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Behavior of *F(R)* Gravity Around a Crossing of the Phantom Divide

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Abatract: We study a model of $F(R)$ gravity in which a crossing of the phantom divide can be realized. In particular, we demonstrate the behavior of $F(R)$ gravity around a crossing of the phantom divide by taking into account the presence of cold dark matter.

Keywords: Gravity, phantom divide expansion.

1. INTRODUCTION

It is observationally supported that the current expansion of the universe is accelerating [1, 2]. The scenarios to account for the current accelerated expansion of the universe fall into two broad categories [3-11]. One is to introduce "dark energy"' in the framework of general relativity. The other is to study a modified gravitational theory, e.g., $F(R)$ gravity, in which the action is represented by an arbitrary function $F(R)$ of the scalar curvature R (for reviews, see [6-11]; and for a new approach, see [12]).

On the other hand, various observational data [13] imply that the ratio of the effective pressure to the effective energy density of the universe, i.e., the effective equation of state (EoS) $w_{\text{eff}} \equiv p_{\text{eff}}/\rho_{\text{eff}}$, may evolve from larger than -1 (non-phantom phase) to less than -1 (phantom phase [14, 15]). Namely, it crosses -1 (the phantom divide) at the present time or in the near past. A number of models to realize the crossing of the phantom divide have been proposed (for a detailed review, see [4]).

There are also several studies for the crossing of the phantom divide in the framework of $F(R)$ gravity [16-22][1] (for related works, see [25]). An explicit model with realizing a crossing of the phantom divide has been constructed in Ref. [26] and its thermodynamics has been examined [27]. Moreover, in Ref. [28] it has been illustrated that multiple crossings of the phantom divide can occur in $F(R)$ gravity as the scalar field theories such as an oscillating quintom model [29] or a quintom with two scalar fields [30] in the framework of general relativity (see also [31, 32]).

In this paper, we study a model of $F(R)$ gravity in which a crossing of the phantom divide can be realized by taking into account the presence of cold dark matter. We demonstrate the behavior of $F(R)$ gravity around a crossing of the phantom divide. In our previous work [26], an analytic solution of $F(R)$ gravity to realize a crossing of the phantom divide without matter has been derived. In this work, as a further investigation, we examine a solution of $F(R)$ gravity to realize a crossing of the phantom divide with cold dark matter. We use units of $k_{\text{B}} = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by $\kappa^2 \equiv 8\pi/M_{\text{Pl}}{}^2$ with the Planck mass of $M_{\text{Pl}} = G^{-1/2} = 1.2\times10^{19}\,\text{GeV}$.

The paper is organized as follows. In Sec. II, we explain the reconstruction method of $F(R)$ gravity proposed in Ref. [33]. In Sec. III, we study a model of $F(R)$ with realizing a crossing of the phantom divide by using the reconstruction method and investigate the behavior of $F(R)$ gravity around a crossing of the phantom divide. Finally, conclusions are given in Sec. IV.

2. RECONSTRUCTION METHOD

First, we explain the reconstruction method of $F(R)$ gravity proposed in Ref. [33]. The action of $F(R)$ gravity with matter is as follows:

$$S = \int d^4x\sqrt{-g}\left[\frac{F(R)}{2\kappa^2} + L_{matter}\right], \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and L_{matter} is the matter Lagrangian. By using proper functions $P(\phi)$ and $Q(\phi)$ of a scalar field ϕ, the action in Eq. (1) can be rewritten to

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[P(\phi)R + Q(\phi)\right] + L_{matter}\right\}. \tag{2}$$

The scalar field ϕ may be regarded as an auxiliary scalar field because it has no kinetic term. From Eq. (1), the equation of motion of ϕ is given by

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[1]The equivalence between $F(R)$ gravity and the scalar-tensor theory has been indicated in Ref. [23]. The crossing of the phantom divide in scalar-tensor theories has been investigated in Ref. [24].

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$$0 = \frac{dP(\phi)}{d\phi} R + \frac{dQ(\phi)}{d\phi}. \quad (3)$$

Substituting $\phi = \phi(R)$ into the action in Eq. (2) yields the expression of $F(R)$ as

$$F(R) = P(\phi(R))R + Q(\phi(R)). \quad (4)$$

From Eq. (2), the field equation of modified gravity is derived as

$$\frac{1}{2} g_{\mu\nu} \left[P(\phi)R + Q(\phi) \right] - R_{\mu\nu} P(\phi) - g_{\mu\nu} \Box P(\phi) + \nabla_\mu \nabla_\nu P(\phi) + \kappa^2 T_{\mu\nu}^{(matter)} = 0, \quad (5)$$

where ∇_μ is the covariant derivative operator associated with $g_{\mu\nu}$, $\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the covariant d'Alembertian for a scalar field, and $T_{\mu\nu}^{(\mathrm{matter})}$ is the contribution to the energy-momentum tensor form matter.

We assume the flat Friedmann-Robertson-Walker (FRW) space-time with the metric,

$$ds^2 = -dt^2 + a^2(t) dx^2, \quad (6)$$

where $a(t)$ is the scale factor. In this background, the components of $(\mu,\nu) = (0,0)$ and $(\mu,\nu) = (i,j)$ $(i,j = 1,\cdots,3)$ in Eq. (5) read

$$-6H^2 P(\phi(t)) - Q(\phi(t)) - 6H \frac{dP(\phi(t))}{dt} + 2\kappa^2 \rho = 0, \quad (7)$$

$$2\frac{d^2 P(\phi(t))}{dt^2} + 4H \frac{dP(\phi(t))}{dt} + \left(4\dot{H} + 6H^2\right) P(\phi(t)) + Q(\phi(t)) + 2\kappa^2 p = 0, \quad (8)$$

where $H = \dot{a}/a$ is the Hubble parameter with $\dot{} = \partial/\partial t$ and ρ and p are the sum of the energy density and pressure of matters with a constant EoS w_i, respectively, with i being some component of matters. After eliminating $Q(\phi)$ from Eqs. (7) and (8), we get

$$\frac{d^2 P(\phi(t))}{dt^2} - H \frac{dP(\phi(t))}{dt} + 2\dot{H} P(\phi(t)) + \kappa^2 (\rho + p) = 0. \quad (9)$$

The scalar field ϕ may be taken as $\phi = t$ if it is redefined properly. By representing $a(t)$ as

$$a(t) = \bar{a} \exp(\tilde{g}(t)) \quad (10)$$

in terms of a constant of $\bar{a}$ and a proper function of $\tilde{g}(t)$ and using $H = d\tilde{g}(\phi)/(d\phi)$, we rewrite Eq. (9) to be

$$\frac{d^2 P(\phi)}{d\phi^2} - \frac{d\tilde{g}(\phi)}{d\phi} \frac{dP(\phi)}{d\phi} + 2\frac{d^2 \tilde{g}(\phi)}{d\phi^2} P(\phi) + \kappa^2 \sum_i (1+w_i) \bar{\rho}_i \bar{a}^{-3(1+w_i)} \exp\left[-3(1+w_i)\tilde{g}(\phi)\right] = 0, \quad (11)$$

where $\bar{\rho}_i$ is a constant. Moreover, from Eq. (7), we obtain

$$Q(\phi) = -6\left[\frac{d\tilde{g}(\phi)}{d\phi}\right]^2 P(\phi) - 6\frac{d\tilde{g}(\phi)}{d\phi}\frac{dP(\phi)}{d\phi} + 2\kappa^2 \sum_i \bar{\rho}_i \bar{a}^{-3(1+w_i)} \exp\left[-3(1+w_i)\tilde{g}(\phi)\right]. \quad (12)$$

We note that if we redefine the auxiliary scalar field ϕ by $\phi = \Phi(\varphi)$ with a proper function Φ and define $\tilde{P}(\varphi) \equiv P(\Phi(\varphi))$ and $\tilde{Q}(\varphi) \equiv Q(\Phi(\varphi))$, the new action

$$S = \int d^4x \sqrt{-g} \left[\frac{\tilde{F}(R)}{2\kappa^2} + L_{matter} \right], \quad (13)$$

$$\tilde{F}(R) \equiv \tilde{P}(\varphi) R + \tilde{Q}(\varphi), \quad (14)$$

is equivalent to the action in Eq. (2) because $\tilde{F}(R) = F(R)$. Here, φ is the inverse function of Φ and we can solve φ with respect to R as $\varphi = \varphi(R) = \Phi^{-1}(\phi(R))$ by using $\phi = \phi(R)$. Consequently, we have the choices in ϕ like a gauge symmetry and therefore we can identify ϕ with time t, i.e., $\phi = t$, which can be interpreted as a gauge condition corresponding to the reparameterization of $\phi = \phi(\varphi)$ [26]. Thus, if we have the relation $t = t(R)$, in principle we can obtain the form of $F(R)$ by solving Eq. (11) with Eqs. (4)) and (12).

We also remark that a crossing of the phantom divide cannot be described by a naive model of $F(R)$ gravity. To realize the crossing, $F(R)$ needs to be a double-valued function, where the cut could correspond to $w_{\mathrm{eff}} = -1$. However, the crossing can be performed by the extension of $F(R)$ gravity, whose action is given by $P(\phi)R + Q(\phi)$.

3. MODEL

Next, we examine a model of $F(R)$ with realizing a crossing of the phantom divide by using the reconstruction method and investigate the behavior of $F(R)$ gravity around a crossing of the phantom divide.

3.1. Crossing of the Phantom Divide

To illustrate the behavior of $F(R)$ with realizing a crossing of the phantom divide, we consider the case in which the Hubble rate $H(t)$ is expressed as [34]

$$H = n\left(\frac{1}{t} + \frac{1}{t_s - t}\right), \quad (15)$$

where n is a positive constant and t_s is the time when the Big Rip singularity [35] appears as will be shown later[2].

[2]Other kinds of finite-time future singularities have been studied in Ref.[36]

In the FRW background, the effective energy density and pressure of the universe are given by $\rho_{\text{eff}} = 3H^2/\kappa^2$ and $p_{\text{eff}} = -\left(2\dot{H} + 3H^2\right)/\kappa^2$, respectively. The effective EoS $w_{\text{eff}} = p_{\text{eff}}/\rho_{\text{eff}}$ is defined as [6]

$$w_{\text{eff}} \equiv -1 - \frac{2\dot{H}}{3H^2}, \tag{16}$$

which implies that a crossing of the phantom divide occurs when the sign of $\dot{H}$ changes.

When $t \to 0$, i.e., $t << t_s$, $H(t)$ in Eq. (15) behaves as $H(t) \sim n/t$ and therefore $\dot{H} \sim -n/t^2 < 0$. In this limit, it follows from Eq. (16) that the effective EoS is given by $w_{\text{eff}} = -1 + 2/(3n) > -1$. Such behavior is identical with that in the Einstein gravity with matter whose EoS is greater than -1. This is the non-phantom phase. On the other hand, when $t \to t_s$, we find $H(t) \sim n/(t_s - t)$ and hence $\dot{H} \sim n/(t_s - t)^2 > 0$. We only consider the period $0 < t < t_s$ because H should be positive. In this case, the scale factor is given by $a(t) \sim \bar{a}\left(t_s - t\right)^{-n}$. Thus, when $t \to t_s$, $a \to \infty$, namely, the Big Rip singularity appears. In this limit, the effective EoS is given by $w_{\text{eff}} = -1 - 2/(3n) < -1$. Such behavior is identical with the case in which there is a phantom matter with its EoS being smaller than -1. This is the phantom phase. Moreover, from Eq. (16), we see that the effective EoS w_{eff} becomes -1 when $\dot{H} = 0$. Solving $w_{\text{eff}} = -1$ with respect to t by using Eq. (15), we find that the effective EoS crosses the phantom divide at $t = t_c$ given by $t_c = t_s/2$. As a consequence, in case of Eq. (15), a crossing of the phantom divide can occur. We show the time evolution of w_{eff} in Fig. (**1**) with $\tilde{t} \equiv t/t_0$, where t_0 is the present time. In all figures, we take $n = 10$ and $t_s \equiv \alpha t_0$ with $\alpha = 2.0$. In this case, $t_c = t_0$. From Fig. (**1**), we see that at the present time, a crossing of the phantom divide from the non-phantom phase ($w_{\text{eff}} > -1$) to the phantom one ($w_{\text{eff}} < -1$) can be realized.

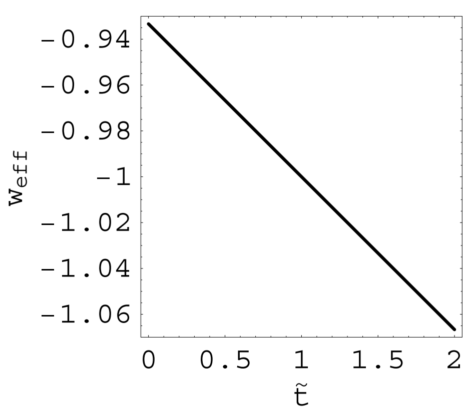

Fig. (1). Time evolution of w_{eff} for $n = 10$ and $\alpha = 2.0$ with $\tilde{t} = t/t_0$.

3.2. Behavior of *F(R)* Gravity Around a Crossing of the Phantom Divide

In what follows, we take $\phi = t$. From Eqs. (10) and (15), $H = d\tilde{g}(t)/(dt)$ and $R = 6\left(\dot{H} + 2H^2\right)$, we obtain

$$\tilde{g}(t) \;=\; n \ln\left(\frac{t}{t_s - t}\right), \tag{17}$$

$$a(t) \;=\; \left[\frac{(\alpha - 1)t}{t_s - t}\right]^n, \tag{18}$$

$$R \;=\; \frac{6nt_s}{t^2\left(t_s - t\right)^2}\left[(2n-1)t_s + 2t\right], \tag{19}$$

where we have taken $\bar{a} = (\alpha - 1)^n$ so that the present value of the scale factor should be unity.

We define $X \equiv t/t_s$ and solve Eq. (19) with respect to X. If n is much larger than unity, we can neglect the second term on the right-hand side of Eq. (19) and therefore obtain the approximate solutions

$$X(\tilde{R}) \approx \frac{1 \pm \sqrt{1 - 4\tilde{R}^{-1/2}}}{2}, \tag{20}$$

where

$$\tilde{R} \;=\; \frac{t_s^2 R}{6n(2n-1)}. \tag{21}$$

For the lower sign and the upper one in Eq. (20), X varies as $0 < X \le 1/2$ and $1/2 \le X < 1$, respectively.

For simplicity, we consider the case in which there exists a matter with a constant EoS $w = p/\rho$. In this case, Eqs. (11) and (12) are rewritten to

$$\begin{aligned} &4\tilde{R}^{5/2}\left(1 - 4\tilde{R}^{-1/2}\right)\frac{d^2P(\tilde{R})}{d\tilde{R}^2} \\ &+2\tilde{R}^2\left[\left(3 - 10\tilde{R}^{-1/2}\right) \mp n\sqrt{1 - 4\tilde{R}^{-1/2}}\,\right]\frac{dP(\tilde{R})}{d\tilde{R}} \pm 2n\tilde{R}\sqrt{1 - 4\tilde{R}^{-1/2}}\,P(\tilde{R}) \\ &+t_s^2\kappa^2(1+w)\bar{\rho}\left[(\alpha - 1)\left(\frac{1 \pm \sqrt{1 - 4\tilde{R}^{-1/2}}}{1 \mp \sqrt{1 - 4\tilde{R}^{-1/2}}}\right)\right]^{-3n(1+w)} = 0 \end{aligned} \tag{22}$$

and

$$\begin{aligned} \tilde{t}_s^{\,2}Q(\tilde{R}) \;=\; &-\frac{n}{2n-1}\tilde{R}P(\tilde{R}) \mp \frac{2}{2n-1}\tilde{R}^2\sqrt{1 - 4\tilde{R}^{-1/2}}\,\frac{dP(\tilde{R})}{d\tilde{R}} \\ &+\frac{1}{3n(2n-1)}t_s^2\kappa^2\bar{\rho}\left[(\alpha - 1)\left(\frac{1 \pm \sqrt{1 - 4\tilde{R}^{-1/2}}}{1 \mp \sqrt{1 - 4\tilde{R}^{-1/2}}}\right)\right]^{-3n(1+w)}, \end{aligned} \tag{23}$$

respectively, where $\tilde{t}_s \equiv t_s/\left[6n(2n-1)\right]$. Here, $\bar{\rho}$ corresponds to the present energy density of the matter. In particular,
we use the present value of cold dark matter with $w = 0$ for $\bar{\rho}$, i.e., $\bar{\rho} = 0.233\rho_c$ [1], where $\rho_c = 3H_0^2/(8\pi G) = 3.97 \times 10^{-47}\,\text{GeV}^4$ is the critical energy

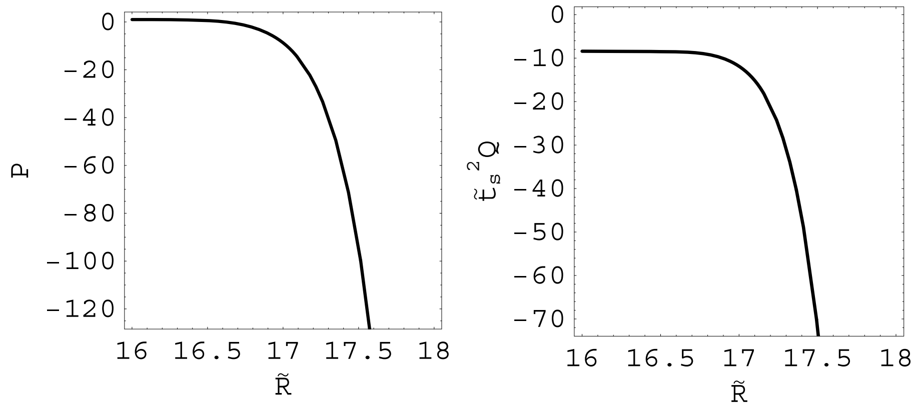

Fig. (2). $P(\tilde{R})$ and $\tilde{t}_s^{\,2}Q(\tilde{R})$ as functions of $\tilde{R}$ for $0 < t < t_c(= t_0)$. We have taken $n = 10$, $\alpha = 2.0$ and $\bar{\rho} = 0.233\rho_c$.

density and $H_0 = 2.13h \times 10^{-42}$ GeV [37] with $h = 0.70$ [38], [39] is the present Hubble parameter. From Eqs. (4) and (21), we have

$$\frac{F(\tilde{R})}{2\kappa^2} = \frac{1}{2\kappa^2 \tilde{t}_s^{\,2}}\left(P(\tilde{R})\tilde{R} + \tilde{t}_s^{\,2} Q(\tilde{R})\right). \tag{24}$$

We examine $F(\tilde{R})$ by solving Eqs. (22)-(24) numerically.

In Figs. (**2** and **4**), we depict $P(\tilde{R})$ and $\tilde{t}_s^{\,2}Q(\tilde{R})$ as functions of $\tilde{R}$. We have used $t_0 \approx 1/H_0$. Figs. (**2** and **4**) show the case of $0 < t < t_c(= t_0)$, i.e., $0 < X < 1/2$ and that of $t_c(= t_0) < t < t_s$, i.e., $1/2 < X < 1$, respectively. We have numerically solved Eq. (22) in the range of $\tilde{R}$ as $16.0001 \le \tilde{R} \le 18.0$. Here, we have taken the initial conditions as $P(\tilde{R} = 16.0001) = 1.0$ and $dP(\tilde{R} = 16.0001)/(d\tilde{R}) = 0$ so that around the time when a crossing of the phantom divide occurs t_c, $F(R)/\left(2\kappa^2\right)$ could contain the term $R/\left(2\kappa^2\right)$, namely, the ordinary Einstein-Hilbert action. We note that at $t = t_c$, i.e., $X = 1/2$ and hence $\tilde{R} = 16.0$, we cannot solve Eq. (22) numerically. We therefore investigate the behavior of $F(R)$ gravity around a crossing of the phantom divide. By using Eq. (24), we show the behavior of $F(\tilde{R})/\left(2\kappa^2\right)$ in Figs. (**3** and **5**), which show the case of $0 < t < t_c(= t_0)$, i.e., $0 < X < 1/2$ and that of $t_c(= t_0) < t < t_s$, i.e., $1/2 < X < 1$, respectively. For $\alpha = 2.0$, because $t_c = t_0$, $0 < X < 1/2$ and $1/2 < X < 1$ correspond to the past and the future, respectively. Furthermore, we illustrate $w_{\mathrm{eff}}(\tilde{R}) = -1 + \left[2/(3n)\right]\left(1 - 2X(\tilde{R})\right)$ in Fig. (**6**). The time evolution of $\tilde{R}$ is given in Fig. (**7**). From Figs. (**6** and **7**), we see that a crossings of the phantom divide can be realized. The results in the all figures are shown by dimensionless quantities.

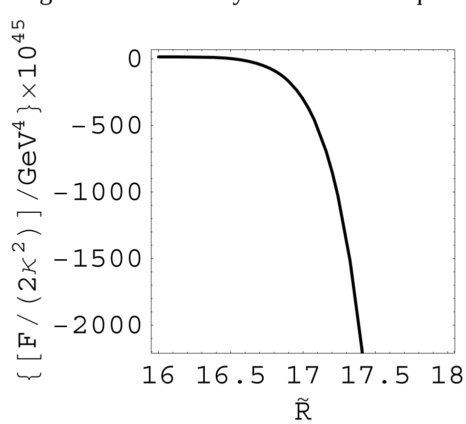

Fig. (3). Behavior of $F(\tilde{R})/\left(2\kappa^2\right)$ as a function of $\tilde{R}$ for $0 < t < t_c(= t_0)$. Legend is the same as Fig. (**2**).

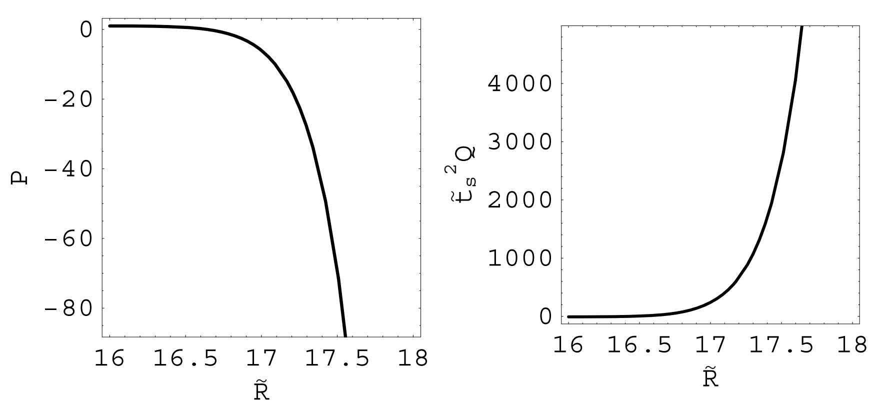

Fig. (4). $P(\tilde{R})$ and $\tilde{t}_s^{\,2}Q(\tilde{R})$ as functions of $\tilde{R}$ for $t_c(= t_0) < t < t_s$. We have taken $n = 10$, $\alpha = 2.0$ and $\bar{\rho} = 0.233\rho_c$.

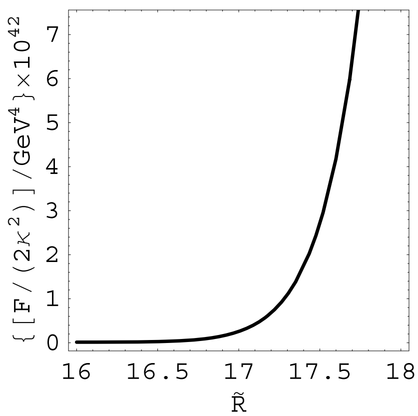

Fig. (5). Behavior of $F(\tilde{R})/(2\kappa^2)$ as a function of $\tilde{R}$ for $t_c(=t_0) < t < t_s$. Legend is the same as Fig. (**4**).

From Figs. (**3** and **5**), we see that before a crossing of the phantom divide, $F(\tilde{R})$ decreases in terms of $\tilde{R}$, while that after the crossing, $F(\tilde{R})$ increases in terms of $\tilde{R}$. The latter behavior is reasonable because in the Hu-Sawicki model [40] of $F(R)$ gravity, which passes the solar system tests, $F(R)$ increases around the present curvature. For $\alpha = 2.0$, the time when a crossings of the phantom divide is the present time, and it follows from Eqs. (19) and (21) that the present value of $\tilde{R}$ is $\tilde{R}_0 = 16.0$. We remark that such behavior is typical for a general class of viable $F(R)$ gravities introduced in Ref. [41] to which the Hu-Sawicki model belongs. This class of modified gravities can satisfy the solar system tests and unify inflation with the late-time cosmic acceleration. As viable models of $F(R)$ gravity, e.g., the models in Refs. [41-47] are also proposed. In addition, investigations to solve the problem of a curvature singularity in $F(R)$ gravity [17, 46, 48-52] have recently been executed in Refs. [53-59]. Theories without a singularity were also constructed in Ref. [17].

Finally, we mention the stability for the obtained solutions of the phantom crossing under a quantum correction coming from the conformal anomaly. In Ref. [26], it has been shown that the quantum correction of massless conformally-invariant fields could be small when a crossings of the phantom divide occurs and therefore the solutions of the phantom crossing could be stable under the quantum correction, although the quantum correction becomes important near the Big Rip singularity.

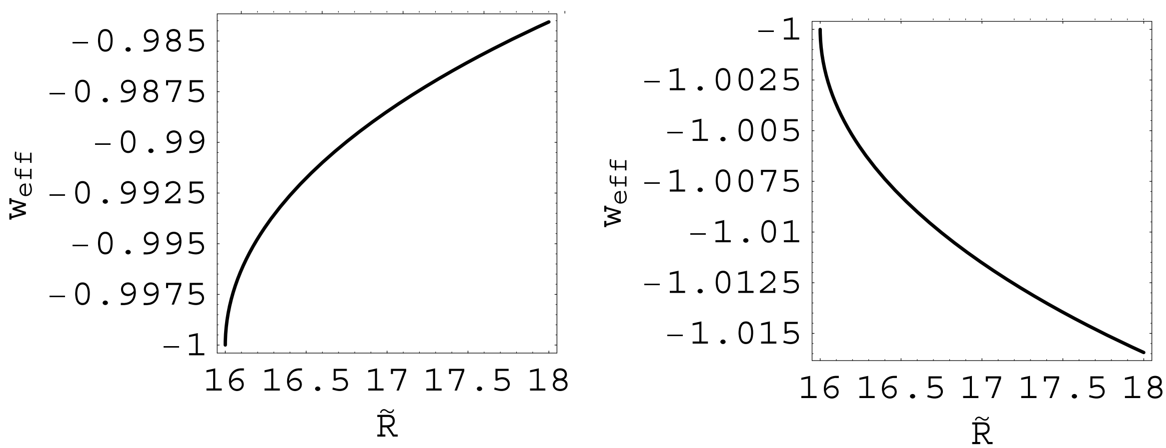

Fig. (6). Behavior of $w_{\rm eff}(\tilde{R})$ as a function of $\tilde{R}$. The left panel and the right one show the case of $0 < t \le t_c(=t_0)$, i.e., $0 < X \le 1/2$ and that of $t_c(=t_0) \le t < t_s$, i.e., $1/2 \le X < 1$, respectively. Legend is the same as Fig. (**1**).

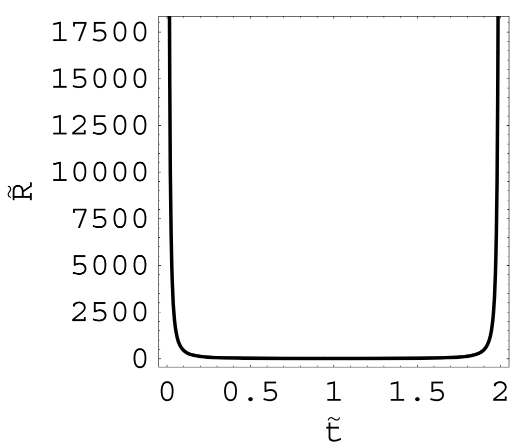

Fig. (7). Time evolution of $\tilde{R}$. Legend is the same as Fig. (**1**).

4. CONCLUSION

In the present paper, we have investigated a model of $F(R)$ gravity in which a crossing of the phantom divide can be realized. In particular, we have illustrated the behavior of $F(R)$ gravity around a crossing of the phantom divide by taking into account the presence of cold dark matter. The demonstration in this work can be interpreted as a meaningful step to consider a more realistic model of $F(R)$ gravity, which could correctly describe the expansion history of the universe.

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