and a PC/104 module with a 133 MHz CPU, which form a 2.029 m long single baseline.

Table **1** lists the different satellite elevation and azimuth angles of three epochs during the experiment. These angles did not change significantly in a short period. Different double difference combinations from the observed satellites are listed in Table **2**. In these combinations, the first and second ones include the satellite with highest elevation angle, while the third and fourth not. They include the satellites with the 2nd and 3rd highest elevation angles. The ADOP values of these four combinations in different epochs are shown in Fig. (**2**). From this figure, we can conclude that the combination with the highest elevation angle may not minimize the ADOP value while the combination with lower elevation angles may obtain smaller ADOP value. Therefore, when selecting the satellite combination for double difference, we may not always choose the one with the highest elevation angle.

Figure (**3**) demonstrates the elevation angles and SNR of the observed satellites. As depicted in this figure, the SNR of each satellite is generally proportional to its elevation angle. However, there exist some exceptions. For instance, although PRN 14 has the highest elevation, its SNR is almost the same as PRN 31 which is lower than PRN 30 with the 2nd elevation angle. Therefore, based on Eq. (10) we should choose PRN 30 as the reference satellite for double difference.

Figure (**4**) presents the comparison of the attitude determination results using the proposed satellite selection method and the traditional one. From this figure and Table **3**, we know that the accuracy of the result using the proposed method is increased compared to the one using the traditional method.

Table 2. Different Satellite Combinations for Double Difference

Combination	PRN
1	14, 30, 31, 32
2	14, 30, 5, 22
3	30, 31, 32, 5
4	31, 32, 5, 22

5. CONCLUSIONS

In attitude determination, carrier phase measurements include many errors which are related to the satellites used for double difference. Therefore, satellite signals with different elevation and azimuth angles and SNRs significantly influence the carrier phase measurements. In this contribution, a satellite selection method based on the angle information and SNR of satellites is proposed. This method considers the influence of elevation and azimuth angles on the attitude determination, and chooses the satellite

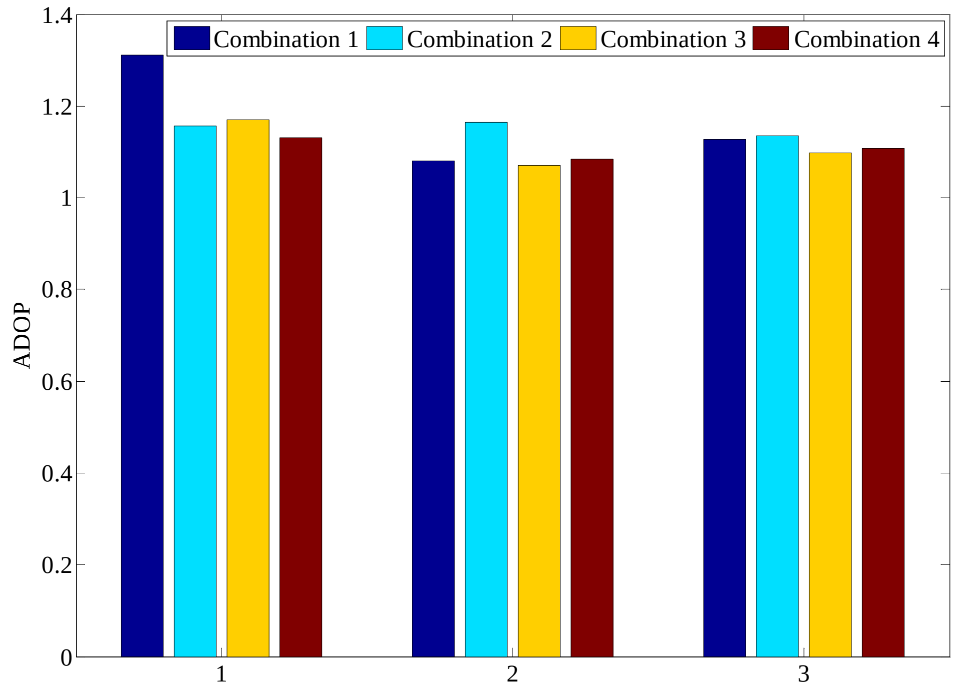

Fig. (2). ADOP of different combinations.

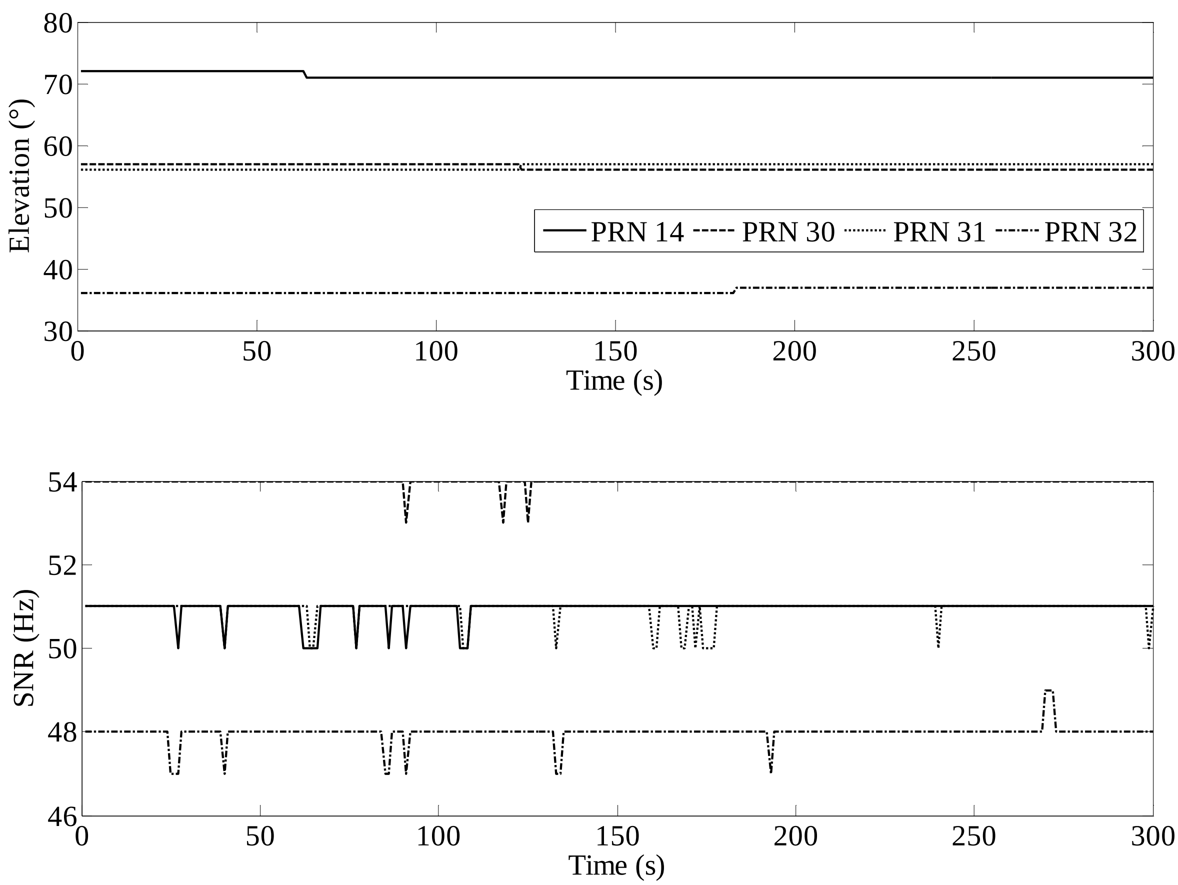

Fig. (3). Elevation angles and SNRs of satellites.

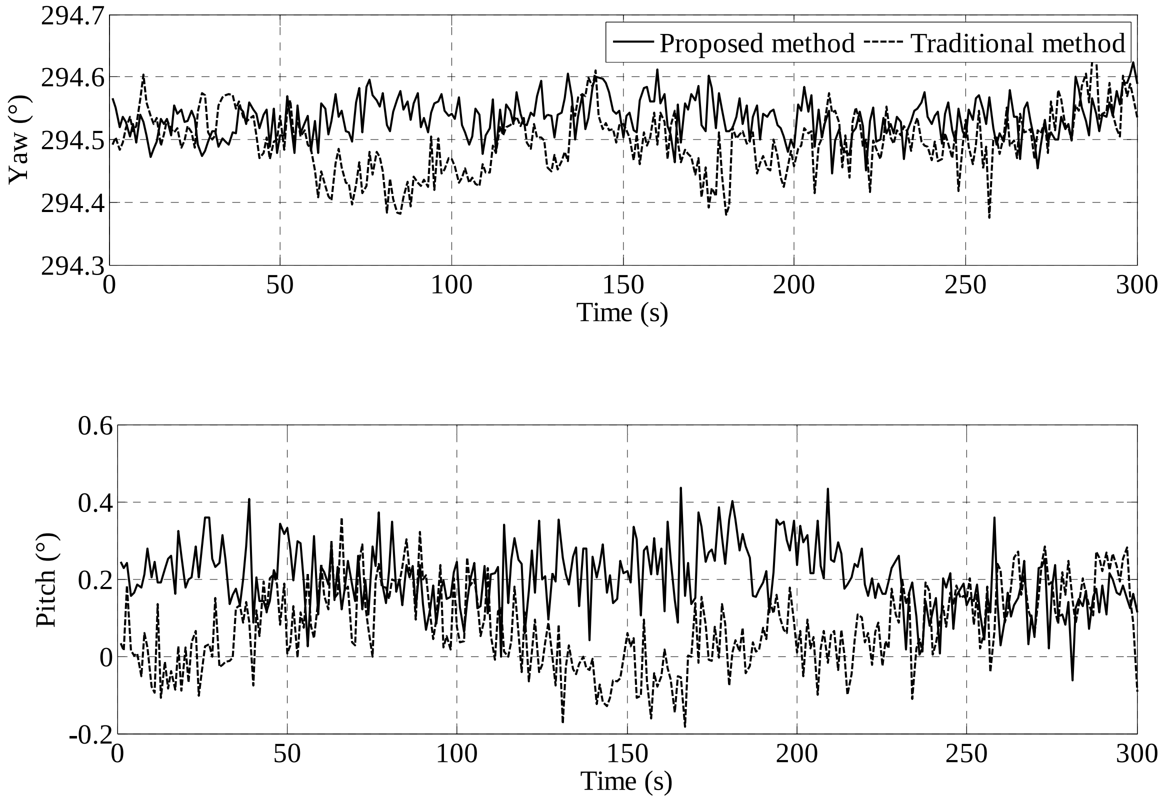

Fig. (4). Attitude determination results.

combination based on the geometry relationship. The reference satellite is not selected as the one with the highest elevation angle but the one that takes both elevation angle and SNR into account simultaneously. In the double difference carrier phase observation equation, the elevation angles are used to set the weight of measurements. The quality of satellite signal is considered to improve the accuracy of attitude determination.

Table 3. Comparison of the Two Methods

Method	Standard Deviation/Degree	
	Yaw	Pitch
Proposed method	0.0319	0.0842
Traditional method	0.0486	0.1059

Experimental results show that the proposed method is more suitable for the attitude determination using GPS. It lowers the noise influence of the double difference carrier phase measurements and increases the accuracy of attitude determination result.

REFERENCE

[1] Alban, S., S.M. Rock, J.D. Powell: US20087454290 (**2008**).

[2] Cohen CE. Global positioning system: theory and applications. AIAA 1996; 519-38.

[3] Li Y, Zhang K, Roberts C, Murata M. On-the-fly GPS-based attitude determination using single-and double-differenced carrier phase measurements. GPS Solutions 2004; 8: 93-102.

[4] Wang B, Miao L, Wang S, Shen J. A fast integer ambiguity repair method for the land vehicle attitude determination using GPS. in: *Proc. IEEE Conference on Intelligent Transportation System* Beijing: China, October 2008; 717-22.

[5] Tiberius C, Kenselaar F. Variance component estimation and precise GPS positioning: case study. J Surv Eng 2003: 129: 11-8.

[6] Sutton, E.: US20016256583 (**2001**).

[7] Williams, D.S.: US20087468695 (**2008**).

[8] Wang B, Miao L, Wang S, Shen J. A constrained LAMBDA method for GPS attitude determination. GPS Solutions 2009; 13: 97-107.

[9] Jin S, Wang J, Park PH. An improvement of GPS height estimations- Stochastic modeling. Earth Planets Space 2005; 57: 253-9.

[10] King RW, Bock Y. Documentation for the GAMIT GPS analysis software. Cambridge: Mass Inst of Technol 1999.

Received: June 30, 2009 Revised: July 27, 2009 Accepted: August 7, 2009