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Mansouri-Sexl Test for Light Speed Anisotropy *via* Cosmic Aberration

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Abstract: The Mansouri-Sexl theory is a well known test theory of relativity. In the current paper we will derive the Mansouri-Sexl formalism for the light aberration and we will show how to improve on the theoretical and experimental basis by constraining both the Mansouri-Sexl parameter $a(v)$ and the parameter $b(v)$. In the process of constraining the Mansouri parameters we devise a novel experiment for measuring and constraining light speed anisotropy as well. An overwhelming number of experiments dealing with light speed anisotropy are laboratory-bound and they are limited to constraining only parameter $a(v)$; we will also take the approach of setting up an astronomical experiment instead of a lab-based one as it befits the relativistic aberration effect. Our paper is organized as follows: in the first section we give a new and more complete derivation of the Mansouri-Sexl aberration effect. In the second part, we apply the newly expanded Mansouri-Sexl aberration formalism in order to devise an astronomical experiment used for constraining both the parameter $a(v)$ and the parameter $b(v)$. This turns the Mansouri-Sexl aberration experiment into a very powerful tool for constraining light speed anisotropy.

Keywords: Mansouri-sexl test theory, relativistic aberration, light speed Anisotropy.

1. THE ROBERTSON-MANSOURI-SEXL TEST THEORY

The test theories of special relativity differ in their assumptions about the form of the Lorentz transforms. The main test theories of special relativity (SR) are named after their authors, Robertson [1], Mansouri and Sexl [2-4] (RMS). These test theories can also be used to examine potential alternate theories to SR - such alternate theories predict particular values of the parameters of the test theory, which can easily be compared to values determined by experiments analyzed with the test theory [5-20]. The existing experiments put rather strong experimental constraints on any alternative theory. RMS starts by admitting by reduction to absurd that there is a preferential inertial frame Σ in which the light propagates isotropically with the speed c_0. All other frames in motion with respect to Σ are considered non-preferential and the light speed is anisotropic. The light speed in the non-preferential frames can be deduced *via* simple calculations described in [3]. We start with the Mansouri-Sexl transforms:

$$\begin{aligned} x &= b(v)(X - vT) \\ y &= d(v)Y \\ z &= d(v)Z \\ t &= a(v)T + \varepsilon(v)x = (a - b\varepsilon v)T + b\varepsilon X \end{aligned} \tag{1.1}$$

where v is the relative speed between S and Σ, $a(v) \approx 1 + \alpha\frac{v^2}{c^2}, b(v) \approx 1 + \beta\frac{v^2}{c^2}$ and (x, y, z, t) are the coordinates in S and (X, Y, Z, T) represent their correspondents in Σ. Inverting, we obtain:

$$\begin{aligned} X &= (\frac{1}{b} - \frac{v\varepsilon}{a})x + \frac{v}{a}t \\ T &= \frac{t - \varepsilon x}{a} = -\frac{\varepsilon}{a}x + \frac{1}{a}t \end{aligned} \tag{1.2}$$

Exactly as in the original Mansouri-Sexl paper we start with the line element that allows us to calculate the light speed in frame S as a function of the isotropic light speed in frame Σ:

$$\begin{aligned} 0 = X^2 - c_o{}^2T^2 &= ((\frac{1}{b} - \frac{v\varepsilon}{a})^2 - \frac{c_0{}^2\varepsilon^2}{a^2})x^2 \\ &+2xt(\frac{v}{a}(\frac{1}{b} - \frac{v\varepsilon}{a}) + \frac{c_0{}^2\varepsilon}{a}) - t^2\frac{c_0{}^2 - v^2}{a^2} \end{aligned} \tag{1.3}$$

$$\begin{aligned} &((\frac{1}{b} - \frac{v\varepsilon}{a})^2 - \frac{c_0{}^2\varepsilon^2}{a^2})\frac{x^2}{t^2} + 2\frac{x}{t}(\frac{v}{a}(\frac{1}{b} - \frac{v\varepsilon}{a}) + \\ &\frac{c_0{}^2\varepsilon}{a}) - \frac{c_0{}^2 - v^2}{a^2} = 0 \end{aligned} \tag{1.4}$$

The anisotropic speed of light propagating along the x axis in S is:

$$c_\pm = \frac{-(v(\frac{a}{b} - v\varepsilon) + ac_0{}^2\varepsilon) \pm \sqrt{(v(\frac{a}{b} - v\varepsilon) + ac_0{}^2\varepsilon)^2 + (c_0{}^2 - v^2)((\frac{a}{b} - v\varepsilon)^2 - c_0{}^2\varepsilon^2)}}{(\frac{a}{b} - v\varepsilon)^2 - c_0{}^2\varepsilon^2} \tag{1.5}$$

The two separate solutions exist for

$$v(\frac{a}{b} - v\varepsilon) + ac_0{}^2\varepsilon)^2 + (c_0{}^2 - v^2)((\frac{a}{b} - v\varepsilon)^2 - c_0{}^2\varepsilon^2 > 0 \tag{1.6}$$

In SR $a(v) = \frac{1}{\gamma(v)}, b = \gamma(v), \varepsilon = -\frac{v}{c^2}$ so $c_+ = c_- = c_0$.

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Exactly as in the original Mansouri-Sexl paper [4] by transforming the light cone $X^2 - c_o^2T^2 = 0$ into S we obtain

$$\frac{c(\theta)}{c_0} = 1 - \frac{v}{c_0}(1+2\alpha)\cos\theta \tag{1.7}$$

Expression (1.7) is valid if slow clock transport synchronization has been used. According to Mansouri, the one-way light speed is a measurable quantity in this case and it is direction dependent for $\alpha \neq -0.5$. We will exploit this property in the Mansouri-Sexl theory of the aberration experiment constructed later in our paper.

2. THE RMS THEORY FOR ABERRATION

In his 1905 paper, "On the Electrodynamics of Moving Bodies" [16], Einstein produces an interesting blueprint for deriving the general formula for the Doppler effect. He starts by considering a generic electromagnetic wave of phase Φ, frequency $\nu = \omega/2\pi$, and of wave-vector (l,m,n) propagating with speed c towards the origin O of a frame K. From the perspective of frame K, of coordinates (x,y,z,t) the phase is:

$$\Phi = \omega\left(t - \frac{lx+my+nz}{c}\right): \tag{2.1}$$

Let k be a system moving with the speed v along the positive x axis of frame K (see Fig. **1**). We want to determine the form of the phase from the perspective of k, departing from the light source.

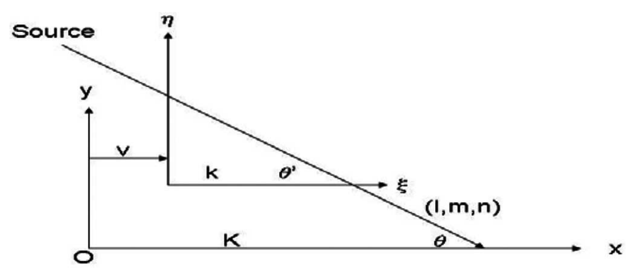

Fig. (1). Reference frames K and k.

Since K and k are in a translation motion along the x axis with respect to each other we replace the Lorentz transformations in Einstein derivation:

$$\begin{aligned} \xi &= \gamma(x - vt) \\ \psi &= y \\ \zeta &= z \\ \tau &= \gamma\left(t - \frac{vx}{c^2}\right) \\ \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} \end{aligned} \tag{2.2}$$

with the corresponding Mansouri-Sexl transforms described by (1.1). In (1.1) (X,Y,Z,T) represent the coordinates in the preferential frame Σ while (x',y',z',t') are the corresponding coordinates in the lab frame S'. In the preferential frame Σ

$$\Phi = \Omega\left(T - \frac{lX+mY+nZ}{c}\right) \tag{2.3}$$

where (l,m,n) are the components of the wave vector. In the lab frame S', where c' is assumed to be the (anisotropic) light speed, the phase ϕ' (all variables in frame S' are lowercase by convention) must have a form similar to the form (2.3) described in the preferential frame Σ:

$$\phi' = \omega'\left(a - b\varepsilon v + \frac{bvl'}{c'}\right)\left(T - cb\frac{\frac{l'}{c'} - \varepsilon}{a - b\varepsilon v + \frac{bvl'}{c'}}\frac{X}{c} - \frac{dm'}{a - b\varepsilon v + \frac{bvl'}{c'}}\frac{Y}{c} - \frac{dn'}{a - b\varepsilon v + \frac{bvl'}{c'}}\frac{Z}{c}\right) \tag{2.4}$$

In the lab frame, by comparing (2.4) with (2.3) we obtain the Mansouri-Sexl aberration formula and the Doppler shifted frequency:

$$l' = \frac{(a - b\varepsilon v)l + b\varepsilon c}{b\left(\frac{c}{c'} - \frac{vl}{c'}\right)} = \frac{c'}{c}\frac{(a - b\varepsilon v)\cos\theta + b\varepsilon c}{b\left(1 - \frac{v}{c}\cos\theta\right)} \tag{2.5}$$

where $l = \cos\theta$.

A quick sanity check shows that in SR $a(v) = \frac{1}{\gamma(v)}, b = \gamma(v), \varepsilon = -\frac{v}{c^2}, c' = c$, so

$$\cos\theta' = \frac{\cos\theta - \frac{v}{c}}{1 - \frac{v}{c}\cos\theta} \tag{2.6}$$

in perfect agreement with Einstein's derivation [16]. Inverting (2.6) we obtain the aberration as a function of the aberration measured in the lab frame:

$$\cos\theta = \frac{\cos\theta' + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta'} \tag{2.7}$$

We will make good use of (2.7) later in the paper.

3. THE EXPERIMENTAL DETERMINATION OF THE MANSOURI-SEXL PARAMETERS $a(v)$ AND $b(v)$

The Mansouri-Sexl parameter ε encapsulates the clock synchronization convention. As such, ε can be expressed as a function of the other two Mansouri-Sexl parameters [2], $a(v)$ and $b(v)$:

$$\varepsilon_E = -\frac{\frac{v}{c^2}a(v)}{\left(1 - \frac{v^2}{c^2}\right)b(v)} \tag{3.1}$$

for "Einstein" clock synchronization and:

$$\varepsilon_T = \frac{1}{b(v)}\frac{da(v)}{dv} = \frac{2\alpha}{b}\frac{v}{c^2} \tag{3.2}$$

for the case of slow clock "transport". It is known that the experimental results are invariant with respect to clock synchronization, so either method will produce the same result, in this particular experiment we will choose the slow clock transport scheme. Inserting (2.7) into (2.5) we obtain:

$$\cos\theta' = (1-(1+2\alpha)\frac{v}{c}\cos\theta')\frac{(a-b\varepsilon v)\dfrac{\cos\theta'+\frac{v}{c}}{1+\frac{v}{c}\cos\theta'}+b\varepsilon c}{b\dfrac{1-\frac{v^2}{c^2}}{1+\frac{v}{c}\cos\theta'}} \tag{3.3}$$

where θ' is the angle as measured in the lab frame S'. Using the Mansouri [2] expressions:

$$a(v) = 1+\alpha\frac{v^2}{c^2}+.... \qquad b(v) = 1+\beta\frac{v^2}{c^2}+.... \tag{3.4}$$

and neglecting the term in $\frac{v^3}{c^3}$ expression (3.3) becomes:

$$\cos\theta'(1-\frac{v^2}{c^2})(1+\beta\frac{v^2}{c^2}) = (1-(1+2\alpha)\frac{v}{c}\cos\theta')((1+\alpha\frac{v^2}{c^2})\cos\theta'+(1+2\alpha)\frac{v}{c}) \tag{3.5}$$

The two parameters α and β can be now constrained using expression (3.5) *via* two independent measurements, at θ_1'. and θ_2'. The experiment is best executed in a geostationary satellite in order to avoid any refraction effects from the Earth's atmosphere.

It is interesting to contrast the method described in our paper with the recent experiments based on studying the angular fluctuations of the light speed with respect to the apex of the dipole of the cosmic microwave background at the European Synchrotron Radiation Facility [22, 23]. The method uses a direct approach of expressing the CMB temperature anisotropy (see eq. 1 in [22]). There is nothing stopping the authors of [22, 23] from reformulating the theory of their experiment in the parametric form of the Mansouri test theory.

CONCLUSIONS

We derived the Mansouri-Sexl formalism for the light aberration and we demonstrated a means to improve on the theoretical and experimental basis such as to constrain both the Mansouri-Sexl parameter $a(v)$ and the parameter $b(v)$ thus setting up a novel experiment for measuring and constraining light speed anisotropy. An overwhelming number of current experiments dealing with light speed anisotropy are laboratory-bound [21-24] and they are limited to constraining only parameter "$a(v)$", so we took the novel approach of setting up instead an astronomical experiment based on the relativistic aberration that constrains both parameters.

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