Retraction Notice

As per Bentham Science's policy, the following article has been retracted at the request of the Author and approval of the Editor-in-Chief of "The Open Astronomy Journal":

Title: "Advances in Black Hole Gravitational Physics and Cold Dark Matter Modelling. The Gravity of Dark Matter"

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According to Dr. Andrew Worsley (author of the article) the reason for retraction is that the paper itself is very condensed and has a number of new ideas in it- which would be better set out separately.

The intention is to withdraw the paper and submit 4 or perhaps 5 papers in the journal in its stead, each then original in their own right and each setting out the new ideas separately.

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Advances in Black Hole Gravitational Physics and Cold Dark Matter Modelling. The Gravity of Dark Matter

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Abstract: One of the most important issues of current physics relates to the presence of cosmological cold dark matter (CDM). In this paper CDM at the Galactic centre is modelled using a previously reformulated type of dynamic Newtonian advanced gravity (DNAg). This DNAg gives results which are technically exactly the same as the general theory of relativity (GTR), in the low and medium mass densities. However, in the high mass densities of supermassive black holes, it predicts a dynamic increase in the force of gravity, which explains the apparent presence of CDM at the galactic centre, and at the cosmological level. Thus on the galactic and cosmological mass scales, these equations allow the apparent presence of CDM to be modelled, and accurately calculated. These dynamic Newtonian advanced gravitational equations also offer readily testable gravitational predictions.

Key Words: Dark matter, gravitation, black hole physics, large scale structure of the Universe.

1. INTRODUCTION

Cosmological Dark Matter

The presence of cosmological cold dark matter (CDM) is important in the understanding of the large scale structure of the Universe. Some models collectively introduce a solution to the presence of CDM, by the reformulation of the laws gravity [1]. One such model is known as modified Newtonian dynamics (MOND), and is a candidate theory for the missing dark matter of galaxies [2]. One other possibility is that there is some other form of post Newtonian gravity which is not dissimilar to the general theory of relativity (GTR), that can explain CDM. One such relativistic extension of MOND is known as Tensor Vector Scalar theory, TeVeS [3].

Here, a more recently described mathematical model of gravity, called dynamic Newtonian advanced gravitation (DNAg), is examined as having a potential to account for presence of dark matter [4]. This advanced Newtonian gravity mathematically predicts the presence of a dynamic increase in the force of gravity around gravitational bodies. Importantly, using worked examples it has been possible to show that the results of this advanced Newtonian gravity, are technically exactly the same as GTR [4, 5], particularly in low and medium mass density objects (see Appendix). Equally in moderate density objects such as binary pulsars these advanced equations agree very closely with the data analysis on these systems [4-6].

Crucially, here it is shown that the equations for DNAg also allow readily testable predictions relating to the gravitation of high mass density objects, such as black holes. Indeed, this modelling leads to a greater understanding of the apparent presence of dark matter at galactic centres. DNAg allows the accurate calculation of the dynamic increase in the force of gravity around these supermassive black holes. Given the association between galactic supermassive black holes and the presence of CDM [1], this represents a model of gravitation, which can thus account for the apparent presence of CDM at the galactic core. Moreover, this increase in gravity can be tested by observing the predicted increase in the effects of gravity in the proximity of a supermassive black hole.

As previously demonstrated, this advanced gravity represents a reformulation of the description of gravity from the general relativistic curvature of space-time, back into describing a dynamic Newtonian increase in the force of gravity [4]. This is based on the algebraic term for the perihelion advance of Mercury, given by Straumann [6, 7]:

Straumann's equation

$$\Delta\varphi \approx \tan\Delta\varphi = \frac{6\pi m^2}{L^2} \tag{1}$$

where $m = GM/c^2$, and $a(1\text{-}e^2) = L^2/m$, G is the gravitational constant and c is the speed of light, a is the semi-major axis and e is the eccentricity

Importantly, Straumann's equation leads to the reformulation of gravity in terms of an adaptation to the Newtonian *force* of gravity [4, 5]. However, strictly speaking the equations required should relate to the acceleration experienced by a mass in a gravitational field, rather than a force. Thus, whilst the equation for the force of gravity remains very useful, the acceleration due to gravity is perhaps more of a reflection of the actual effects of a gravitating body, on the surrounding space-time itself. Particularly as the effects

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of this acceleration then become (largely) independent of the particular mass that is being attracted. This is very useful when considering the increased effects of advanced Newtonian gravity, particularly in relationship to the presence of cosmological CDM.

It has been possible to show that the results of advanced Newtonian gravity, using the equations for acceleration due to gravity, are technically exactly the same as GTR [4, 5], where GTR has been extensively tested (see Appendix). Accurate results are also achieved with binary pulsar data [4-6]. However, in high mass density objects such as black holes, advanced Newtonian gravity predicts a dynamic increase in the acceleration due to gravity, in the proximity of a black hole. Moreover, the association between the presence of CDM at the galactic core is then explained by the presence of a supermassive black hole at the galactic centre which exerts this dynamic increase in the acceleration due to gravity.

Additionally, the total mass contained within a measured volume of space-time, if sufficiently large, will also produce an extra gravitational acceleration, within that volume. As regards the apparent presence of cosmological dark matter, the apparent additional mass is dependant upon the total amount of baryonic mass contained within the observable Universe. Thus an increased acceleration of gravity, and in turn the apparent presence of dark matter, will be deduced from any vantage point within the observable Universe. By this token it has already been possible to model the apparent cosmological CDM of the Universe, and derive an accurate result for the total of this cosmological dark matter [4, 5].

In this paper it is possible to go further by modelling the gravity at the centre of the galaxy, where it has been shown that a super-massive black hole exists [8]. Using similar measurements it is clear that future experiments will enable a thorough test of these advanced Newtonian equations and the expected increase in gravity, using stellar infrared interferometry. This will enable experiments to confirm the presence of a dynamic increase in the force of gravity in the proximity of a black hole, by measuring the orbit of stellar objects around the galactic supermassive black hole.

2. METHODS

Mathematical Methods

All mathematical calculations follow strict standard algebraic and standard mathematical rules. The gravitational equations used [Eqs. (3 & 4)], are those of the previously published Dynamic Newtonian Advanced gravity (DNAg), as detailed in the introduction [4, 5].

The principle physics proofs are based upon standard physical formulae. The proofs offer a high degree of agreement with currently known values in GTR. The paper details worked examples which compare DNAg to full general relativity (see Appendix). These technically give exactly the same answers as full GTR, in low and medium mass density objects. The paper also proposes observational experimental methods for the experimental verification of the findings using observations of black holes, as listed in the results and conclusions (see Table **1**).

3. RESULTS

Dynamic Newtonian Advanced Gravity (DNAg)

In this paper, we address the question: what is the fundamental nature of the cold dark matter (CDM) at the centre of galaxies, and can this be understood and resolved by an advanced adaptation of gravitation? Some authors have written on this subject but the answer remains obscure [9-12]. It has previously been shown that the cosmological CDM within the cosmological event horizon, can arise from a reformulation of the equations of gravity using a dynamic Newtonian advanced formulation of gravity (DNAg) [4, 5]. In order to model CDM the advanced Newtonian force equations have been reformulated, to describe an increase in the acceleration due to gravity [Eqs. (2 & 3)].

Dynamic Newtonian advanced gravitational (DNAg) acceleration equations (a_g)

$$a_g = \frac{GM}{R^2}\left[1+3GM/Rc^2\right]^2 \tag{2}$$

Averaged for elliptical orbits,

$$a_g = \frac{GM}{R^2}\left[1+3GM/\ell c^2\right]^2 \tag{3}$$

where M is the larger mass, c is the speed of light and G the gravitational constant, R is the distance, (normally taken as the radius) and $\ell = a\left(1-e^2\right)$, where a is the semi major axis and e is the eccentricity.

According to these equations, this extra increase in acceleration occurs according to the mass and density of a gravitating body. It has thus been shown that these equations technically give exactly the same answers as full general relativity, in low and medium mass density objects, as seen with the advance in the perihelion of Mercury, of 42.98 arc sec per century [4, 5]. For further proof, we can do the same calculation, for the change in the perihelion of the Earth around the Sun and we again, get exactly the same answer as general relativity, 3.84 arc sec per century (see Appendix). Recent evidence confirms that this is the same as the experimentally determined advance in the perihelion of Earth, 3.84'' ± 0.1 arc sec/cy [13]. A similar calculation may be performed for any gravitational body in this mass density range. Recent experiments have been able to estimate the advance in the perihelion of Mars, and we again get a result which agrees with general relativity and the experimental advance in the perihelion of Mars, 1.35 ± 0.1 arc sec/cy [13]. To calculate this with GTR, would normally take an in depth knowledge of tensor calculus and reams of calculations, a few lines of algebra can achieve the same results, using advanced Newtonian gravity (see Appendix).

Similarly accurate results can be obtained for binary pulsars with advanced Newtonian gravity [4, 5]. In advanced Newtonian gravity, the parameterization is fundamentally mathematically the same as the DD model [14, 15]. In his recent textbook, Straumann [6], examined the so called DD model for the binary pulsar PSR B 1534 +12. Using data from Stairs, *et al.* [16], an analysis of the data published by Straumann showed that the results are strikingly accurate for the gravitational parameters examined. Thus, the results from binary pulsar data are also in agreement with the DNAg presented here. Indeed a re-analysis of the results presented for gravitational radiation damping, tend to favour the DNAg model presented here [4-6].

By the same mechanism these results are also able to explain the apparent presence of cosmological CDM. Firstly let us address the problem at the event horizon of a black hole singularity. A black hole conventionally becomes an infinitely dense singularity. There is also effectively an infinite force at the event horizon [17]. However, in advanced Newtonian gravity, the acceleration generated at the Schwarzschild radius would only be the normal acceleration due to Newtonian gravity, multiplied by a constant factor. This factor would be 6.25 that of normal Newtonian gravity, see Eq. (5) below.

Acceleration due to Gravity at the Schwarzschild Radius (R_s*)*

$$a_g = \frac{GM}{R^2}\left[1 + 3GM/Rc^2\right]^2 \quad (2)$$

and

$$R_S = 2GM/c^2 \quad (4)$$

$$a_g = \frac{GM}{R^2}\left[1 + \tfrac{3}{2}\right]^2 \; = \; a_g = \frac{GM}{R^2} x\left[6.25\right] \quad (5)$$

where M is the larger mass, m is the smaller mass, c is the speed of light and G the gravitational constant, R is the distance, (normally taken as the radius)

It is also possible to predict that the forces of gravity and the acceleration produced in the proximity of the black hole, and at the Schwarzschild radius. The increases above that of normal gravity seen in DNAg, is dynamic. So the increased gravity experienced as the object approaches a black hole, rises in a dynamic fashion (see Table **1**).

Importantly, the same factor of increased acceleration applies, however big the mass of the black hole is. Importantly, at 1.0 Schwarzschild radius the force would be exactly 6.25 multiplied by that of standard Newtonian gravity, as in Eq. (5). Clearly, the effect could be most readily tested in the case of the supermassive black hole at the centre of the Milky Way Galaxy.

Interestingly, it is possible to show that these advanced Newtonian equations allow the exact calculation of the increased forces of gravity seen in the proximity of a black hole and at the event horizon. This is of particular relevance with regards the supermassive black hole that is present in the centre of galaxies, as this additional gravity can therefore explain the predominance of apparent CDM at the galactic core. This can be tested and demonstrated with a straightforward algebraic treatment of gravity, using Eq. (3).

Using this equation, the dynamic increase in gravity depends on the proximity to the Schwarzschild radius (R_S). For an object at interstellar distances then gravity would appear to be that of conventional gravity. However, the mathematics of dynamic Newtonian advanced gravitation indicates an

Table 1. Acceleration Due to Gravity (a_g), in the Proximity of a Black Hole and Within a Black Hole, Using Dynamic Newtonian Advanced Gravitation (DNAg)

1). $R = 48R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + \tfrac{1}{32}\right]^2$	$= \frac{GM}{R^2} x\left[1.0635\right]$
2). $R = 24R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + \tfrac{1}{16}\right]^2$	$= \frac{GM}{R^2} x\left[1.1289\right]$
3). $R = 12R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + \tfrac{1}{8}\right]^2$	$= \frac{GM}{R^2} x\left[1.2656\right]$
4). $R = 6R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + \tfrac{1}{4}\right]^2$	$= \frac{GM}{R^2} x\left[1.5625\right]$
5). $R = 3R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + \tfrac{1}{2}\right]^2$	$= \frac{GM}{R^2} x\left[2.25\right]$
6). $R = 1.5R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + 1\right]^2$	$= \frac{GM}{R^2} x\left[4\right]$
7). $R = 1R_S$:	$a_g = \frac{GM}{R^2} x\left[1 + \tfrac{3}{2}\right]^2$	$= \frac{GM}{R^2} x\left[6.25\right]$
8). $R = 0.75R_S$:	$a_g = \frac{GM}{R^2} x\left[3\right]^2$	$= \frac{GM}{R^2} x\left[9\right]$
9). $R = 0.5R_S$:	$a_g = \frac{GM}{R^2} x\left[4\right]^2$	$= \frac{GM}{R^2} x\left[16\right]$
10). $R = 0.375R_S$:	$a_g = \frac{GM}{R^2} x\left[5\right]^2$	$= \frac{GM}{R^2} x\left[25\right]$
11). $R = 0.3R_S$:	$a_g = \frac{GM}{R^2} x\left[6\right]^2$	$= \frac{GM}{R^2} x\left[36\right]$
12). $R = .0.25R_S$:	$a_g = \frac{GM}{R^2} x\left[7\right]^2$	$= \frac{GM}{R^2} x\left[49\right]$

Where *R* is the radius of the in-falling object from the black hole, R_S is the Schwarzschild radius, *G* is the gravitational constant, *M* is the mass of the black hole. Answers given to 4 decimal places.

increase in the gravitational force in the proximity of a black hole (see Table **1**). At $48R_S$ radii, the apparent increase in gravity is a factor of 1.0635 multiplied by that of Newtonian gravity. At $24R_S$ radii, the increase in the ratio to standard

Newtonian gravity is 1,129. At $12R_S$ radii, there begins to be a significant increase in the ratio of dynamic gravity to standard Newtonian gravity, which is 1.2650 times that of Newton. This increase of the apparent effects of gravity further increases as we approach the Schwarzschild radius. So, at $3R_S$ radii, the increase in the ratio of gravity to standard Newtonian gravity is 2.25. At $1.5R_S$ radii the ratio is exactly 4.0, and at the Schwarzschild radius the increase in the ratio to standard Newtonian gravity is exactly 6.25.

Importantly these observations offer readily predictable changes in the apparent force of gravity. These would be particularly apparent in the proximity of the supermassive black hole in the centre of the Milky Way galaxy, where there is a 3 million solar mass black hole [8]. From the perspective of the external observer there would be a significant increase in the force of gravity as an object approached within $12R_S$ radii of the galactic supermassive black hole. Moreover, at distances closer than that, the apparent force of gravity would increase dynamically and considerably. This would increase up to a maximum of 6.25 times that of normal Newtonian gravity, at the Schwarzschild radius itself.

What is important is the ability to model the gravitational physics in the proximity of a black hole, which will enable the modelling of CDM. Specifically, the apparent CDM at the centre of a galaxy, where a supermassive black hole almost invariably exists, can be accurately modelled. Equally well, this is where experiment defines an increase in the amount of galactic CDM, by gravitational lensing [1]. This is entirely in keeping with the DNAg presented here, (see Table **1**).

It is also theoretically possible to predict the increased forces of gravity within a black hole (see Table **1**), which will enable the physics within a black hole to be determined. It is *stressed* that the presence of CDM in galactic halos, is the subject of a further paper, and is thus not fully addressed here. However, the same principles that govern the nature of the cosmological CDM, apply to CDM in the galactic halo, and can also be answered by using advanced Newtonian gravity. Again we would expect to see an increase in the apparent force of gravity of the CDM, particularly if the galactic halo contained (primordial) black holes [18]. Again, this also dovetails, with the amount of cosmological cold dark matter present in the observable Universe [4, 5]. This is because the equations remain accurate for small and greater mass density objects, such as stars, binary pulsars and black holes. They also remain accurate for greater overall masses, including the mass of the observable Universe.

Indeed in the range where general relativity has been widely tested, then the answers for advanced Newtonian gravity are technically the same. So in advanced Newtonian gravity the answers given are equivalent to GTR and give answers that are technically in exact agreement to GTR for Mercury and other planets in the solar system (see Appendix). For mass densities involved in binary pulsars, then dynamic Newtonian advanced gravity is also accurate [4-6]. It is in the mass densities involved in black holes, where the answers are significantly different. Indeed it is precisely in this respect that conventional gravitation begins to produce infinities, specifically in the case of the black hole singularity, whereas advanced Newtonian gravity does not.

The elegance of the advanced Newtonian approach is that we can solve the problem of singularities that arise with a black hole. We can thus get rid of the concept of infinitely dense singularities that exist within the observable Universe. This can be achieved using the equations of advanced Newtonian gravity [Eqs. 2-5)]. These equations are technically in exact agreement with GTR at low and medium mass densities. Importantly, this advanced gravity technically gives exactly the same answers as general relativity, where GTR applies (for worked examples see Appendix). Specifically, with low and medium density masses, where GTR has been thoroughly tested, then the answers agree very closely.

Additionally, an analysis of the data for cosmological CDM, shows that the advanced Newtonian equations can also readily and accurately account for the presence of this CDM. Thus, the principle advantage of advanced Newtonian gravity starts to be important in our treatment of objects with the mass density of black holes. That difference also enters the equations when we are dealing with the mass of the observable Universe, and advanced Newtonian gravity accurately predicts the apparent presence of cosmological CDM at this level [4, 5].

Importantly, using advanced Newtonian gravity we should be able to explain and accurately account for the presence of cosmological CDM. This can be done on the same basis as with the explanation of a black hole. To recap, with dynamic Newtonian advanced gravity, the equation for the acceleration due to gravity at the event, or light horizon, of a black hole is equivalent to 6.25 times the acceleration exerted in standard Newtonian gravitation [Eq. (9)]. Importantly the equivalent force of gravity is the same for different observers. In the case of black holes where gravity is concentrated in a central mass, then the force of gravity is 6.25 times that of normal Newtonian gravity at the Schwarzschild radius, and continues to increase inside a black hole, in accordance with the dynamic Newtonian gravitational equations (see Table **1**).

Interestingly, 6.25 is also the observed ratio of ordinary (baryonic) matter to the amount of Cosmological dark matter at the cosmological event horizon [4, 5]. The reason for this effect will be discussed in the subsequent paragraphs. Suffice it to say that within the cosmic event horizon itself, where there is a large mass which is evenly distributed, then the increased force of gravity can explain the presence of cosmological CDM for observers within their cosmological horizon. To fully explain, it should first be noted that the known radius of the cosmic event horizon (using proper distance) is approximately 13.7 billion ly (1.296×10^{26} m). Using conventional calculations [Eq. (8)], the Schwarzschild radius of the Universe, given the latest WMAP estimated mass density of 9.9×10^{-27} kg/m^3, is also approximately equal to 13.7 billion ly. Thus in this case, the Schwarzschild radius actually describes the cosmological light horizon of our Universe. This cosmological observation thus makes an important test of advanced Newtonian gravitation. In this case advanced Newtonian gravitation, predicts that the amount of CDM within the cosmic event horizon of the Universe is exactly 6.25 that of ordinary baryonic matter.

To recap, as one would logically expect, the age of the Universe is 13.7 billion years and thus the cosmic event horizon is 13.7 billion light years. The corollary is, that using advanced quantum gravity we can account for our cosmological observations, and we can estimate the amount of dark matter within the cosmological event horizon from the equations for the additional acceleration and force of gravity exerted at this horizon [Eq. (5)]. This additional acceleration would be 6.25 that of normal gravity, from whichever vantage point different observers have from within the cosmological event horizon of the Universe [Eq. (5)]. Thus, these equations accurately account for the additional amount of apparent dark matter of the Universe at 6.25 times the observable matter in the Universe. Specifically 4% *x* 6.25, gives a total of 25%, accounting for both baryonic and dark matter [4, 5].

The remaining 1% of dark matter probably consists of energetic neutrinos, and remnant primordial black holes in the outer regions (halo regions) of galaxies [1, 18]. Overall, observation of the cosmic microwave background radiation yields a dark matter content at the edge of the cosmic event horizon, at the cosmological Schwarzschild radius, of 6.25 multiplied by the baryonic mass content of the Universe in keeping with current observations [1].

Additionally, experimental observations indicate that the predominance of CDM in the visible galaxy, lays in the centres of the galaxies. The presence of the additional gravity predicted by dynamic Newtonian advanced gravity (DNAg), particularly in supermassive black holes, can therefore explain the predominance of CDM at the galactic core. Thus the equations for DNAg, can resolve the problems related to singularities and in turn galactic dark matter, and by the same mechanism can accurately resolve the apparent presence of cosmological CDM within the cosmological event horizon.

4. CONCLUSIONS AND DISCUSSION

The gravitational Principles of Cold Dark Matter

In this paper the possibility that CDM fundamentally arises from a reformulation of gravity, is examined. The principle findings are, that using the previously published equations for dynamic Newtonian advanced gravitation [4, 5], it is possible to reformulate the equations for gravity and develop these equations in a way, which accounts accurately for cosmological CDM. At the same time these equations technically agree exactly with general relativity, in low and medium density mass objects. They also agree with the results of data from binary pulsars [4-6].

However, in high mass density objects such as black holes, it is also possible to resolve the problems relating to black hole singularities, using advanced Newtonian gravitation. Indeed, a similar mechanism, using the same advanced gravitational equations, makes it possible to account for the presence CDM in the galactic core. In a similar way dynamic Newtonian advanced gravity (DNAg), can account for the apparent presence of cosmological CDM. Conventionally, the matter in the observable Universe is divided into two main components: visible or baryonic matter (4%), and dark matter (22%) [1]. Thus the total of the baryonic matter and dark matter combined, consist of 26% of the total energy contained within the Universe. The currently favoured model for the missing dark matter is that it consists of cold dark matter (CDM). In this paper, it is shown that this apparent CDM, particularly that which is present in the centre of the galaxy, can fundamentally arise from a reformulation of gravity.

Alternative hypotheses for dark matter involve the modification of the theories of gravity. MOND is one such theory in which there is an increase in the force of gravity. MOND can theoretically explain the apparent missing CDM of Galaxies [2]. However, it cannot by itself readily explain the missing cosmological CDM [19, 20]. Needless, to say the most accurate description of gravity we have to date is still the general theory of relativity (GTR) [21]. So where in the low and medium mass range, where GTR has been fully tested, the answers should agree closely with GTR.

The introduction of dynamic Newtonian advanced gravity, is related to general relativity [4, 5]. The same algebraic equation for the advance in the perihelion of Mercury that was used to develop general relativity, has been used to develop DNAg [21]. Indeed, similarly to GTR, it is possible to readily calculate the advance in the perihelion of Mercury, to a very good degree of accuracy (for worked examples see Appendix). We can also by the same means, using a worked examples, calculate the advance of the perihelion of the Earth around the Sun and we technically get exactly the same answer as general relativity, 3.84 arc sec per century. Indeed recent experimental evidence for the advance perihelion of Earth, agrees with these findings, 3.84 ± 0.1 arc sec [13]. These results suggest a straight line correlation between advanced Newtonian gravity and standard GTR, for low and medium mass density gravitational bodies. A similar calculation may be performed for any gravitational body such as Mars (see Appendix). These results further confirm a straight line correlation between advanced Newtonian gravity and standard GTR, at these mass densities.

Further evidence for the validity of this approach at greater mass densities, arises from a re-analysis of data, for binary pulsars [3]. The use of supercomputers allowed the calculation of the orbit of binary pulsars, using what was the so-called DD-model [14, 15]. Although the DD model does not reformulate the gravitational equations in terms of a force or an acceleration due to gravity, the post Newtonian parameters (PPN) from DD model are equivalent to the advanced Newtonian model presented here [4, 5]. In his most recent textbook, Straumann [6], showed that there was no significant deviation from results in the DD model, even at these very accurate levels of measurement for the parameters. A reanalysis of the data which described the gravitational radiation damping also showed that the DNAg model was surprisingly accurate for this parameter [4, 5].

It is also important to note that whilst advanced Newtonian gravity, agrees very closely with GTR in the low and medium mass densities such as the solar system and binary pulsars, it does not break down in high density gravitational objects. In high mass density objects, like black holes, the major difficulty that arises is that infinite densities or "singularities" appear from the standard GTR. One reason for the production of the singularities in general relativity is that in the equations, an extra term, which is in the order of recurring terms of $2GM/ac^2$, is applied to the gravitating mass. So in low mass density objects, the difference is very small, but in high density mass objects when the Schwarzschild radius is reached (see Eq. 5), this difference mathematically results in the production of infinities and the formation of black hole singularities. This effect of producing an infinite density singularity can be avoided by the use of DNAg

In the DNAg model for advanced gravity presented here, the Schwarzschild radius describes the event horizon, the horizon for the escape velocity of light, but it is not necessarily the "limit" for the formation of a singularity. In dynamic Newtonian advanced gravitation, the problem of the singularity does not arise (see Eq. 5), and it is possible to begin to model the physics of what might be occurring in the proximity of a black hole (Table **1**).

In DNAg we can calculate the effects of gravity at a particular radius and the same force law applies, whatever radius we choose. Equally well this force law applies as much for an observer, for instance, at the event horizon, as it does for a distant observer - both observers will determine the same force at any particular radius, so the laws of gravity are maintained for all observers. From the point of view of observers at the event horizon and observers distant to the event horizon, they will both measure a total force that is 6.25 times greater than the standard Newtonian force, at the event horizon. In particular, in the case of the distant observer, the effects of gravitation will increase dynamically in the proximity of a black hole, and that observer will therefore infer the apparent presence of dark matter.

Additionally, the other benefit of advanced Newtonian gravity is that it is relatively straight forward to use. With advanced Newtonian gravity it is entirely possible to resolve a number of gravitational problems including the presence of cosmological CDM and CDM at the galactic centre using Eqs. (2-5), and these greatly ease the mathematics involved (for worked examples see Appendix). They also accurately predict the acceleration due to gravity in the proximity of a black hole (see Table **1**). Thus advanced Newtonian gravity can be used to determine the physics of a black hole and using the same equations explain CDM at the galactic core *and* the apparent presence of cosmological CDM [Eqs. (2-5)].

Additionally one anomaly, which remains unresolved relates to the Pioneer 10 & 11, Galileo and Ulysses missions. In each case there appears to be an additional acceleration *towards* the Sun acting upon these probes of ~ 8 x 10^{-10} m/sec^2. At least some, if not all of this effect on gravity *can* be accounted for by dynamic Newtonian advanced gravity, from the known mass of the Sun and from the increased gravitational effects of this (and the close flybys of planets such as Jupiter and Saturn) on the trajectory of these probes. In advanced dynamic gravity we can calculate the *cumulative* effects of this increased force of gravity, which allows us a good estimate of this effect. The results of more precise telemetry, however would be needed for accurate confirmation.

Equally this advanced adaptation of gravity also helps explain the apparent missing mass in the galactic halo. Some evidence suggests that MOND, by an increased effect of gravity, may explain the apparent behaviour of galaxies and galaxy clusters without invoking particulate cold dark matter [19, 20]. Certainly the observational data we do have on galactic CDM suggests it may be due to gravitational effects. Thus the increased apparent dark matter in the centre of the galaxy are readily be explained by the increased effects of gravity presented in this paper (see Table **1**). This paper does not fully address the possibility of dark matter in the galactic halo, which is the subject of a further paper. However, this missing mass may be due to the presence of dark matter, particularly on the form of primordial black holes in the galactic halo [18]. Moreover, if this form of galactic CDM does consist of primordial black holes, then the gravitational effects described by the advanced Newtonian gravity presented here, would account for their increased gravitational effect.

It is also possible to begin to explain such anomalies as the black hole singularity, galactic CDM, and account for the presence of cosmological CDM. Advanced Newtonian gravity can also explain the additional gravitational pull that Pioneer and Voyager are experiencing as they leave the solar system. It resolves the problems related to the formation of singularities, by using gravity without encountering the difficulty relating to the formulation of singularities [Eq. (5)]. As regards cosmological CDM, the apparent prevalence of CDM will follow the equations of advanced Newtonian gravity using the same equations. Thus observation yields a dark matter content, at the cosmological horizon, of 6.25 multiplied by the baryonic mass content of the Universe.

Additionally, advanced Newtonian gravity can be progressed, in order to explain the presence of the recently discovered dark energy [22]. By using the principles of advanced Newtonian gravity, it is also possible to begin to define the properties and the field equations of dark energy, and in turn formulate advanced quantum gravity in terms of quintessence [5]. The presence of this quintessence describes space-time in terms of the 9 space dimensions as in conventional string theory [23, 24].

The other advantage of this work is that we can also readily test its validity. Advanced Newtonian gravity has already been shown to be very accurate for objects in the solar system (see Appendix), and for binary pulsars using data from Straumann [6]. It is also accurate in predicting the amount of Cosmological dark matter present in the observable Universe [4, 5]. It can also be used to predict the gravitational physics of the centre of the Milky Way galaxy. At the centre of the galaxy exists a 3 million solar mass black hole, that lays within a Schwarzschild radius of 6 x 10^6 km. That is equivalent to 3 million solar masses, within a radius that is only 10 times the radius of the Sun. Although this is extreme gravity, advanced Newtonian gravity makes some very precise predictions about the gravitational physics in the proximity of a super-massive black hole, which can readily be tested, by measuring the apparent increase in gravity in the proximity of a black hole (see Table **1**).

In this paper, overall it has been shown that advanced Newtonian gravity can explain the physical phenomena of gravity in a way which technically agrees exactly with the general theory of relativity, in low and medium mass density objects. Advanced Newtonian gravity predicts, as does standard GTR, an increase in the gravitational field around gravitational bodies. Under high mass density conditions, such as with binary pulsars, a re-analysis of the results shows that advanced Newtonian gravitation gives very accurate results. Dynamic Newtonian advanced gravity can additionally resolve the difficulties associated with the formation of singularities. Moreover by the same mechanism, dynamic Newtonian advanced gravity (DNAg), can accurately account for the apparent presence of cosmological dark matter, and also account for the presence of CDM at the galactic centre.

ACKNOWLEDGEMENTS

The assistance of John Duffield in the revision of this paper is acknowledged.

APPENDIX

Advances in the perihelia of solar system bodies.

1). *Advance in the Perihelion of Mercury (worked example).*

$$\Delta_{circ} = \frac{3GMs}{c^2\, a(1-e^2)} = 7.987 \times 10^{-8}$$

multiplied by the No. of orbits in a century,

$$= 3.316 \times 10^{-5}$$

the ratio of circumference to arc second

$$= 360 \times 3600 = 1.296 \times 10^{6}$$

calculated advance in the perihelion of Mercury per century

$3.316 \times 10^{-5} \times 1.296 \times 10^{6}$ = <u>42.98 arc sec</u>.

Equivalent *general relativistic* value per century

= 42.98 arc sec

Experimentally estimated advance in the perihelion of Mercury per century [19].

= 43 ± 0.1 arc sec

where Δ_{circ} is the change in circumference of the orbit of Mercury, G is the gravitational constant, Ms the mass of the Sun, c the speed of light, a is the semi major axis of Mercury 's orbit (in meters), e is the eccentricity.

2). *Advance in the Perihelion of Earth (worked example).*

$$\Delta_{circ} = \frac{3GMs}{c^2\, a(1-e^2)} = 2.961 \times 10^{-8}$$

in a century $= 2.961 \times 10^{-6}$

the ratio of circumference to arc second

$$= 360 \times 3600 = 1.296 \times 10^{6}$$

calculated advance in the perihelion of Earth per century

$2.961 \times 10^{-6} \times 1.296 \times 10^{6}$ = <u>3.84 arc sec</u>.

Equivalent *general relativistic* value per century

= 3.84 arc sec

Experimentally estimated advance in the perihelion of Earth per century [19].

= 3.84 ± 0.1 arc sec

where Δ_{circ} is the change in circumference of the orbit of Earth, G is the gravitational constant, Ms the mass of the Sun, c the speed of light, a is the semi major axis of Earth 's orbit (in meters), e is the eccentricity.

3). *Advance in the Perihelion of Mars (worked example).*

$$\Delta_{circ} = \frac{3GMs}{c^2\, a(1-e^2)} = 1.9595 \times 10^{-8}$$

multiplied by the No. of orbits in a century

$$= 1.0416 \times 10^{-6}$$

the ratio of circumference to arc second

$$= 360 \times 3600 = 1.296 \times 10^{6}$$

calculated advance in the perihelion of Mars per century

$1.0416 \times 10^{-6} \times 1.296 \times 10^{6}$ = <u>1.35 arc sec</u>.

Equivalent *general relativistic* value per century

= 1.35 arc sec

Experimentally estimated advance in the perihelion of Mars per century [19].

$$= 1.35 \pm 0.1 \text{ arc sec}$$

where Δ_{circ} is the change in circumference of the orbit of Mars, G is the gravitational constant, Ms the mass of the Sun, c the speed of light, a is the semi major axis of Mars orbit (in meters), e is the estimated eccentricity.

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Received: November 17, 2010 Revised: January 03, 2011 Accepted: January 13, 2011