Escape of the Atmosphere of Rotating Planet

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Abstract: The problems of escape of a planetary atmosphere is reviewed. Formulae are derive for the rates of loss of mass and angular momentum from a unit area and from the whole planetary surface. The resulting formulae are applied to find the residence times of H, O, N_2, O_2 and CO_2 in Martian atmosphere. All constituents are so stable while hydrogen is never stable and cannot be retained to my extent.

Keywords: Escape of a planetary atmosphere, loss of mass-residence times.

INTRODUCTION

The classical theory of escape has been developed by [1] who found that an atmosphere will be stable for astronomical periods (greater than 10^9 years) if the root mean square velocity of the particle is less than the escape velocity V_e. Valuable discussions were given by [2-4]. [5] treated the problem taking into account the effect of orbital motion. The effect of rotation of the planet was introduced by [6] who derived formulae for the rates of loss of mass and angular momentum per unit area of the surface. In this paper formulae are derived to find expressions for the rates of loss of mass and angular momentum per unit area f the surface (including higher order terms than obtained by Burke to applicable for higher rotational velocities), and for the total rates of loss of mass and angular momentum from the whole planetary surface.

The two necessary conditions for escape are:

1) the spatial velocity V_s should be larger than the escape velocity V_e.

2) Negligible chance of undergoing collisions. This condition is satisfied at the critical level (base of the exosphere).

If r_o is the radius of the planet, the critical velocity required for the outgoing particles at a height z above the planet's surface to escape is

$$V_e = r_o(2g_o/r)^{1/2} \tag{1}$$

Where g_o is the gravity at the level r_o and $r = z + r_o$

The rotation of the planet has two effects on the flux of the outgoing particles, (i) the reduction of gravity by the induced centrifugal force, resulting in an effective velocity of escape given by $V_e'^2 = V_e^2 - V_r^2$; and (ii) particles in the same direction as the rotational motion will have more chance to escape than those in the opposite direction. Thus in addition to the most energetic particles those with the greatest forward angular momentum will have also increased

chance to attain escape speeds, resulting in an escaping flux primarily in the forward direction.

Rates of Loss of Mass and Angular Momentum

Consider a system of rectangular coordinates (x_1, x_2, x_3). Let the plane x_1x_2 be tangent to the planetary surface with x_2 in the direction of the velocity of rotation V_r, and x_3 normal to the planetary surface. The rate of loss of mass per unit area will be

$$\frac{dm}{dt} = \iiint \mu N(V) V_3 dV \tag{2}$$

where μ is the mass per particle, $N(V)dV$ is the number of particles at the critical level (r_c) with velocities between V and $V + dV$.

The rate of loss of angular momentum is

$$\frac{dl}{dt} = \iiint \mu N(V)(V_r + V_2)RV_3 dV \tag{3}$$

R is the normal distance between the direction of V_2 and the axis of rotation (i.e. the radius of the parallel of latitude at the point over which escape is considered but measured to the vertical height z_c). $N(V)dV$ will be taken as maxwellian distribution function

$$N(V)dV = \frac{n_c}{II^{3/2}U^3} \exp\left(-\frac{V^2}{U^2}\right) dV_1 dV_2 dV_3 \tag{4}$$

$$U = \left(\frac{2kT_c}{\mu}\right)^{\frac{1}{2}} = 0.12895\left(\frac{T_c}{\mu'}\right)^{\frac{1}{2}} \text{ km.s}^{-1}$$

where n_c and T_c are the total number of particles and temperature respectively at the critical level, and μ' is molecular weight expressed in grams.

Transforming to a polar coordinate system at rest with respect to the center of the planet and with origin momentarily coincident with the former, we get.

$$V_1 = V_s \sin\theta\cos\varphi,$$

$$V_2 = V_s \sin\theta\sin\varphi - V_r, \tag{5}$$

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$V_3 = V_s \cos\theta.$

and the distribution function becomes

$$N(V)dV = \frac{n_c}{\Pi^{3/2}U^3} \exp[(-V_s^2 + 2V_r V_s \sin\theta \sin\varphi - V_r^2)/U^2] \qquad (6)$$

$$\cdot V_s^2 \sin\theta \, dV_s \, d\theta \, d\varphi$$

Substituting the distribution function given by eqn. (6) in eqns. (2) and (3) we get

$$\frac{dm}{dt} = \frac{\mu n_c}{\Pi^{3/2}} \, U \, I_m \qquad (7)$$

and

$$\frac{dl}{dt} = \frac{\mu n_c}{\Pi^{3/2}} \, R \, U \, I_1 \qquad (8)$$

where

$$I_m = \exp(-u_r^2)\exp(-u_s^2 + 2u_r u_s \sin\theta \sin\varphi)$$

$$u_s^3 \cos\theta \sin\theta \int_{u_e}^{\infty} du_s \int_0^{2\pi} d\varphi \int_0^{2\pi} d\theta \qquad (9)$$

$$I_1 = \exp(-u_r^2)\exp(-u_s^2 + 2u_r u_s \sin\theta \sin\varphi)$$

$$u_s^4 \cos\theta \sin\theta \int_{u_e}^{\infty} du_s \int_0^{2\pi} d\varphi \int_0^{2\pi} d\theta \qquad (10)$$

and

$$u_s = V_s/U, \ u_e = V_e/U, \ u_r = V_r/U \qquad (11)$$

It is clear that the rotational velocity is contained only in u_r. Considering u_r as a measure of the effect of rotaion on the escape process, we can divide the planets into three groups:

(i)   Planets for which rotation can be neglected, these are Mercury and Venus

(ii)   Planets for which $u_r \ll 1$ and hence higher powers of u_r can be neglected these are the Earth and Mars.

(iii) Planets with $u_r \gg 1$, these are the major planets.

Considering the case when $u_r \to 0$, eqns. (9) and (10) can be integrated easily to yield

$$Im = \frac{\pi}{2} \exp(-u_e^2)(1+u_e^2) \qquad (12)$$

and

$$I_1 = 0$$

The Value of $I_1 = 0$ is clear since we have assumed in advance that the planet does not rotate. The first of eqns. (12), combined with eqn. (7) give the rate of loss of mass per unit area when the planetary rotation is negligible.

Considering the second case, i.e. $u_r \ll 1$. Set $y = \sin\varphi$, and $b = 2u_r u_s$. Since $u_s \approx 1$, then $b < 1$. Carrying out the θ interaction, eqns (9) and (10) yield

$$I_m = \exp(-u_r^2) \int_{u_e}^{\infty} du_s \int_0^{2\pi} d\varphi \exp(-u_s^2) u_s^3 \, f_m \qquad (13)$$

and

$$I_1 = \exp(-u_r^2) \int_{u_e}^{\infty} du_s \int_0^{2\pi} d\varphi u_s^2 \exp(-u_s^2) u_s^3 \, f_1 \qquad (14)$$

where

$$f_m = \sum_{n=0}^{\infty} \frac{1}{(n+2)n!} b^n y^n \qquad (15)$$

$$f_1 = \sum_{n=0}^{\infty} \frac{1}{(n+3)n!} b^n y^{n+1} \qquad (16)$$

Substituting f_m and f_1 in (13) and (14) respectively, retaining only terms up to the fifth power in b and u_r, writing φ sinfor y and performing the φ and u_s integration result in

$$I_m = \frac{\Pi}{2} \exp(-u_e^2)\left\{1 + u_e^2 + \frac{1}{2}u_r^2 u_e^4 - \frac{1}{4}u_r^4 u_e^4\left(1 - \frac{1}{3}u_e^2\right)\right\} \qquad (17)$$

and

$$I_1 = \frac{\Pi}{2} \exp(-u_e^2)u_r \left\{1 + u_e^2 + \frac{1}{2}u_e^4 + \frac{1}{6}u_r^2 u_e^6 - \frac{1}{2}u_r^4 u_e^6\left(1 - \frac{1}{4}u_e^2\right)\right\} \qquad (18)$$

It is clear that in the limit as $u_r \to 0$, eqns. (17) and (18) reduce to eqns. (12). Neglecting terms of order higher than the second in ur we obtain those of [6].

In the third case $u_r \gg 1$ we can not expand f_m and f_1 in powers of u_r and hence eqns. (13) and (14) have to be integrated numerically.

Total Rates of Loss from the Whole Surface

Assuming that u_r is computed at the equator $r = r_o$, then at any other latitude D its value will be $u_r \cos D$. The rates of loss from circular element of the surface parallel to the equator and of width $r_o \, dD$ will be given by f.cos $2\Pi r_o^2 \, D \, dD$. f, where f represents dm/dt or dl/dt. Further

$$R = (r_o + z)\cos D \qquad (19)$$

where z is the height of the following particle. It is to be noted that for a particle of height z, u_r may be larger than that of the same thermal velocity. But since the particle may not be strictly constrained to rotate with the same angular velocity as the planet, it seems sufficient to consider the value of u_r at the surface.

The total rates of mass and angular momentum from the whole planet will be

$$\frac{dM}{dt} = \frac{4n}{\Pi^{1/2}} u \, r_o^2 \, I_M$$

$$\frac{dL}{dt} = \frac{4n}{\Pi^{3/2}} u^2 \, r_o^2 (r_0 + z) I_L \qquad (20)$$

where

$$I_M \int_0^{\pi/2} I_m(u_r \ \cos D)\cos D \ dD$$

$$I_L \int_0^{\pi/2} I_l(u_r \ \cos D)\cos^2 D \ dD \qquad (21)$$

In case of large rotational velocities $u_r \gg 1$, $(u_r \cos D)$ and $I_l(u_r, \cos D)$ are first obtained by numerical integration of formulae (13) and (14) for different values of D (i.e. for

values of $u_r \cos D$ from 0, at the pole, up to u_r, at the equator) and then performing numerically the integrations in equations (21).

When $u_r < 1$, substituting from eqns. (17) and (18) in eqns. (21) and integrating we get

$$I_M = \frac{II}{2}\exp(-u_e^2)\left\{1 + u_e^2 + \frac{1}{3}u_r^2 u_e^4 - \frac{2}{15}u_r^4 u_e^4\left(1 - \frac{1}{3}u_e^2\right)\right\} \quad (22)$$

$$I_L = \frac{II}{3}\exp(-u_e^2)u_r\left\{1 + u_e^2 + \frac{1}{2}u_e^4 + \frac{2}{15}u_r^2 u_e^6 - \frac{2}{35}u_r^4 u_e^6\left(1 - \frac{1}{4}u_e^2\right)\right\} \quad (23)$$

Again the classical formulae is obtained by neglecting terms including u_r:

$$I_M = \frac{II}{2}\exp(-u_e^2)(1 + u_e^2)$$

Eqns. (20), (22), and (23) give the total rates of loss of mass and angular momentum from the whole planet assuming that colisions beyond the critical level, at r_c, are negligible, otherwise they will merely represent the flux of outgoing particles. The escaping particles are provided by diffusion from below, if the rate of escape is much higher than that of diffusion, then diffusion will control the rate of escape. Further, the net rate of loss will be smaller than those given by the above eqns. due to the input of particles from the interplanetary medium in addition to the existence of some collisions above the critical level which cause some particles to return back below critical level. Also near the critical level collisions become so rare that Maxwellian distribution will hold less exact.

2. HISTORY OF MARS ATMOSPHERE

It is widely belived that the climate on Mars was much different than the present one [7, 8]. The criteria led to this belief :

(1)   Valley networks suggest that liquid water was once existant [9, 10],

(2)   There are indications that erosion rates 3.5-4 Gyrs. ago was much higher than it is now.

For the erosion to be effective, liquid water must exist on surafce, while the present mean equatorial temperature (about 250° K) and the mean surface pressure (6-7 mb) cause ice to sublime without passing with the liquid state. Hence the surface pressure at some early stage must have been above 30 mb [11]. The importance of this point increases by the presence of evidences from the degradation of impact crateres on the oldest surfaces which suggests that erosion processes were more efficient up untill 3.5 by age [12].

Some authers go to saying that the Mars early climate even had some similarity to the earth`s climate at the epoch when life was first forming, and as the solar output was their loss by 30% than at present [13] this opened the posibility that life may have formed on early Mars [14, 15].

Conducted a 3-dimensional simulation of HDO in the Mars atmosphere which has important clues to understanding climate evolution [16]. The study revealed that the Mars atmosphere is, at present enriched in denterium

concentration providing an indirect evidence of a metter climate in the past.

If the above were true, that is the early Mars climate was substantially different from the presently existing one then this would be a large evidence that the atmospheric constituents undergo much time variations and points to the importance of analyzing the important loss and regeneration processes. These can be outlimed as: Impact erosion [17], Sputtering by Solar wind [18, 19], Weathering [11], incorporation in the polar caps and like formations [20], and photochemical processes. Clearly if we stress only in the loss processes, these means become minor component to the escape first outgoing particles, which is object of present work.

3. THE RESIDENCE TIME

The residence time t may be defined as the time interval during which a certain constituents decreases by factor e^{-1}, where e is the base of Napierian logrithms.

If N_o is the total number of particles at time t_o, and F the escaping flux then

$$t = \frac{N_o}{F} \quad (24)$$

4. THE CRITICAL LEVEL

The height z_c of the critical level can be determined from the condition that a fraction e^{-1} of the fast upward moving particles at z_c will experince no collisions during their ascent, that is, the probability of collisions equals e-1.

With this definition, the height of the critical level is that at which

$$n_c = (4II^a 2H_c)^{-1} \quad (25)$$

where a is the radius of a representative particle level is that at which

$$H_c = RT /\cdot\mu g$$

Since the mean free path is proportional to $(4II^a 2H_c)^{-1}$, we see that at the critical level the mean free path for the particle moving fast in a horizontal direction is equal to the scale height.

For Mars, a, can be taken as that of a molecule of CO_2, or about 1.6×10^{-8} cm. Using data reported by [21], and computing the right and left hand sides of eqn. (25) for heights up to 260 kms. the critical level is estimated to occur at about 208 kms. where the temperature is about 400° K and the number density is 1.5×10^8 cm^{-3}.

5. ESCAPE OF MARS ATMOSPHERIC CONSTITUENTS

In this section the escape of H, O, N_2, O_2, and CO_2 will be considered. The loss of mass and angular momentum and escape flux of particles per unit areas, and from the whole surface of the planet will be computed using formulae (7), (8), (17), (18), (20), and (22), and then the residence times of these constituents are computed using eqn. (24) and assuming, for simplicity, that the total number of particles is given by

Table 1. The Results (The Residence Time)

Gas	H	O	N_2	O_2	CO_2
n_c	10^7	10^7	10^7	10^6	10^8
n_o	2×10^8	---	8.3×10^{15}	1.8×10^{14}	1.9×10^{17}
N_o	1.4×10^{34}	---	2.4×10^{40}	4.2×10^{38}	3×10^{41}
dm\dt	3×10^3	16×10^{-17}	6.2×10^{-35}	8×10^{-42}	1.6×10^{-56}
dl\dt	10^{17}	2.4×10^{-3}	1.4×10^{-20}	2×10^{-27}	4.1×10^{-43}
dM\dt	3×10^{22}	58	2×10^{-16}	2.6×10^{-23}	3.8×10^{-39}
dL\dt	4×10^{35}	7.3×10^{15}	4×10^{-2}	5.8×10^{-9}	1.1×10^{-24}
dn\dt	5×10^{26}	9.8×10^4	2.2×10^{-13}	2.5×10^{-20}	3.7×10^{-35}
dN\dt	3×10^{45}	3.6×10^{23}	7.3×10^5	8.1×10^{-2}	8.5×10^{-18}
T	5×10^{-12}	2.3×10^{15}	3.3×10^{34}	5.2×10^{39}	5.4×10^{58}

$$N_o = 4IIr^2 n_o H_o$$

where n_o, and H_o are, respectively, the number density and scale height of each constituent near the surface.

The value of n_o, and the number densities of O, N_2, O_2, and CO_2 at the critical level are given by [21]; Tois taken equal 225^oK. The number density of Hydrogen molecules near the surface is assumed equal to the number of Oxygen atoms (about 10^7 cm^{-3}). The results are given in Table **1**. Small letters are used to indicate loss from unit area (at the equator) and capital letters to indicate loss from the whole planetary surface. The residence time is in units of 10^9 yrs. The table reveals while hydrogen is never stable and could not be retained to any extent.

Finally setting the effect of orbital motion to zero in [5] the present formulas result, indicating the validity of both.

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