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Gravity Effects in Inclined Air Showers Induced by Cosmic Neutrinos

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Abstract: The Randall-Sundrum model with a small curvature is considered in which five-dimensional Planck scale lies in the TeV region. The cross sections for interactions of ultra-high energy cosmic neutrinos with nucleons are calculated. It is shown that effects related with Kaluza-Klein graviton excitations can be detected in deeply penetrating inclined air showers induced by these neutrinos. The expected number of the inclined air showers at the Auger Observatory is estimated as a function of two parameters of the model.

1. WARPED EXTRA DIMENSION WITH THE SMALL CURVATURE

One of the most important problems of the modern particle physics is the hierarchy problem, i.e. unnaturally large ratio of the gravity scale (10^{19} GeV) to the electroweak scale (10^2 GeV). To solve this problem, a number of theories with large spacial extra dimensions have been proposed [1]. However, they could only explain the huge value of the Planck mass by introducing another large scale, namely, the size of extra flat dimensions. Thus, the hierarchy problem was not really solved, but reformulated in terms of this new scale.

The model which does solve the problem most economically is the Randall-Sundrum (RS) model [2] with a single extra dimension and warped background metric [3]:

$$ds^2 = e^{2\kappa(\pi r c-|y|)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dy^2. \qquad (1)$$

Here $y = r_c\,\theta$ ($-\pi \le \theta \le \pi$), r_c is the "radius" of the extra dimension, while $\{x^\mu\}$, $\mu = 0, 1, 2, 3$, are the coordinates in four-dimensional space-time. The parameter κ defines the scalar curvature in five dimensions. Note that the points (x^μ, y) and (x^μ, $-y$) are identified, and the periodicity condition, $(x^\mu,\ y)=(x_\mu, y + 2\pi r_c)$, is imposed. The tensor $\eta_{\mu\nu}$ is the Minkowski metric.

It is assumed that there are two 3-dimensional branes with equal and opposite tensions located at the points $y = 0$ (called the Plank brane) and $y = \pi r_c$ (referred to as the TeV brane). All SM fields are confined to the TeV brane, while the gravity propagates in five dimensions. The following relation between the 4-dimensional (reduced) Planck mass, $\bar{M}_{\rm P1}$, and (reduced) gravity scale in five dimensions, $\bar{M}_5$, can be derived:

$$\bar{M}_{\rm P1}^2 = \frac{\bar{M}_5^3}{k}\left(e^{2\pi k r_c} - 1\right). \qquad (2)$$

The masses of the Kaluza-Klein (KK) graviton excitations are proportional to the curvature parameter κ:

$$m_n = x_n\kappa,\ \text{n} = 1, 2 \ldots, \qquad (3)$$

where x_n are zeros of the Bessel function $J_1(x)$. On the TeV brane, the zero graviton mode, $h_{\mu\nu}^{(0)}$, and massive graviton modes, $h_{\mu\nu}^{(n)}$, are coupled to the energy-momentum tensor of the matter, $T^{\mu\nu}$, as follows:

$$\mathcal{L}_{\rm int} = -\frac{1}{\bar{M}_{Pl}}T^{\mu\nu}h_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi}T^{\mu\nu}\sum_{n=1}^{\infty}h_{\mu\nu}^{(n)} \qquad (4)$$

with

$$\Lambda_\pi = \left(\frac{\bar{M}_5^3}{k}\right)^{1/2} \qquad (5)$$

being the physical scale on this brane. Let us note that the metric (1) differs from that presented in the original paper [2] in which both $\bar{M}_5$ and κ have to be taken as large as the Planck mass ($\bar{M}_5 \sim \kappa \sim \bar{M}_{\rm P1}$). Moreover, the size of the warped extra dimension should be extremely small ($r_c \simeq 60 l_{\rm Pl}$) [3]. Thus, in order to explain the huge value of $\bar{M}_{\rm P1}$ in such a scheme, one has to introduce new mass scales of the same order, namely, $\bar{M}_5$, κ, and r_c^{-1}.

However, the hierarchy problem can be successfully solved in the RS scenario, but with the metric (1). The equation (2) allows us to consider the *small curvature option* of the RS model [3-5]:[1]

$$\kappa \sim 1\ \text{GeV},\ \bar{M}_5 \sim 1\ \text{TeV}. \qquad (6)$$

In such a case, we get an almost continuous spectrum of low-mass graviton excitations with the lightest mass equal to 3.83 κ, and small mass splitting $\Delta m \simeq \pi\kappa$. Note that in the standard scenario of the RS model [2] one has a series of KK graviton resonances with the lightest one having the mass around 1 TeV.

The RS model with the large extra dimension has been checked by the DELPHI Collaboration [6]. The gravity

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[1]For numerical estimates, the region 0.5 GeV < κ < 1.5 GeV will be used (see Fig. **3**).

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effects were searched for by studying photon energy spectrum in the process $e^+e^- \to \gamma + \not{E}_\perp$. The limit on M_5 obtained [6] is:

$$\bar{M}_5 > 0.92 \text{ TeV}. \tag{7}$$

The search for the large extra dimensions in the diphoton channel using data collected by the CDF and DØ Collaborations at $\sqrt{s}$ = 1.96 TeV is presented in Refs. [7]. The measured $p_\perp$-distributions are in a good agreement with the SM background, that allowed us to obtain the bound [8]:

$$\bar{M}_5 > 0.81 \text{ TeV}. \tag{8}$$

The discovery limit of the LHC in the two-photon production (requiring a 5σ effect) has also been derived for two values of the integrated luminosity $\mathcal{L}$ [8]:

$$\bar{M}_5 = \begin{cases} 6.3\,\text{TeV}, & \mathcal{L} = 100\ \text{fb}^{-1} \\ 5.1\,\text{TeV}, & \mathcal{L} = 30\ \text{fb}^{-1} \end{cases} \tag{9}$$

Previously, the gravity effects in the RS model with the small curvature were already looked for in a number of different processes (see Refs. [8-10]). In the present paper we will estimate the upper bound on $\bar{M}_5$ which can be reached by the Auger ground array in detecting quasi-horizontal air showers induced by ultra-high energy (UHE) cosmic neutrinos.

2. GRAVITY EFFECTS IN INTERACTIONS OF COSMIC NEUTRINOS WITH ATMOSPHERIC NUCLEONS

A promising possibility to detect effects induced by low-mass KK gravitons (3) is to look for their contributions to the scattering of the SM fields in the trans-Planckian kinematical region:

$$\sqrt{s} \gtrsim \bar{M}_5 >> -t, \tag{10}$$

with $\sqrt{s}$ being the colliding energy and $t = q_\perp^2$ four-dimensional momentum transfer. It is also assumed that inequality $\kappa \ll \bar{M}_5$ (6) is satisfied. As we will see below, in the trans-Planckian region the gravity contribution to the scattering of UHE cosmic neutrinos off the atmospheric nucleons can dominate the SM contribution.

In the eikonal approximation which is valid in the kinematical region (10), elastic scattering amplitude is given by the sum of gravi-Reggeons, i.e. reggeized gravitons in the *t*-channel. Because of the presence of extra dimension, the Regge trajectory of the graviton is splitting into an infinite sequence of trajectories enumerated by the KK number *n* [11]:

$$\alpha_n(t) = 2 + \alpha_g' t - \alpha_g' m_n^2, \quad n = 0,1,... \tag{11}$$

In string theories, the slope of the gravi-Reggeons is universal, and $\alpha_g' = M_s^{-2}$, where M_s is the string scale. For more details, see Refs. [11].

Correspondingly [3], the gravity Born amplitude for the neutrino scattering off *a point-like* particle looks like[2]:

$$A^{\text{B}}_{\text{grav}}(s,t) = \frac{\pi\alpha_g' s^2}{2\Lambda_\pi^2} \sum_{n\neq 0} \left[i - \cot\frac{\pi\alpha_n(t)}{2} \right] \left(\frac{s}{\bar{M}_5} \right)^{\alpha_n(t)-2}. \tag{12}$$

The differential neutrino-proton cross section is of the form:

$$\frac{d\sigma}{dy} = \frac{1}{16\pi s} \left| A_{\nu\text{p}}(s,t) \right|^2. \tag{13}$$

The inelasticity $y = -t/s$ defines the fraction of the neutrino energy transferred to the nucleon. $A_{\nu\text{p}}$ is the neutrino-proton amplitude which is related to the eikonal:

$$A_{\nu p}(s,t) = 4\pi i\ s \int_0^\infty db b\, J_0(bq_\perp)\, \{1 - \exp[i\chi(s,b)]\}. \tag{14}$$

In its turn, the eikonal is given by the equation:

$$\chi(s,b) = \frac{1}{4\pi s} \int_0^\infty dq_\perp q_\perp J_0(q_\perp b) A^{\text{B}}_{\nu p}(s,t). \tag{15}$$

The calculations show that the imaginary part of the eikonal is negligible with respect to its real part, since $\text{Im}\chi/\text{Re}\chi = \text{O}(\kappa/\bar{M}_5)$. That is why we can omit the contribution from inelastic interactions.

The *hadronic* Born amplitude in (15) is defined by the gravity amplitude (12) and skewed (*t*-dependent) parton distributions $F_i(x, t)$:[3]

$$A^{\text{B}}_{\nu p}(s,t) = \sum_{i=q,\bar{q},g} \int_0^1 dx A^{\text{B}}_{\text{grav}}(xs,t) F_i(x,t). \tag{16}$$

The *t*-dependent distributions have the Regge-like form [3]:

$$F_i(x,t) = f_i(x) \exp[t(r_0^2 - \alpha_P' \ln x)], \tag{17}$$

where α_0' is the Pomeron slope, while $f_i(x)$ is the distribution of the parton of the type *i* inside the proton. The values of the parameters are [12]:

$$r_0^2 = 0.62\ \text{GeV}^{-2},\ \alpha_P' = 0.094\ \text{GeV}^{-2}. \tag{18}$$

We will use the set of parton distribution functions $f_i(x)$ from Ref. [13].

In Fig. (**1**) and Fig. (**2**) we present total neutrino-nucleon cross sections calculated by using Eqs. (12)-(16) for two values of the curvature κ and different values of the reduced fundamental gravity scale $\bar{M}_5$. For comparison, the SM prediction for the neutrino total cross section is presented in both figures which was calculated by using the following expression from Ref. [14][4].

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[2]Remember that the KK gravitons interact universally with the SM fields (4).

[3]Since $A^{\text{B}} \sim s^2$, the integral converges rapidly at $x = 0$.

[4]This expression is valid for 10^7 GeV $\le E_\nu \le 10^{12}$ GeV within 10% [14].

$$\sigma_{\rm tot}(\nu N) = 7.84 \cdot 10^{-36}\,{\rm cm}^2 \left(\frac{E_\nu}{1{\rm GeV}}\right)^{0.363} \tag{19}$$

It varies from $2.72 \cdot 10^{-33}$ cm^2 at $E_\nu = 10^7$ GeV up to $1.78 \cdot 10^{-31}$ cm^2 at $E_\nu = 10^{12}$ GeV.

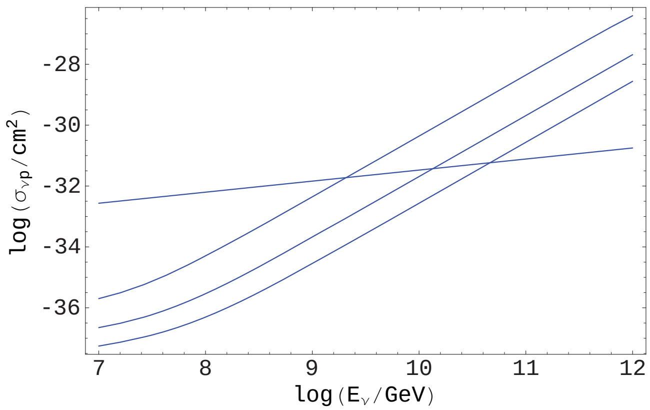

Fig. (1). The total neutrino-proton cross section as a function of the neutrino energy. The curves correspond (from above) to $\bar{M}_5 = 3$, TeV, 5 TeV, and 7 TeV. The parameter κ is equal to 100 MeV. The straight line: SM total cross section.

Previously, low-scale gravity effects in cosmic neutrino interactions were calculated in models with compactified extra dimensions (see [15, 16] and references therein). Recently, the gravity effects on the neutrino-nucleon cross sections in the eikonal approximation were estimated for the case of infinitely thin branes embedded in five extra dimensions [17]. The black hole production cross sections in cosmic neutrino interactions were also calculated (see, for instance, Refs. [18]).

By comparing Figs. (**1**, **2**) with figures from Refs. [17], one can see that the neutrino cross sections in the small curvature scenario of the RS model and those in the ADD model have different energy dependence. The formers are significantly smaller at $E_\nu \lesssim 10^9$ GeV, but exceed the ADD cross sections at $E_\nu \gtrsim 10^{10}$ GeV (at comparable values of gravity scale $\bar{M}_5$ in both models).

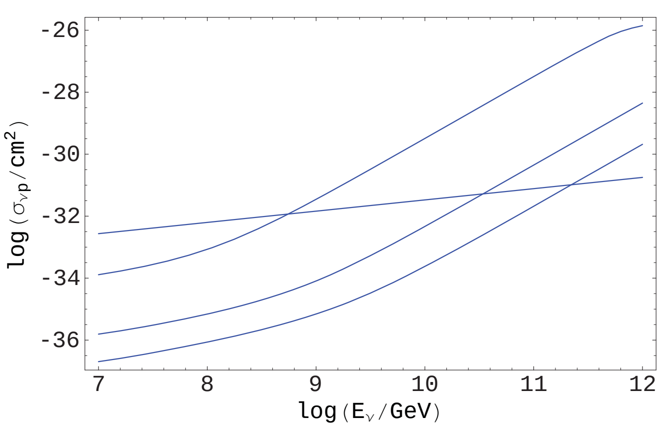

Fig. (2). The total neutrino-proton cross section as a function of the neutrino energy. The curves correspond (from above) to $\bar{M}_5 = 1$, TeV, 3 TeV, and 5 TeV. The parameter κ is equal to 1 GeV. The straight line: SM total cross section.

The number of neutrino induced air showers is given by

$$\frac{dN_{\rm ev}}{dt} = \int_{E_{\rm th}}^{E_{\rm max}} dE_\nu \int_0^1 dy\,\theta(E_{\rm sh} - E_{\rm th}) \frac{d\sigma(E_\nu)}{dy} \Phi(E_\nu) A_{\rm eff}(E_{\rm sh}, E_\nu), \tag{20}$$

where E_ν is the energy of the cosmic neutrino, $\Phi(E_\nu)$ denotes its flux, and

$$E_{\rm sh} = yE_\nu \tag{21}$$

is the air shower energy. The effective aperture for the UHE neutrinos is defined by the neutrino flux attenuation att(E_ν) and detector efficiency P ($E_{\rm sh}$):

$$A_{\rm eff}\,(E_{\rm sh}, E_\nu) = {\rm att}(E_\nu)\, P\,(E_{\rm sh})\, A_{\rm p}(E_{\rm sh}). \tag{22}$$

The attenuation att(E_ν) depends (besides neutrino-nucleon total cross section) on $X_{\rm obs}$, the depth within which air shower is visible for the ground array detector, and $X_{\rm uno}$, the minimum atmospheric depth a neutrino must reach in order to induce an observable shower to these detectors.

In order to isolate neutrino-induced events at the Auger Observatory, deeply penetrating quasi-horizontal air showers should be looked for [19, 20]. We impose the following bounds on the zenith angle of the incoming neutrino: $75^\circ \leq \theta_{\rm zenith} \leq 90^\circ$. The functions P ($E_{\rm sh}$) and $A_{\rm p}(E_{\rm sh})$, as well as values of the parameters $X_{\rm obs}$ and $X_{\rm uno}$, are taken from Ref. [21]. In particular, the deeply penetrating events must satisfy the condition $X_{\rm uno} \geq 1700$ g/cm^2 . Since, on average, ultra-high energy air shower develops to its maximum after traversing 800 g/cm^2, [5] it is set $X_{\rm obs} = 1300$ g/cm^2 (see [20, 21] for more details).

The threshold energy in (20) is taken to be $E_{\rm th} = 5 \cdot 10^7$ GeV, and the maximum energy $E_{\rm max} = 10^{12}$ GeV. The result of our calculations for the Waxman-Bahcall neutrino flux [22] is presented in Fig. (**3**). It shows the rate of the inclined air showers at the Auger detector as a function of two parameters of the model.

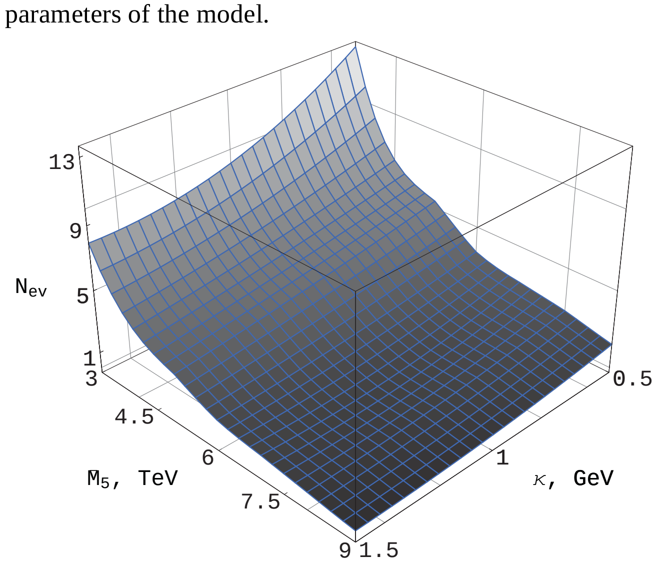

Fig. (3). The expected rate of the neutrino induced inclined air showers ($75^\circ \leq \theta_{\rm zenith} \leq 90^\circ$) at the Auger Observatory for the Waxman-Bahcall flux (in yr^{-1}).

[5] It corresponds to Xmax ≥ 2500 g/cm^2, where Xmax is the shower maximum.

The number of the inclined air showers at $\bar{M}_5$ = 7 TeV, $\bar{M}_5$ = 8 TeV, and $\bar{M}_5$ = 9 TeV are presented in Table **1** for three different values of the parameter κ. Our SM prediction is 0.13 events per year. It can be compared with the estimate from Ref. [15], 0.22 SM events per year for $\theta_{\text{zenith}} \geq 70^{\circ}$.[6]

Table 1. The Number of the Inclined Neutrino Induced Air Showers (in yr^{-1}) at Several Values of the Parameters $\bar{M}_5$ and κ

	$\bar{M}_5$ = 7 TeV	$\bar{M}_5$ = 8 TeV	$\bar{M}_5$ = 9 TeV
κ = 0.5 GeV	2.69	2.53	1.54
κ = 1.0 GeV	1.32	1.17	0.68
κ = 1.5 GeV	1.18	0.77	0.37

Taking into account that systematic errors of SM cross section can be factor 3, we conclude that the search limit of the Auger Observatory for the 5-dimensional Planck scale $\bar{M}_5$ varies from 7 TeV to 9 TeV when κ varies from 1.5 GeV to 0.5 GeV. These values are closed to the discovery limit of the LHC which was derived recently in the framework of the same scenario (see Eq. (9)).

Since our scheme has only one extra dimension, the sum over KK excitations is UV-finite, contrary to the ADD scheme with two or more extra dimensions (see, for instance, Sec. II.C of the first reference in [18]). In any case, a possible UV cutoff is irrelevant, since it should be equal to (or larger than) the scale $\bar{M}_5$, while the masses of the first KK states are much smaller than $\bar{M}_5$.

Thus, our predictions (Figs. **1**-**3**) depend only on the 5-dimensional gravity scale $\bar{M}_5$ and curvature parameter κ.

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[6]Remember that we imposed stronger condition θzenith $\geq 75^{\circ}$ during our calculations.