

# Can Ghost Condensate Decrease Entropy?

Shinji Mukohyama\*

*Institute for the Physics and Mathematics of the Universe (IPMU), The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8582, Japan*

**Abstract:** By looking for possible violation of the generalized second law, we might be able to find regions in the space of theories and states that do not allow holographic dual descriptions. We revisit three proposals for violation of the generalized second law in the simplest Higgs phase of gravity called ghost condensate. Two of them, (i) analogue of Penrose process and (ii) semiclassical heat flow, are based on Lorentz breaking effects, by which particles of different species can have different limits of speed. We show that processes in both (i) and (ii) are always slower than accretion of ghost condensate and cannot decrease the total entropy before the accretion increases the entropy. The other proposal is to use (iii) negative energy carried by excitations of ghost condensate. We prove an averaged null energy condition, which we conjecture prevents the proposal (iii) from violating the generalized second law in a coarse-grained sense.

## INTRODUCTION

The second law of thermodynamics defines the arrow of time, stating that entropy does not decrease. Similarly, the attractive nature of gravity defines the arrow of time for a black hole at least classically. Nothing can escape from a black hole and, as a result, the area of a black hole horizon cannot decrease [1]. This classical-mechanical statement is known as the area law or the second law of black hole thermodynamics, and it is believed that a black hole has entropy proportional to the horizon area [2]. Quantum mechanically, however, a black hole emits Hawking radiation [3] and, hence, the area of a black hole horizon can decrease. Therefore, the second law of black hole thermodynamics does not hold quantum mechanically. Instead, it is believed that the total entropy, i.e. the sum of black hole entropy and matter entropy outside black hole, does not decrease [4]. This statement about a system including a black hole and matter is called the generalized second law. One must, however, be aware that the generalized second law has been proven only in some limited situations [5].

As already stated, a black hole is believed to have entropy proportional to the horizon area. The proportionality coefficient can be determined by substituting the Hawking temperature for black hole temperature in the first law of black hole thermodynamics, a relation analogous to the first law of thermodynamics. The black hole entropy  $S_{bh}$  is then determined as

$$S_{bh} = \frac{k_B c^3}{4G_N \hbar} A_h, \quad (1)$$

where  $A_h$  is the horizon area,  $k_B$  is the Boltzmann constant,  $\hbar$  is the Planck constant,  $G_N$  is the Newton constant, and

$c$  is the speed of light. The fact that this formula includes  $G_N$ ,  $\hbar$  and  $k_B$  hints some deep relations among gravity, quantum mechanics and statistical mechanics. For this reason, many people believe that black hole entropy is a key concept towards our understanding of quantum gravity.

The AdS/CFT correspondence [6], being one of the most outstanding recent triumphs of string theory, stemmed from research in microscopic counting of black hole entropy. It is a concrete realization of the so called holographic principle that insists equivalence between a gravitational theory in  $d+1$  dimensions and a non-gravitational field theory in  $d$  dimensions. While the AdS/CFT correspondence applies to gravity with a negative cosmological constant, it is not known whether there exists a holographic principle applicable to gravitational theories with a zero or positive cosmological constant. For example, it is thought that the so called dS/CFT correspondence [7], if it really exists, would lead to a non-unitary CFT. Moreover, it is believed that a de Sitter spacetime in string theory is only meta-stable and should decay into more stable configurations after a certain timescale [8] (see also [9]). How this could be understood in a field theory dual to de Sitter gravity is not clear. Since our universe today is thought to have a positive cosmological constant, it is obviously important to investigate whether there really exists a holographic principle applicable to gravitational theories with positive cosmological constant<sup>1</sup>. Note, however, that absence of holographic dual would not necessarily imply inconsistencies of a theory or/and a state.

As a first step towards this outstanding problem, it is intriguing to try to find a way to identify regions in the space of theories and states that do not allow holographic dual descriptions. One possible strategy is to use the generalized second law. In theories and states that have holographic dual descriptions, a black hole is presumably dual to a thermal excitation and, thus, the generalized second law is expected

\*Address correspondence to this author at the Institute for the Physics and Mathematics of the Universe (IPMU), The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8582, Japan; Tel: 04-7136-4946; Fax: 04-7136-4941; E-mail: shinji.mukohyama@ipmu.jp

<sup>1</sup>See [10] for an attempt towards a holographic description of a theory with exactly zero cosmological constant in background with a negative spatial curvature.

to be dual to the ordinary second law of thermodynamics. Therefore, violation of the generalized second law would indicate lack of holographic descriptions since the ordinary second law of thermodynamics should hold in non-gravitational theories. For this reason, by looking for possible violation of the generalized second law, we might be able to find regions in the space of theories and states that do not allow holographic dual descriptions.

This approach could be particularly useful for theories which cannot be embedded in asymptotically anti de Sitter (AdS) spacetime, where the AdS/CFT correspondence is well formulated. If a gravitational theory is formulated in asymptotically AdS spacetime then we can analyze and possibly constrain the theory by using properties of the CFT which is expected to be dual to it. However, if a theory cannot be embedded in asymptotically AdS spacetime, then we cannot use this strategy and should look for other ways. As explained in the previous paragraph, the generalized second law may provide such a possibility.

Actually, it is known that the simplest Higgs phase of gravity called ghost condensate [11-13] cannot be embedded in asymptotically AdS spacetime. The reason is very simple: the coefficient of canonical time kinetic term for excitations around ghost condensate vanishes if the condensate is spacelike. On the other hand, in Minkowski and de Sitter backgrounds, ghost condensate is timelike and gives a healthy time kinetic term to excitations around the condensate. It should also be noted that, because of Jeans-like infrared instability, ghost condensate is not eternal unless cosmological constant is positive. For this reason, ghost condensate presumably provides a good testing ground for our strategy using the generalized second law as a criterion for existence/non-existence of holographic dual.

In the literature there are already three proposals to violate the generalized second law by ghost condensate. The purpose of this paper is to revisit those proposals to see if they really violate the generalized second law. The conclusion is, unfortunately from the viewpoint of developing the strategy explained above, that they do not.

The rest of this paper is organized as follows. Sec. 0 explains a black hole solution in ghost condensate. Sec. 0 revisits the proposal by Eling, Foster, Jacobson and Wall [14] based on a classical process analogous to Penrose process and shows that it does not decrease the total entropy. In Sec. 4 we review the analysis in [15], showing that semiclassical heat flow proposed by Dubovsky and Sibiryakov [16] does not violate the generalized second law. In Sec. 5 we prove a spatially averaged version of the null energy condition, which we conjecture prevents negative energy from violating the generalized second law in a coarse-grained sense. Sec. 6 is devoted to a summary of this paper.

## 2. BLACK HOLE AND GHOST CONDENSATE

The Schwarzschild metric with the horizon radius  $r_g$  is written in the Lemaître reference frame as

$$g_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau, R)} + r^2(\tau, R) d\Omega^2, \quad r(\tau, R) = \left[ \frac{3}{2} \sqrt{r_g} (R - \tau) \right]^{2/3}. \quad (2)$$

The vector  $\xi^\mu$  defined by

$$\xi^\mu = \left( \frac{\partial}{\partial \tau} \right)^\mu + \left( \frac{\partial}{\partial R} \right)^\mu \quad (3)$$

satisfies the Killing equation,  $L_\xi g_{\mu\nu} = 0$ , and is normalized as

$$g_{\mu\nu} \xi^\mu \xi^\nu = -f(r), \quad f(r) \equiv 1 - \frac{r_g}{r}. \quad (4)$$

A particle following a radial geodesic is characterized by the mass  $m$  and the conserved energy  $E$  associated with the Killing vector  $\xi^\mu$  as

$$m^2 = -g^{\mu\nu} p_\mu p_\nu = (p_\tau)^2 - \frac{r}{r_g} (p_R)^2, \quad E = -\xi^\mu p_\mu = -p_\tau - p_R, \quad (5)$$

where  $p_\mu$  is the 4-momentum covector of the particle. These equations have two branches of solutions:

$$p_\tau = -\frac{E}{f} \left[ 1 \pm \sqrt{1-f} \sqrt{1-f \frac{m^2}{E^2}} \right], \quad p_R = -E - p_\tau. \quad (6)$$

For  $E > 0$ , the “+” sign corresponds to out-going geodesics and the “-” sign corresponds to in-coming geodesics. For  $E < 0$ , the “-” sign corresponds to out-going geodesics and the “+” sign corresponds to in-coming geodesics.

Ghost condensate in the Schwarzschild background [17] is approximated by

$$\phi = M^2 \tau, \quad (7)$$

where  $\phi$  is the scalar field responsible for ghost condensate and  $M$  plays the role of the order parameter of spontaneous Lorentz breaking. To be precise, the Schwarzschild metric is just an approximate solution valid within a certain time scale. Actually, ghost condensate slowly accretes towards the black hole and, as a result, the black hole mass evolves as [17]

$$M_{bh} \approx M_{bh0} \approx \left[ 1 + \frac{9\alpha M^2}{4M_{pl}^2} \left( \frac{3M_{pl}^2 \nu}{4M_{bh0}} \right)^{2/3} \right], \quad (8)$$

where  $M_{bh}$  is the Misner-Sharp energy evaluated at the apparent horizon,  $\nu$  is the advanced null time coordinate normalized at infinity (the ingoing Eddington-Finkelstein-type null coordinate),  $M_{bh0}$  is the initial value of  $M_{bh}$  at  $\nu = 0$ , and  $\alpha$  ( $= O(1) > 0$ ) is a coefficient of a higher derivative term. Note that the positivity of  $\alpha$  stems from stability of spatially inhomogeneous excitations of ghost condensate and, thus, must be respected. The same formula (as well as the positivity of  $\alpha$ ) applies to the gauged ghost condensate [18].

## 3. ANALOGUE OF PENROSE PROCESS

Eling, Foster, Jacobson and Wall (EFJW) [14] proposed a process analogous to Penrose process to violate the

generalized second law in theories with Lorentz violation. (See also [19]). By applying the EFJW process to a black hole in ghost condensate, one might think that the generalized second law would be violated. On the contrary, in this section we shall show that the EFJW process is inefficient and always dominated over by accretion of ghost condensate.

EFJW consider two classes A and B of particles. Particles in the class A follow geodesics of the metric  $g_{A\mu\nu}$ , and those in the class B follow geodesics of the metric  $g_{B\mu\nu}$ , where  $g_{A,B\mu\nu}$  are defined up to constant conformal factors as

$$g_{A,B\mu\nu} = -u_\mu u_\nu + c_{A,B}^{-2} (g_{\mu\nu} + u_\mu u_\nu), \quad (9)$$

$c_{A,B}$  ( $c_A \neq c_B$ ) are positive constants representing limits of speed, and  $u_\mu$  is a unit timelike vector representing the preferred time direction. Without loss of generality, we can assume that

$$c_A < c_B. \quad (10)$$

Note that, in the following discussions, ambiguities due to undetermined constant conformal factors can be absorbed into normalization of mass and energy of particles in each class. Moreover, only dimensionless quantities such as  $E/m$  and  $f$  are important in the following discussions and such ambiguities will cancel with each other in any physical statements.

In the case of ghost condensate, the preferred direction is specified as  $u_\mu = \partial_\mu \phi / \sqrt{X}$ , where  $\phi$  is the scalar field responsible for the ghost condensate and  $X = -\partial^\mu \phi \partial_\mu \phi$ . Here, it is assumed that  $\partial_\mu \phi$  is non-vanishing and timelike. The order parameter of the spontaneous Lorentz breaking is  $M$  defined by the vacuum expectation value of  $X$  as

$$\langle X \rangle = M^4. \quad (11)$$

For example, see (7). Any Lorentz breaking effects, such as deviation of  $c_{A,B}$  from unity, are induced by non-vanishing  $M$  and should vanish in the limit  $M^2/M_{Pl}^2 \rightarrow 0$ . Therefore, we have<sup>2</sup>

$$c_{A,B} = 1 + O\left(\frac{M^2}{M_{Pl}^2}\right). \quad (12)$$

This is in accord with the fact that quantum corrections via gravitational interactions generate direct couplings<sup>3</sup> of matter fields to  $\partial_\mu \phi$  unless protected by symmetry and induce Lorentz breaking effects. Since the strength of

gravitational interaction is  $G_N = M_{Pl}^{-2}$  and the background value of  $\partial_\mu \phi$  is proportional to  $M^2$ , such induced effects must be proportional to some positive powers of  $M^2/M_{Pl}^2$  at leading order. The constant conformal factors and the normalization of mass and energy, mentioned in the previous paragraph, are also  $1 + O(M^2/M_{Pl}^2)$ .

We suppose that  $g_{\mu\nu}$  is the Schwarzschild metric shown in (2) and that  $u_\mu = \partial_\mu \tau$  (see (7)). In this case, both  $g_{A\mu\nu}$  and  $g_{B\mu\nu}$  are Schwarzschild metric with different horizon radii:

$$g_{A,B\mu\nu} dx^\mu dx^\nu = -d\tau^2 + \frac{r_{gA,B} dR^2}{r_{A,B}(\tau, R)} + r_{A,B}^2(\tau, R) d\Omega^2, \quad (13)$$

where

$$r_{A,B}(\tau, R) = c_{A,B}^{-1} r(\tau, R) = \left[ \frac{3}{2} \sqrt{r_{gA,B}} (R - \tau) \right]^{2/3}, \quad (14)$$

$$r_{gA,B} = c_{A,B}^{-3} r_g.$$

The vector  $\xi^\mu$  defined by (3) still satisfies the Killing equation for  $g_{A,B\mu\nu}$ ,  $L_\xi g_{A,B\mu\nu} = 0$ , and is normalized as

$$g_{A,B\mu\nu} \xi^\mu \xi^\nu = -f_{A,B}(r), \quad f_{A,B}(r) \equiv 1 - \frac{r_{gA,B}}{r_{A,B}} = 1 - c_{A,B}^{-2} \frac{r_g}{r}. \quad (15)$$

Let us prepare two in-coming massive particles outside the horizons: one in the class A with mass  $m_A$  and the other in the class B with mass  $m_B$ . Since they are prepared outside the horizon, they have positive energies,  $E_A > 0$  and  $E_B > 0$ . We suppose that these particles meet at  $r = r_0$  satisfying  $f_A(r_0) < 0 < f_B(r_0)$ , i.e. in the region between the horizon for the class A and that for the class B, and split into two massless particles: one in the class A in-coming with negative energy  $E_{A'} < 0$  and the other in the class B out-going with positive energy  $E_{B'} > 0$ . Note that a particle in the class A can have negative energy at  $r = r_0$  since it is inside the horizon for this class. On the other hand, a particle in the class B cannot have negative energy at  $r = r_0$  since it is outside the horizon for this class. Since the total energy conserves and  $E_{A'}$  is negative,  $E_{B'} = E_A + E_B - E_{A'}$  is larger than the initial total energy  $E_A + E_B$ . This is the EFJW process.

Actually, the EFJW process is kinematically forbidden unless  $E_A/m_A = O(\sqrt{f_B}) \ll 1$ . To see this, note that EFJW invokes the conservation of momentum covector, which is summarized as

$$E_A + E_B = E_{A'} + E_{B'},$$

$$P_A + P_B = P_{A'} + P_{B'}. \quad (16)$$

<sup>2</sup>Since  $M_{Pl}$  is defined by  $G_N = M_{Pl}^{-2}$ , odd powers of  $M_{Pl}$  do not show up. Odd powers of  $M$  do not show up either since  $M$  is defined by  $\langle \partial_\mu \phi \rangle \propto M^2$ .

<sup>3</sup>Note that  $\phi$  appears only via its derivatives because of the shift symmetry.

Here,

$$p_{A,B} = -\frac{E_{A,B}}{f_{A,B}} \left[ 1 - \sqrt{1-f_{A,B}} \sqrt{1-f_{A,B} \frac{m_{A,B}^2}{E_{A,B}^2}} \right],$$

$$p_{A',B'} = -\frac{E_{A',B'}}{f_{A,B}} \left[ 1 + \sqrt{1-f_{A,B}} \right], \quad (17)$$

and  $f_{A,B} \equiv f_{A,B}(r_0)$ . Note that (12) and  $f_A < 0 < f_B$  imply that

$$f_{A,B} = O\left(\frac{M^2}{M_{Pl}^2}\right). \quad (18)$$

It is easy to solve equations (16) with respect to  $E_{A'}$  as

$$E_{A'} = \frac{C_A E_A + C_B E_B}{C}, \quad (19)$$

where

$$C_A = |f_A| + f_B + |f_A| \sqrt{1-f_B} - f_B \sqrt{1+|f_A|} \sqrt{1+|f_A| \frac{m_A^2}{E_A^2}},$$

$$C_B = |f_A| \sqrt{1-f_B} \left[ 1 + \sqrt{1-f_B \frac{m_B^2}{E_B^2}} \right],$$

$$C = |f_A| \left[ 1 + \sqrt{1-f_B} \right] + f_B \left[ 1 + \sqrt{1+|f_A|} \right]. \quad (20)$$

Here, we have re-expressed  $f_A$  as  $-|f_A|$ . EFJW suppose that  $E_{A,B} > 0$  and  $E_{A'} < 0$ . This indeed requires that  $C_A < 0$ , since  $C_B$  and  $C$  are positive definite. This necessary condition is rewritten as

$$\frac{E_A^2}{f_B m_A^2} < \frac{1+|f_A|}{2(1+\sqrt{1-f_B}) - f_B + \frac{|f_A|}{f_B} (1+\sqrt{1-f_B})^2} < O(1), \quad (21)$$

and shows that the EFJW process is kinematically forbidden unless  $E_A / m_A = O(\sqrt{f_B}) \ll 1$ . This also shows that the initial massive particle in the class A must be released from a point very close to the horizon where  $r = r_g \times [1 + O(\sqrt{f_B})]$ . Therefore, before starting the process, we need to keep such a particle at rest in the vicinity of the horizon.

It is also easy to see that (19) combined with  $E_A > 0$  and  $E_B > 0$  implies that

$$-\frac{E_{A'}}{\sqrt{f_B} m_A} < \frac{\sqrt{f_B} \sqrt{1+|f_A|}}{C} \sqrt{\frac{E_A^2}{m_A^2} + |f_A|}. \quad (22)$$

This, combined with (21), leads to

$$-\frac{E_{A'}}{\sqrt{f_B} m_A} < O(1). \quad (23)$$

This means that the amount of negative energy carried by the final massless particle in the class A is rather low:  $|E_{A'}| / m_A = O(\sqrt{f_B}) \ll 1$ .

EFJW treats all particles participating the process as test particles. Therefore, in order to justify this treatment their gravitational backreaction to the geometry must be small enough. Let  $\Delta r = (|f_A| + f_B) r_g$  be the difference between horizon radii for the class A and the class B. The corresponding proper distance is  $\Delta l = 2\sqrt{r_g \Delta r}$ . In order to justify the test particle treatment,  $\Delta l$  must be sufficiently longer than the gravitational radius of each massive particles. This requires that

$$m_{A,B} \ll \frac{1}{2} M_{Pl}^2 \Delta l \approx 2M_{bh} \sqrt{|f_A| + f_B}. \quad (24)$$

As already stated at the end of the paragraph before the last, before starting the process, we need to keep the initial particle with mass  $m_A$  at rest at  $r = r_g \times [1 + O(\sqrt{f_B})]$ . By demanding that this initial condition should not disturb the geometry in the vicinity of the horizon, we obtain a similar condition on  $m_A$ .

$$m_A \ll M_{bh} \times O(\sqrt{f_B}). \quad (25)$$

Using (18) in (23) and (24) (or (25)), we obtain

$$|E_{A'}| \ll M_{bh} \times O\left(\frac{M^2}{M_{Pl}^2}\right). \quad (26)$$

This shows that the amount of negative energy sent to the black hole by the EFJW process is rather low. As EFJW states, it takes time of order  $r_g$  to perform this process since particles need to travel this distance<sup>4</sup>. During this time scale, ghost condensate accretes into the black hole, according to the formula (8). This amounts to the increase of the black hole mass given by

$$\Delta M_{bh}|_{v=r_g} \sim M_{bh} \times \frac{M^2}{M_{Pl}^2}. \quad (27)$$

This is much larger than the amount of negative energy (26). Therefore, we conclude that the EFJW process is too inefficient to decrease black hole entropy in ghost condensate. This conclusion trivially extends to gauged ghost condensate since the accretion rate is the same.

#### 4. SEMICLASSICAL HEAT FLOW

For the Schwarzschild background (2), the effective metric  $g_{A\mu\nu}$  for particles in the class A and the effective metric  $g_{B\mu\nu}$  for particles in the class B have different surface gravity and, thus, different Hawking temperatures  $T_{bhA}$  and  $T_{bhB}$ . Without loss of generality, we can assume that  $T_{bhA} < T_{bhB}$ .

By using the semiclassical heat flow due to Hawking radiation, Dubovsky and Sibiryakov (DS) [16] proposed a

<sup>4</sup>As already stated, the initial massive particle in the class A should be kept at rest near the horizon. We need to supply it from somewhere else and somehow stop it near the horizon. This already takes time scale of order  $r_g$ .

process to violate the generalized second law in ghost condensate. DS consider two shells surrounding the black hole, one with temperature  $T_{shellA}$  interacting with particles in the class A only and the other with temperature  $T_{shellB}$  interacting with particles in the class B only. By tuning these temperatures of the shells, one can satisfy

$$T_{bhA} < T_{shellA} < T_{shellB} < T_{bhB}, \quad (28)$$

and ensure that the net energy flux from the shell A to the black hole is equal to the net energy flux from the black hole to the shell B. In this case, energy is transferred from the shell A to the shell B via the black hole while black hole mass remains unchanged. Since the shell A has lower temperature than the shell B, this process appears to violate the generalized second law. This is the DS process.

As shown in [15], however, the DS process is suppressed by the factor  $M^2 / M_{pl}^2$  and, as a result, is much slower than the Jeans instability of ghost condensate. Indeed, it is even slower than accretion of ghost condensate<sup>5</sup>. Therefore, black hole entropy increases due to accretion before the DS process starts operating. Here, we shall briefly review the argument of [15].

As already stated, the scale  $M$  is the order parameter of spontaneous Lorentz breaking and the Lorentz symmetry should recover in the  $M^2 / M_{pl}^2 \rightarrow 0$  limit. This is the reason why the deviation of limits of speed from unity is suppressed by  $M^2 / M_{pl}^2$ , as shown in (12). This implies that differences among various temperatures in (28) are suppressed by  $M^2 / M_{pl}^2$ . In particular, we have

$$|T_{shellA,B} - T_{bhA,B}| = T_{bh} \times O\left(\frac{M^2}{M_{pl}^2}\right), \quad (29)$$

where  $T_{bh}$  is the temperature of the metric  $g_{\mu\nu}$ , and the net energy flux from the shell A or B to the black hole is

$$F_{shell \rightarrow bh} \sim \pm T_{bh}^2 \times O\left(\frac{M^2}{M_{pl}^2}\right), \quad (30)$$

where the “+” sign is for the shell A and the “−” sign is for the shell B. Note that the net energy flux from each shell to the black hole vanishes when temperatures of the black hole and the shell agree, i.e. in the limit  $M^2 / M_{pl}^2 \rightarrow 0$ .

DS argue that the sum of entropy of the shell A and entropy of the shell B decreases since energy moves from the shell A with lower temperature to the shell B with higher temperature. However, the temperature difference is again suppressed by  $M^2 / M_{pl}^2$  and, thus, the rate of decrease of shells' entropy is

$$\frac{dS_{shells}}{dt} = \left(\frac{1}{T_{shellB}} - \frac{1}{T_{shellA}}\right) |F_{shell \rightarrow bh}| \sim -\frac{|F_{shell \rightarrow bh}|}{T_{bh}} \times O\left(\frac{M^2}{M_{pl}^2}\right). \quad (31)$$

<sup>5</sup>In [15], comparison with the time scale of accretion was explicitly illustrated for gauged ghost condensate only, but it holds also for ungauged ghost condensate since the quoted accretion rate is common for gauged and ungauged ghost condensate.

Combining this with (30), we obtain

$$\frac{dS_{shells}}{dt} \sim -T_{bh} \times O\left(\frac{M^4}{M_{pl}^4}\right). \quad (32)$$

This is highly suppressed. Indeed, it takes the time scale  $t_{DS}$  defined by

$$t_{DS} \sim T_{bh}^{-1} \times \frac{M_{pl}^4}{M^4} \quad (33)$$

for the DS process to decrease shells' entropy just by one unit.

Now, the formula (8) tells us that the black hole entropy significantly increases by accretion of ghost condensate in the time scale  $t_{DS}$ . On the other hand, the shells' entropy can decrease just by one unit in this time scale. Therefore, the total entropy including the black hole entropy increases and the DS process does not violate the generalized second law [15]. This conclusion trivially extends to gauged ghost condensate since the accretion rate is the same.

## 5. NEGATIVE ENERGY

As a yet another proposal to violate the generalized second law, let us consider negative energy carried by excitations of ghost condensate [20]. In this section we shall see that an averaged energy condition holds, and we conjecture it protects the generalized second law.

Let us consider the action of the form

$$I = \int dx^4 \sqrt{-g} P(X), \quad X = -\partial^\mu \phi \partial_\mu \phi. \quad (34)$$

The stress-energy tensor is

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}, \quad (35)$$

where

$$\rho = 2P'X - P, \quad u_\mu = \frac{\partial_\mu \phi}{\sqrt{X}}. \quad (36)$$

The equation of motion is

$$\nabla^\mu J_\mu = 0, \quad (37)$$

where

$$J_\mu \equiv -2P' \partial_\mu \phi \quad (38)$$

is the current associated with the shift symmetry and the corresponding charge is conserved.

Ghost condensate is characterized by a non-vanishing timelike vacuum expectation value of  $\partial_\mu \phi$  as in (11). In the language of the action (34), it corresponds to the value of  $X$  where  $P' = 0$ . Actually,  $P' = 0$  is a dynamical attractor of the system in an expanding universe. In the flat Friedmann-Robertson-Walker (FRW) background the equation of motion (37) for a homogeneous  $\phi$  is  $\partial_t(a^3 P' \partial_t \phi) = 0$  and implies that

$$P' \partial_t \phi \propto \frac{1}{a^3} \rightarrow 0 \quad (a \rightarrow \infty), \quad (39)$$

where  $a$  is the scale factor of the universe. There are two choices:  $P' \rightarrow 0$  or  $\partial_t \phi \rightarrow 0$ . The later corresponds to the trivial Lorentz invariant background and the former corresponds to the ghost condensate. The ghost condensate is, in this sense, a dynamical attractor of the system. Fluctuations around the ghost condensate background obtain a time kinetic term with the correct sign if  $P'' > 0$ . On the other hand, the leading spatial kinetic term comes from higher derivative terms such as  $(W\phi)^2$ . For the correct sign of the higher derivative term, those fluctuations are stable and have a healthy low energy effective field theory. Hence, ghost condensate is characterized by the background  $X = M^4$  ( $> 0$ ) satisfying  $P'(M^4) = 0$  and  $P''(M^4) > 0$ . The scale  $M$  is the order parameter of spontaneous Lorentz breaking and also plays the role of the cutoff scale of the low energy effective field theory for excitations of ghost condensate<sup>6</sup>.

In the ghost condensate background,  $\rho + P = 2P'X$  vanishes and thus  $T_{\mu\nu}$  acts as a cosmological constant. This is the reason why Minkowski and de Sitter spacetimes are exact solutions in ghost condensate<sup>7</sup>. If we consider fluctuations around the ghost condensate background then we notice that  $\rho + P = 2P'X; 2M^4 P''(M^4) \delta X + O(\delta X^2)$ , where  $\delta X = X - M^4$ . This means that  $\rho + P$  is positive for  $\delta X > 0$  and negative for  $\delta X < 0$ . Therefore, the null energy condition can be violated by excitations of ghost condensate.

In the following, however, we shall prove a spatially averaged version of the null energy condition.

The Lagrangian  $P$  is expanded as

$$P = M^4 \left[ p_0 + \frac{1}{2} p_2 \chi^2 + O(\chi^3) \right], \quad \chi \equiv \frac{X}{M^4} - 1, \quad (40)$$

where  $p_0 = P(M^4)/M^4$  and  $p_2 = M^4 P''(M^4) = O(1) > 0$ . Thus, we obtain

$$\rho + P - M^2 J_\mu u^\mu = 2P'X \left( 1 - \frac{M^2}{\sqrt{X}} \right) = M^4 [p_2 \chi^2 + O(\chi^3)]. \quad (41)$$

In the regime of validity of the effective field theory,  $|\chi| \ll 1$  and the right hand side is non-negative. Therefore, by integrating ((41)) over a spacelike hypersurface orthogonal to  $u^\mu$ , we obtain

$$\int d\Sigma(\rho + P) \geq M^2 Q, \quad (42)$$

where  $Q$  is the conserved charge associated with the shift symmetry:

$$Q = \int d\Sigma J_\mu u^\mu. \quad (43)$$

As stated in the third paragraph of this section,  $P' = 0$  is a dynamical attractor in an expanding universe. Thus, it is very natural to set  $Q = 0$ . Moreover, if  $Q < 0$  today then  $P'$  was negative with large absolute values in the past. Since a large negative  $P'$  leads to UV instabilities of the system, negative  $Q$  is strongly disfavored. Actually, with the exact shift symmetry, it is impossible to have a negative  $Q$  without introducing UV instabilities<sup>8</sup>. Therefore, it is necessary to assume that the shift charge  $Q$  initially vanishes or starts with a positive value<sup>9</sup>. In this case we obtain

$$\int d\Sigma(\rho + P) \geq 0. \quad (44)$$

This is the averaged null energy condition.

The averaged energy condition states that negative energies are always accompanied by larger positive energies. This is somehow similar to the so called quantum inequalities and the quantum interest conjecture [23] in ordinary quantum field theory: even in Minkowski spacetime the ordinary field theory can have negative local energy, but negative energy is always accompanied by larger positive energy. Since direct couplings between ghost condensate and matter fields are suppressed by the Planck scale (cf. the third paragraph of Sec. 0), excitations of ghost condensate interact with ordinary matter only gravitationally. This implies that those positive and negative energies cannot be separated from each other by hand. Therefore, although one could decrease black hole entropy by gravitationally exciting a lump of negative energy and sending it into a black hole, larger positive energy should follow it and eventually increase the black hole entropy. For this reason, we conjecture that the averaged energy condition protects the generalized second law in a coarse-grained sense<sup>10</sup>.

## 6. SUMMARY

We have revisited three proposals to violate the generalized second law by ghost condensate: (i) analogue of Penrose process, (ii) semiclassical heat flow, and (iii) negative energy. The proposals (i) and (ii) are based on Lorentz breaking effects, by which particles of different species can have different limits of speed. We have shown that processes in both (i) and (ii) are always slower than accretion of ghost condensate and cannot decrease the total entropy before the accretion increases the entropy. We have also proved an averaged null energy condition, which we

<sup>6</sup>Extra modes due to higher time derivative terms have frequencies around or above  $M$  and, thus, are outside the regime of validity of the low energy effective field theory.

<sup>7</sup>Anti de Sitter spacetime is not a solution in ghost condensate with higher derivative terms, essentially because there is no flat FRW slicing in anti de Sitter spacetime. Spacelike condensate leads to instability of excitations around the condensate.

<sup>8</sup>If the shift symmetry is softly broken then it is possible to have a negative  $Q$  without instabilities in expanding universe [21].

<sup>9</sup>Negative energy solutions presented in [22] are inconsistent with any initial conditions with  $Q \geq 0$  and thus are excluded by this condition.

<sup>10</sup>In ordinary thermodynamics, entropy can fluctuate both upwards and downwards by order unity but will eventually increase. Thus, in ordinary thermodynamics the second law holds only in a coarse-grained sense.

conjectured prevents the proposal (iii) from violating the generalized second law in a coarse-grained sense.

### ACKNOWLEDGEMENTS

The author would like to thank Brian Feldstein and Simeon Hellerman for useful discussions. The work of the author was supported in part by MEXT through a Grant-in-Aid for Young Scientists (B) No. 17740134, by JSPS through a Grant-in-Aid for Creative Scientific Research No. 19GS0219, and by the Mitsubishi Foundation. This work was supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

### REFERENCES

- [1] Hawking SW. Gravitational radiation from colliding black holes. *Phys Rev Lett* 1971; 26: 1344.
- [2] Bekenstein JD. Black holes and entropy. *Phys Rev* 1973; D7: 2333.
- [3] Hawking SW. Particle creation by black holes. *Commun Math Phys* 1975; 43: 199 [Erratum-ibid. 46, 206 (1976)].
- [4] Bekenstein JD. Generalized second law of thermodynamics in black hole Physics. *Phys Rev* 1974; D9: 3292.
- [5] Wall AC. Ten proofs of the generalized second law. *JHEP* 2009; 0906: 021 [arXiv:0901.3865 [gr-qc]].
- [6] Maldacena JM. The large N limit of superconformal field theories and super-Gravity. *Adv Theor Math Phys* 1998; 2: 23; [*Int J Theor Phys* 38, 1113 (1999)] [arXiv:hep-th/9711200].
- [7] Strominger A. The dS/CFT correspondence. *JHEP* 2001; 0110: 034 [arXiv:hep-th/0106113].
- [8] Kachru S, Kallosh R, Linde A, Trivedi SP. De Sitter vacua in string Theory. *Phys Rev* 2003; D 68: 046005 [arXiv:hep-th/0301240].
- [9] Goheer N, Kleban M, Susskind L. The trouble with de Sitter space. *JHEP* 2003; 0307: 056 [arXiv:hep-th/0212209].
- [10] Sekino Y, Susskind L. Census taking in the hat: FRW/CFT duality. arXiv:0908.3844 [hep-th].
- [11] Arkani-Hamed N, Cheng HC, Luty MA, Mukohyama S. Ghost condensation and a consistent infrared modification of gravity. *JHEP* 2004; 0405: 074 [arXiv:hep-h/0312099].
- [12] Arkani-Hamed N, Cheng HC, Luty MA, Mukohyama S, Wiseman T. Dynamics of gravity in a higgs phase. *JHEP* 2007; 0701: 036 [arXiv:hep-ph/0507120].
- [13] Mukohyama S. Towards a higgs phase of gravity in string theory. *JHEP* 2007; 0705: 048 [arXiv:hep-th/0610254].
- [14] Eling C, Foster BZ, Jacobson T, Wall AC. Lorentz violation and perpetual motion. *Phys Rev* 2007; D 75: 101502 [arXiv:hep-th/0702124].
- [15] Mukohyama S. Ghost condensate and generalized second law. arXiv:0901.3595[hep-th].
- [16] Dubovsky SL, Sibiryakov SM. Spontaneous breaking of Lorentz invariance, black holes and perpetual motion of the 2nd kind. *Phys Lett* 2006; B 638: 509 [arXiv:hep-th/0603158].
- [17] Mukohyama S. Black holes in the ghost condensate. *Phys Rev* 2005; D 71: 104019 [arXiv:hep-th/0502189].
- [18] Cheng HC, Luty MA, Mukohyama S, Thaler J. Spontaneous Lorentz breaking at high energies. *JHEP* 2006; 0605: 076 [arXiv:hep-th/0603010].
- [19] Kant E, Klinkhamer FR, Schreck M. Lorentz violation and black-hole thermodynamics: Compton scattering process. arXiv:0909.0160 [hep-th].
- [20] Arkani-Hamed N. Talk at Perimeter Institute conference IR Modifications. November 2006.
- [21] Creminelli P, Luty MA, Nicolis A, Senatore L. Starting the universe: Stable violation of the null energy condition and non-standard cosmologies. *JHEP* 2006; 0612: 080 [arXiv:hep-th/0606090].
- [22] Feldstein B. Spontaneous Lorentz violation, negative energy and the second law of thermodynamics. arXiv:0904.1212 [hep-th].
- [23] Ford LH, Roman TA. The quantum interest conjecture. *Phys Rev* 1999; D 60: 104018 [arXiv:gr-qc/9901074].

Received: September 6, 2009

Revised: September 11, 2009

Accepted: September 11, 2009

© Shinji Mukohyama; Licensee *Bentham Open*.

This is an open access article licensed under the terms of the Creative Commons Attribution Non-Commercial License (<http://creativecommons.org/licenses/by-nc/3.0/>) which permits unrestricted, non-commercial use, distribution and reproduction in any medium, provided the work is properly cited.