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Vibrations of Fluid-Conveying Pipes Resting on Two-parameter Foundation

Kameswara Rao Chellapilla[*,a] and H.S. Simha[b]

[a]*Narsimha Reddy College of Engineering, Dhulapally, Medchel, Andhra Pradesh, India*

[b]*Design & Engineering Division, Indian Institute of Chemical Technology, Hyderabad-500 007, India*

Abstract: The problem of vibrations of fluid-conveying pipes resting on a two-parameter foundation model such as the Pasternak-Winkler model is studied in this paper. Fluid-conveying pipes with ends that are pinned-pinned, clamped-pinned and clamped-clamped are considered for study. The frequency expression is derived using Fourier series for the pinned-pinned case. Galerkin's technique is used in obtaining the frequency expressions for the clamped-pinned and clamped-clamped boundary conditions. The effects of the transverse and shear parameters related to the Pasternak-Winkler model and the fluid flow velocity parameter on the frequencies of vibration are studied based on the numerical results obtained for various pipe end conditions. From the results obtained, it is observed that the instability caused by the fluid flow velocity is effectively countered by the foundation and the fluid conveying pipe is stabilized by an appropriate choice of the stiffness parameters of the Pasternak-Winkler foundation. A detailed study is made on the influence of Pasternak-Winkler foundation on the frequencies of vibration of fluid conveying pipes and interesting conclusions are drawn from the numerical results presented for pipes with different boundary conditions.

Keywords: Fluid conveying pipes, pasternak foundation, two-parameter foundation, frequencies.

INTRODUCTION

The design of a pipeline transporting fluids involves not only strength calculations as per the specified codes, but also analysis of the behaviour of the pipeline under different operating conditions. The latter part is not fully covered in the various design codes. Vibration and stability analysis of such pipelines is an important part of the design process. The technology of transporting fluids, through long pipelines, covering different types of terrain, has to take into consideration the dynamic aspects of the system, most fundamental of which is the natural frequency. It is known from previous work that as the fluid velocity is increased to its critical value, the natural frequency of the pipeline tends to zero. Literature abounds with various analysis techniques for different end conditions and different models of the fluid-conveying pipeline. A brief survey of the relevant literature, is presented in a recent paper by Chellapilla and Simha [9], in 2007. Of particular interest are the papers by Gregory and Païdoussis [1], in 1966 and Païdoussis and Issid [2] in 1974, which have dealt with the issues of stability of pinned-pinned, clamped-clamped and cantilevered fluid-conveying pipes, even in the presence of a tensile force and a harmonically perturbed flow field. However, all the above studies did not consider elastic foundation conditions.

In practice, long, cross-country pipelines rest on an elastic medium such as a soil, and hence, a model of the soil medium must be included in the analysis. The Winkler model, in which soil is represented by a series of constant stiffness, closely spaced linear springs, is a very popular model of the soil employed in many studies, perhaps because it is simple a linear model. Many researchers, Stein & Tobriner [3], Lottati and Kornecki [4], Dermendjian-Ivanova [5] and Raghava Chary *et al*. [6] studied fluid-conveying pipes resting on elastic foundation. Recently, in 2002, Doaré *et al*. [7] studied instability of fluid conveying pipes on Winkler type foundation. In all these studies, the soil was modeled as a Winkler foundation model.

However, a real soil medium is more complex in its elastic behaviour. To address the deficiencies of a Winkler model, the two-parameter foundation models were developed in which, an interaction between the springs of the Winkler model is included to obtain a more realistic model of the soil. The Pasternak model is one such formulation. In the Pasternak model, an incompressible shear layer is introduced between the Winkler springs and the pipe surface. The springs are connected to this shear layer, which is capable of resisting only transverse shear, thus allowing for "shear interaction" between the Winkler springs. There is a good amount of literature on the analysis of fluid conveying pipes resting on one-parameter elastic foundation models like the Winkler model, and also on the behaviour of beams on two-parameter foundations. Elishakoff & Impollonia [8], in 2001, analysed the stability of fluid conveying pipes on partial elastic foundation, considering both Winkler and rotary foundations. Very recently, Chellapilla and Simha [9], in 2007, studied the effect of a Pasternak foundation on the critical velocity of a fluid-conveying pipe.

In this paper, the above work is extended to the study of the effect of the Pasternak foundation on the natural frequen-

*Address correspondence to this author at the Narsimha Reddy College of Engineering, Dhulapally, Medchel, Andhra Pradesh, India; Tel: 0091-40-23811623; E-mail: chellapilla95@gmail.com

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cies of the pipeline for the pinned-pinned, clamped-clamped and clamped-pinned boundary conditions. Two-term Fourier series solution is obtained for the pinned-pinned condition while the two-term Galerkin method has been employed to get results for the other two cases. The paper is organized in the following way: First, fundamental frequencies of the pipe without fluid flow and resting on a two-parameter elastic foundation are obtained and compared with those of a similar beam. Next, fluid with flow velocity is introduced and analysed for different conditions. Results are presented showing the variation of natural frequencies for various values of the foundation stiffness parameters, flow velocities and mass ratios.

THEORETICAL DEVELOPMENT

In the development of the equations governing the motion of the pipeline, the assumptions made are the following – The pipe is long and straight, thus facilitating the use of Euler-Bernoulli beam theory; the motions are small so that the system can be analysed by the linear theory; and, the effects of internal pressure and external forces are neglected in the analysis. The equation of motion for a fluid-conveying pipe of length L with lateral displacement w, resting on a two-parameter foundation shown in Fig. (**1**) of reference [9] is given by:

$$EI\frac{\partial^4 w}{\partial x^4} + M\frac{\partial^2 w}{\partial t^2} + \left(\rho A v^2 - k_2\right)\frac{\partial^2 w}{\partial x^2} + 2\rho A v\frac{\partial^2 w}{\partial x \partial t} + k_1 w = 0 \tag{1}$$

In the above equation, $A\rho$ is the mass of pipe/unit length, v is the steady flow velocity of fluid, E is the modulus of elasticity of the pipe material, I is it's moment of inertia, M is the total mass of pipe plus fluid/unit length, k_1 represents the Winkler foundation stiffness parameter and k_2 represents the additional shear constant parameter of the foundation. In Eq. (1) above, the first term accounts for the elastic force, the second term represents the inertia force due to the acceleration of the pipe with fluid, the third, the inertia force of the fluid flowing in a curved path, the fourth term represents the inertia force due to Coriolis acceleration and the last term is due to the Winkler foundation. The free vibration solution for three simple boundary conditions is obtained in what follows.

Pinned-Pinned Pipe

The boundary conditions for a pinned-pinned pipe are

$$w(0,t) = w(L,t) = 0$$
$$\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \tag{2}$$

Taking the solution of Eq. (1) which satisfies the boundary conditions Eq. (2) as

$$w(x,t) = \sum_{n=1,3,5,...} a_n \sin\frac{n\pi x}{L}\sin\omega_j t + \sum_{n=2,4,6,...} a_n \sin\frac{n\pi x}{L}\cos\omega_j t,\ j = 1,2,3,... \tag{3}$$

where ω_j represents the natural frequency of the j^{th} mode of vibration. Following the method given in [6], substituting Eq. (3) in Eq. (1) and expanding in a Fourier series we have an equation of the form:

$$\left[\mathbf{K} - \omega_j^2 \mathbf{MI}\right]\left\{\mathbf{a}\right\} = 0 \tag{4}$$

where **K** is the stiffness matrix whose elements are enumerated in [6], **I** is the identity matrix and $\mathbf{a}^T=\{a_1,a_2,\ldots,a_n\}$. Setting the determinant of the coefficient matrix above equal to zero and retaining the first two terms of the above equation, we get the frequency equation, Eq. (5), making use of the following non-dimensional parameters:

$$\beta = \frac{A}{M};\ \Omega_j = \omega_j L^2\sqrt{\frac{M}{EI}},\ j = 1,2,3,...;$$
$$V = v\,L\sqrt{\frac{\rho A}{EI}};\ \gamma_1 = \frac{k_1 L^4}{EI};\ \gamma_2 = \frac{k_2 L^2}{EI}$$

$$\Omega_j^4 - \left[\left(\frac{256}{9}\beta - 5\ {}^2\right)\left(V^2 - \ {}_2\right) + 17\ {}^4 + 2\ {}_1\right]\Omega_j^2 + \left[\begin{array}{l} 4\ {}^4\left(V^2 - \ {}_2\right)^2 - \left(V^2 - \ {}_2\right) \\ \left(5\ {}^2\ {}_1 + 20\ {}^6\right) + \\ \left(16\ {}^8 + 17\ {}^4\ {}_1 + \ {}_1^2\right) \end{array}\right] = 0 \tag{5}$$

Clamped-Pinned and Clamped-Clamped Pipe

The boundary conditions for a clamped-pinned pipe are

$$w(0,t) = w(L,t) = 0$$
$$\frac{\partial w(0,t)}{\partial x} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \tag{6}$$

And those for a clamped-clamped pipe are

$$w(0,t) = w(L,t) = 0$$
$$\frac{\partial w(0,t)}{\partial x} = \frac{\partial w(L,t)}{\partial x} = 0 \tag{7}$$

We assume the deflection of the pipe to be of the form

$$w(x,t) = \Re\left[\phi_n\left(\frac{x}{L}\right)e^{i\omega t}\right] \tag{8}$$

In Eq. (8), $\Re$ denotes the real part, $\phi_n\left(x/L\right)$ is a series of beam eigen-functions $\psi_r(\xi)$

given by [10]:

$$\begin{aligned}&\psi_r(\xi) = \cosh(\lambda_r\xi) - \cos(\lambda_r\xi) - \\ &\sigma_r\left(\sinh(\lambda_r\xi) - \sin(\lambda_r\xi)\right), \\ &r = 1,2,3,....,n\ ;\ \xi = \left(\frac{x}{L}\right) \\ &\sigma_r = \frac{\cosh\lambda_r - \cos\lambda_r}{\sinh\lambda_r - \sin\lambda_r}\end{aligned} \tag{9}$$

In the above equation, λ_r is the frequency parameter of the pipe without fluid flow, which is considered as a beam, and it's values are:

λ_1 = 3.926602 and λ_2 =7.068583 for the pinned-clamped case and

λ_1 = 4.730041 and λ_2 =7.853205 for the clamped-clamped case.

Substituting Eq. (8) in the equation of motion Eq. (1) gives

$$\begin{aligned}L_n = EI\frac{\partial^4\phi}{\partial x^4} + \left(\rho A v^2 - k_2\right)\frac{\partial^2 w}{\partial x^2} + \\ 2i\omega\rho A v\frac{\partial\phi}{\partial x} + \left(k_1 - M\omega^2\right)\phi = 0\end{aligned} \tag{10}$$

Minimizing the mean square of the residual L_n over the length of the pipe using Galerkin's method,

$$\int L_n(\phi)\psi_r\left(\frac{x}{L}\right)dx = 0, \quad r = 1,2,3,...N \tag{11}$$

Substituting Eq. (9) and using the non-dimensional parameters, we obtain,

$$L_n(\psi) = \sum_{r=1}^{\infty} a_r\begin{bmatrix}\psi_r^{iv} + \left(V^2 - {}_2\right)\psi_r^{''} \\ +2i\beta^{1/2}V\Omega\psi_r^{'} + \left({}_1 - \Omega^2\right)\psi_r\end{bmatrix} \tag{12}$$

where the derivatives of ψ are with respect to ξ. The above equation is multiplied by ψ_s and using the orthogonal property of the eigenfunctions and the values of the resulting integrals from Felgar [10], the following infinite system of equations in a_r is obtained.

$$\begin{aligned}&a_r\left(\lambda_r^4 + \gamma_1 - \Omega^2\right) + \left(V^2 - \gamma_2\right)\sum_{s=1}^{\infty}\left(a_r C_{rs}\right) + \\ &2i\beta^{1/2}V\Omega\sum_{s=1}^{\infty}\left(a_r b_{rs}\right) = 0\end{aligned} \tag{13}$$

Setting the resulting determinant of the coefficient matrix to zero and using only the first two terms, we have the following frequency equations in Ω_j:

For the clamped-pinned case it is:

$$\begin{aligned}&\Omega_j^4 - \begin{bmatrix}\lambda_1^4 + \lambda_2^4 + \left(C_{11} + C_{22}\right)\left(V^2 - {}_2\right) \\ +4\beta\left(V^2 - {}_2\right)b_{12}^2 + 2\ {}_1\end{bmatrix}\Omega_j^2 \\ &+\begin{bmatrix}\left(\lambda_1^4 + {}_1\right)\left(\lambda_2^4 + {}_1\right) + \\ \left(V^2 - {}_2\right)\begin{Bmatrix}\left(\lambda_2^4 + {}_1\right)C_{11} + \\ \left(\lambda_1^4 + {}_1\right)C_{22}\end{Bmatrix} \\ +\left(V^2 - {}_2\right)^2\left(C_{11}C_{22} - C_{12}C_{21}\right)\end{bmatrix} = 0\end{aligned} \tag{14}$$

For the clamped-clamped case it is:

$$\begin{aligned}&\Omega_j^4 - \begin{bmatrix}\lambda_1^4 + \lambda_2^4 + \left(C_{11} + C_{22}\right)\left(V^2 - {}_2\right) \\ +4\beta\left(V^2 - {}_2\right)b_{12}^2 + 2\ {}_1\end{bmatrix}\Omega_j^2 \\ &+\begin{bmatrix}\left(\lambda_1^4 + {}_1\right)\left(\lambda_2^4 + {}_1\right) + \\ \left(V^2 - {}_2\right)\begin{Bmatrix}\left(\lambda_2^4 + {}_1\right)C_{11} + \\ \left(\lambda_1^4 + {}_1\right)C_{22}\end{Bmatrix} \\ +\left(V^2 - {}_2\right)^2\left(C_{11}C_{22}\right)\end{bmatrix} = 0\end{aligned} \tag{15}$$

In Eqs. (14) and (15), the constants C_{11} etc. are integral values, which are taken from Felgar [10] and reproduced in Tables **1** and **2**. The above equations are quadratic in Ω_j^2, and solving for Ω_j, we obtain the fundamental frequencies for the clamped-pinned and clamped-clamped cases respectively.

RESULTS AND DISCUSSION

The results are presented for the following cases: a) No-fluid, no-flow, pipe on two-parameter elastic foundation; b) Fluid conveying pipe – no foundation; c) Fluid conveying pipe – Winkler foundation only and d) Fluid conveying pipe – both Winkler and Pasternak foundations. Comparison has been made with available literature wherever possible and new results have been presented for fluid conveying pipes on two-parameter foundation. For the pinned-pinned boundary condition, numerical results have been obtained considering the first two terms of the equation resulting from using Fourier series. It is assumed that the mode shapes of the pipe will not change with fluid flow and hence, for the clamped-pinned and the clamped-clamped boundary conditions, the modes that are assumed in the present work are for a pipe without fluid flow (beam).

Case 1: No-Flow: γ_1, γ_2 Varying

Results have been obtained for the no-flow condition, where $V = 0$ and $\beta = 0$. This condition constitutes a beam on elastic foundation. Tables **3** and **4** compare values of the

Table 1. Integral Values b_{12}, C_{11}, C_{22}, C_{12}, C_{21} for Clamped-Pinned Beam as Enumerated by Felgar [13]

Parameter	Clamped-Pinned
b_{mn}	$\frac{\lambda_m \lambda_n}{\lambda_n^4 - \lambda_m^4}\left[\begin{array}{l} -(-1)^{m+n}\left(\lambda_n^2+\lambda_m^2\right)\sqrt{(\sigma_n^2+1)(\sigma_m^2+1)} \\ +(-1)^n\left(\lambda_n^2-\lambda_m^2\right)\sqrt{(\sigma_n^2+1)(\sigma_m^2-1)} \\ -(-1)^m\left(\lambda_n^2-\lambda_m^2\right)\sqrt{(\sigma_n^2-1)(\sigma_m^2+1)} \\ +\left(\lambda_n^2+\lambda_m^2\right)\sqrt{(\sigma_n^2-1)(\sigma_m^2-1)} \\ +4\lambda_m\lambda_n \end{array}\right]$
C_{mm}	$\sigma_m\lambda_m\left(1-\sigma_m\lambda_m\right)$
C_{mn}	$\frac{4\lambda_m^2\lambda_n^2}{\lambda_n^4-\lambda_m^4}\left(\sigma_n\lambda_n-\sigma_m\lambda_m\right)$

Table 2. Integral Values b_{12}, C_{11}, C_{22}, C_{12}, C_{21} for Clamped-Clamped Beam as Enumerated by Felgar [13]

Parameter	Clamped-Clamped
b_{mn}	$\frac{4\lambda_m^2\lambda_n^2}{\lambda_n^4-\lambda_m^4}\left(\sigma_n\lambda_n-\sigma_m\lambda_m\right)\left[1+\left(-1\right)^{m+n}\right]$
C_{mm}	$\sigma_m\lambda_m\left(2-\sigma_m\lambda_m\right)$
C_{mn}	$\frac{4\lambda_m^2\lambda_n^2}{\lambda_n^4-\lambda_m^4}\left(\sigma_n\lambda_n-\sigma_m\lambda_m\right)\left[1+\left(-1\right)^{m+n}\right]$

fundamental frequencies with those obtained by Chen, *et al.* [11]. It is seen that the present results are in very good agreement with those of Chen, *et al.* In Fig. (**1**), the fundamental frequency parameter Ω_1 is plotted against Winkler foundation parameter γ_1 for varying values of the Pasternak foundation parameter γ_2, for the pinned-pinned case. The results show that the frequency increases appreciably for values of γ_1 greater than 1000.

Figs. (**2**,**3**) show the results for the clamped-pinned and the clamped-clamped boundary conditions respectively. Here, too, the trend is similar.

Case 2: Fluid Conveying Pipe : No Foundation

Results for a pipe with fluid flow have been presented for no foundation and compared with available literature. Table **5** shows the values of the first two frequency parameters Ω_1 and Ω_2 for a pinned-pinned fluid-conveying pipe. In this table, values of the velocity parameter are varying from zero to the critical value for different values of the mass ratio parameter β, with both γ_1 and $\gamma_2 = 0$. The critical flow velocity for each case has been computed following the method given by Chellapilla & Simha [9]. Fig. (**4**) shows the variation of the frequency parameter Ω_1 with flow velocity parameter V for different values of the mass ratio β for the pinned-pinned boundary condition. For V=0 and for V=V_{cr}, there is no difference in the frequency parameter for any value of β. For intermediate values of V, there is a slight decrease in the frequency parameter for increasing values of β. The results for clamped-pinned and the clamped-clamped cases also follow the same trend and are shown in Tables **6** and **7**. There is very good agreement in the values of Ω_1 with those obtained by Païdoussis & Issid [2], for the pinned-pinned and clamped-clamped cases.

Case 3: Fluid Conveying Pipe : Winkler Foundation Only

Next, results obtained for the condition where $\gamma_2 = 0$ are presented. This represents the presence of only the Winkler foundation. Table **8** shows some representative values of Ω_1 for different values of β and the Winkler foundation parameter γ_1 for all the three boundary conditions. Fig. (**5**) shows the plot of Ω_1 versus V for different values of the mass ratio β and γ_1 for the pinned-pinned boundary condition. As expected, the Winkler foundation has a stabilizing effect in the pipe and increasing values of γ_1 tend to increase both the critical flow velocity V_{cr} and the fundamental frequency Ω_1. Figs. (**6**,**7**) show the plots for clamped-pinned and the clamped-clamped cases respectively. A similar trend is noticed in these cases also. These results compare very well with those of Raghava Chary *et al*. [6].

Table 3. Fundamental Frequency Parameter $\sqrt{\Omega_1}$ (Pinned-Pinned Pipe) for No-Flow Condition

γ_1	γ_2	$\sqrt{\Omega_1}$ exact (Ref. [11])	$\sqrt{\Omega_1}$ present	% Variation
0.0	0.0	3.141	3.141	0.0
	0.01		3.149	
	0.1		3.217	
	0.5	3.476	3.476	0.0
	1.0	3.735	3.736	0.02
	2.5	4.296	4.297	0.02
	10.0		5.721	
	100.0		9.959	
10^2	0.0	3.748	3.748	0.0
	0.01		3.752	
	0.1		3.793	
	0.5	3.960	3.960	0.0
	1.0	4.143	4.143	0.0
	2.5	4.582	4.582	0.0
	10.0		5.850	
	100.0		9.984	
10^4	0.0	10.024	10.024	0.0
	0.01		10.024	
	0.1		10.026	
	0.5	10.036	10.036	0.0
	1.0	10.048	10.048	0.0
	2.5	10.083	10.084	0.009
	10.0		10.257	
	100.0		11.867	
10^6	0.0	31.621	31.623	0.006
	0.01		31.623	
	0.1		31.623	
	0.5	31.622	31.623	0.003
	1.0	31.622	31.624	0.006
	2.5	31.623	31.625	0.006
	10.0		31.631	
	100.0		31.700	

$$\%\text{variation} = \left| \frac{\left(\sqrt{\Omega_1}_{\,present} - \sqrt{\Omega_1}_{\,exact}\right)}{\sqrt{\Omega_1}_{\,exact}} \times 100 \right|$$

Table 4. Fundamental Frequency Parameter $\sqrt{\Omega_1}$ (Clamped-Clamped Pipe) for No-Flow Condition

γ_1	γ_2	$\sqrt{\Omega_1}$ exact (Ref. [11])	$\sqrt{\Omega_1}$ present	% Variation
0.0	0.0	4.730	4.730	0.0
	0.01		4.733	
	0.1		4.769	
	0.5	4.869	4.916	0.965
	1.0	4.994	5.083	1.782
	2.5	5.320	5.505	3.477
	10.0		6.827	
	100.0		11.456	
10^2	0.0	4.950	4.950	0.0
	0.01		4.953	
	0.1		4.984	
	0.5	5.071	5.114	0.847
	1.0	5.182	5.264	1.582
	2.5	5.477	5.649	3.14
	10.0		6.905	
	100.0		11.473	
10^4	0.0	10.123	10.122	0.009
	0.01		10.123	
	0.1		10.126	
	0.5	10.137	10.142	0.049
	1.0	10.152	10.162	0.098
	2.5	10.194	10.222	0.274
	10.0		10.503	
	100.0		12.845	

$$\%\text{variation} = \left| \frac{\left(\sqrt{\Omega_1}_{\,present} - \sqrt{\Omega_1}_{\,exact}\right)}{\sqrt{\Omega_1}_{\,exact}} \times 100 \right|$$

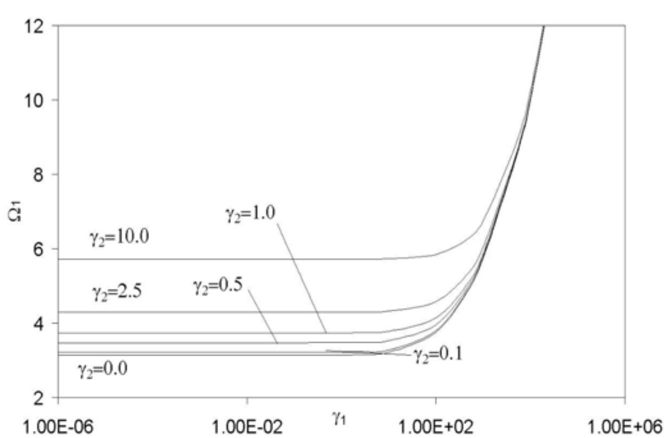

Fig. (1). Pinned-pinned pipe, no-flow condition: Influence of γ_2 on Ω_1 for various values of γ_1.

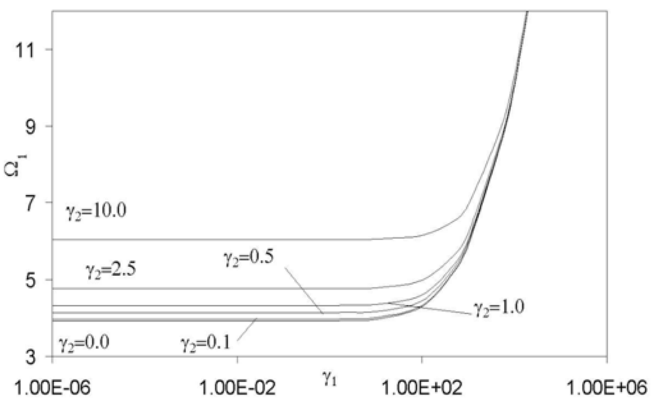

Fig. (2). Pinned-clamped pipe, no-flow condition: Influence of γ_2 on Ω_1 for various values of γ_1.

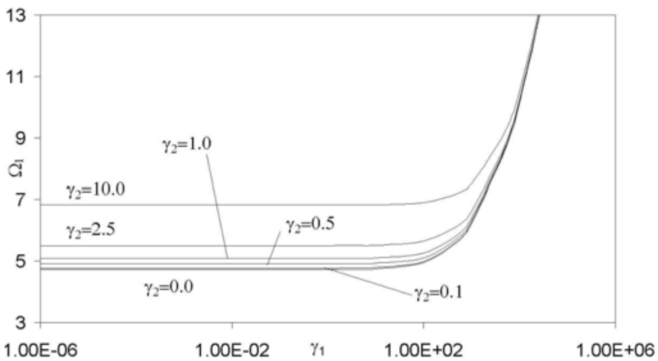

Fig. (3). Clamped-clamped pipe, no-flow condition: Influence of γ_2 on Ω_1 for various values of γ_1.

Table 5. First Two Frequency Parameters Ω_1, Ω_2 for Pinned-Pinned Fluid-Conveying Pipes without Foundation for Various Values of β

V	β = 0.1		β = 0.3		β = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2
0	9.869	39.478	9.869	39.478	9.869	39.478
0.1	9.864	39.473	9.864	39.474	9.864	39.475
0.2	9.849	39.459	9.848	39.463	9.847	39.466
0.3	9.823	39.436	9.821	39.443	9.820	39.450
0.4	9.787	39.404	9.784	39.416	9.781	39.429
0.5	9.741	39.362	9.736	39.382	9.731	39.401
1	9.346	39.013	9.328	39.091	9.310	39.168
1.5	8.652	38.424	8.612	38.599	8.574	38.773
2	7.579	37.583	7.516	37.897	7.455	38.208
2.5	5.935	36.470	5.855	36.967	5.779	37.456
3	2.898	35.057	2.839	35.784	2.784	36.497
3.141	0.0	34.597	0.0	35.399	0.0	36.183

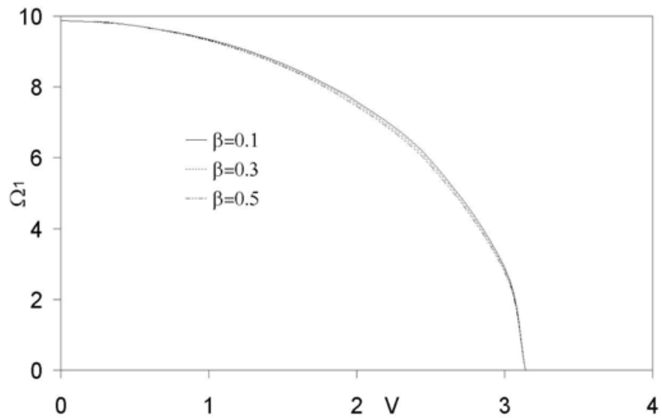

Fig. (4). Pinned-pinned pipe, no foundation: Influence of β on Ω_1 for various values of *V*.

Table 6. First Two Frequency Parameters Ω_1, Ω_2 for Clamped-Pinned Fluid-Conveying Pipes without Foundation for Various Values of β

V	β = 0.1		β = 0.3		β = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2
0	15.418	49.964	15.418	49.964	15.418	49.964
0.1	15.414	49.960	15.414	49.961	15.413	49.962
0.2	15.402	49.949	15.401	49.952	15.400	49.955
0.3	15.383	49.929	15.381	49.936	15.379	49.944
0.4	15.356	49.902	15.352	49.915	15.348	49.927
0.5	15.321	49.867	15.315	49.887	15.309	49.907
1	15.027	49.573	15.003	49.653	14.979	49.732
1.5	14.525	49.080	14.472	49.260	14.420	49.439
2	13.793	48.380	13.702	48.702	13.612	49.021
2.5	12.792	47.465	12.657	47.971	12.526	48.471
3	11.454	46.321	11.275	47.056	11.105	47.778
3.5	9.646	44.929	9.433	45.942	9.234	46.929
4	7.017	43.265	6.806	44.606	6.614	45.904
4.499	0.0	41.294	0.0	43.019	0.0	44.678

Case 4: Fluid Conveying Pipe : Two Parameter Foundation

Finally, new results for varying values of γ_2 have been presented. Table **9** tabulates the fundamental frequency parameter Ω_1 for various values of γ_1, γ_2 and V and for β=0.1 and 0.5, for the pinned-pinned condition. Fig. (**8**) shows the effect of the second foundation parameter on the fundamental frequency as well as on the critical flow velocity for the pinned-pinned boundary condition. A comparison for values of γ_1 = 100, 500 and 1000 shows that with increasing values of γ_2, both Ω_1 and V increase significantly. The figure also

Table 7. First Two Frequency Parameters Ω_1,Ω_2 for Clamped-Clamped Fluid-Conveying Pipes without Foundation for Various Values of β

V	β = 0.1		β = 0.3		β = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2
0	22.373	61.672	22.373	61.672	22.373	61.672
0.1	22.370	61.669	22.370	61.670	22.369	61.671
0.2	22.361	61.659	22.360	61.662	22.359	61.666
0.3	22.347	61.642	22.344	61.650	22.341	61.657
0.4	22.326	61.619	22.322	61.633	22.317	61.646
0.5	22.300	61.589	22.293	61.610	22.285	61.631
1	22.081	61.340	22.051	61.423	22.021	61.507
1.5	21.712	60.921	21.645	61.110	21.578	61.297
2	21.185	60.330	21.067	60.666	20.952	61.000
2.5	20.491	59.560	20.311	60.088	20.136	60.609
3	19.613	58.604	19.360	59.369	19.118	60.120
3.5	18.530	57.451	18.197	58.500	17.885	59.523
4	17.207	56.087	16.793	57.470	16.410	58.809
4.5	15.590	54.496	15.100	56.265	14.657	57.965
5	13.588	52.652	13.040	54.866	12.557	56.974
6	7.317	48.069	6.847	51.371	6.459	54.459
6.378	0.0	45.952	0.0	49.751	0.0	53.279

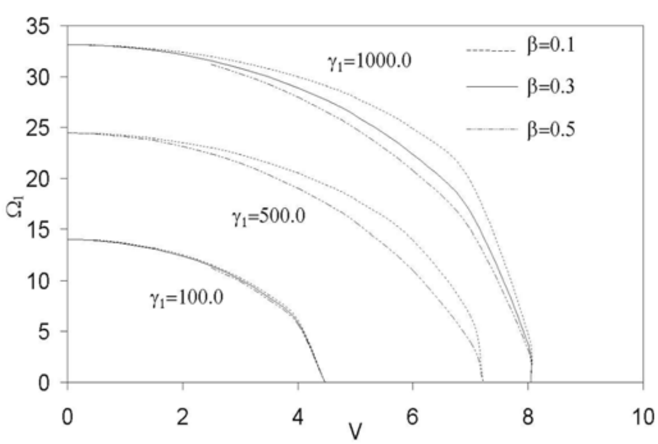

Fig. (5). Pinned-pinned pipe, γ_2 = 0: Variation of Ω_1 with V for various values of γ_1 and β.

shows that higher critical velocity is obtained by only increasing γ_2, without increasing γ_1.

These results are for a mass ratio β=0.1. In Figs. (**9**,**10**), the variation of Ω_1 with V for γ_1 = 0 and different values of γ_2 and β, for the pinned-pinned boundary condition is shown. This corresponds to the case where there is no Winkler component in the foundation and only the shear parameter is present. It is seen that as the shear parameter is increased, the frequency parameter increases appreciably, indicating that the shear constant of the foundation has a significant role in the vibration characteristics of the fluid conveying pipe. This is especially more pronounced as the shear parameter increases beyond 2.5, where a very high increase in the fundamental frequency as well as the critical velocity is observed. It is also seen that for values of γ_2 greater than zero, increasing values of the flow velocity diminishes the effect of mass ratio β.

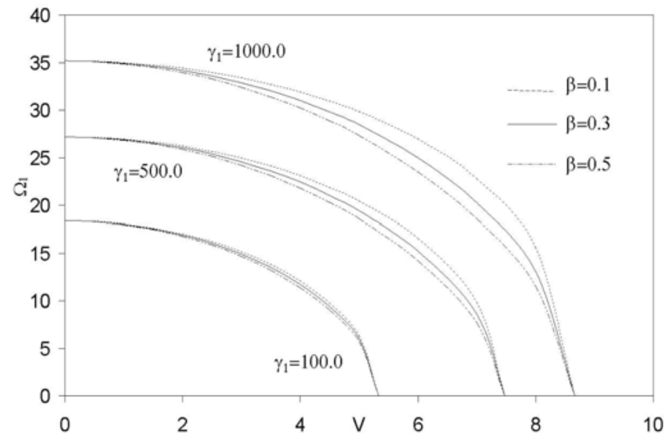

Fig. (6). Clamped-pinned pipe, γ_2 = 0: Variation of Ω_1 with V for various values of γ_1 and β.

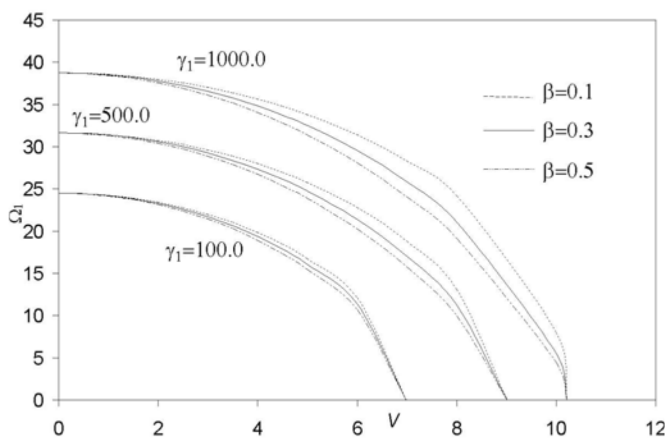

Fig. (7). Clamped-clamped pipe, γ_2 = 0: Variation of Ω_1 with V for various values of γ_1 and β.

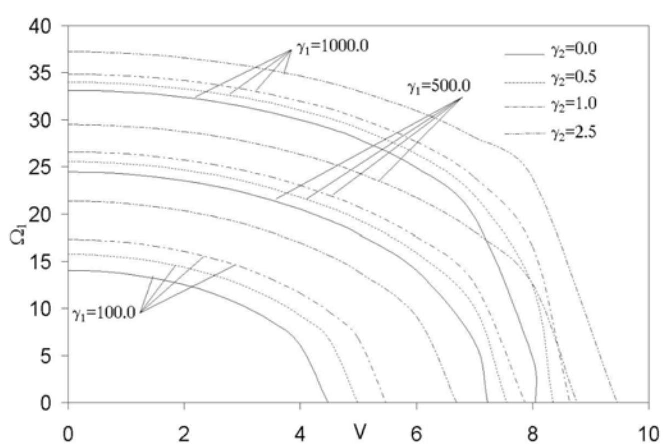

Fig. (8). Pinned-pinned pipe: Variation of Ω_1 with V for various values of γ_1 and γ_2 with β = 0.1.

Table 8. Fundamental Frequency Parameter Ω_1 for Fluid-Conveying Pipes for Various Values of the Winkler Foundation Parameter γ_1 and Mass Ratios β with γ_2= 0

Boundary Condition	β	V	γ_1					
			0.01	0.5	2.5	10.0	10^2	10^3
Pinned-pinned	0.1	0.0	9.870	9.895	9.995	10.364	14.050	33.127
		1.0	9.348	9.374	9.480	9.866	13.681	32.945
		2.0	7.580	7.612	7.741	8.207	12.514	32.390
	0.3	0.0	9.870	9.895	9.995	10.364	14.050	33.127
		1.0	9.329	9.355	9.461	9.847	13.654	32.880
		2.0	7.517	7.549	7.677	8.139	12.411	32.128
	0.5	0.0	9.870	9.895	9.995	10.364	14.050	33.127
		1.0	9.311	9.337	9.442	9.828	13.627	32.816
		2.0	7.456	7.487	7.614	8.073	12.310	31.878
Clamped-pinned	0.1	0.0	15.419	15.434	15.499	15.739	18.377	35.181
		1.0	15.028	15.044	15.111	15.356	18.046	34.989
		2.0	13.794	13.811	13.883	14.149	17.017	34.403
	0.3	0.0	15.419	15.434	15.499	15.739	18.377	35.181
		1.0	15.004	15.020	15.086	15.332	18.018	34.933
		2.0	13.702	13.720	13.791	14.055	16.905	34.179
	0.5	0.0	15.419	15.434	15.499	15.739	18.377	35.181
		1.0	14.980	14.996	15.062	15.307	17.989	34.877
		2.0	13.613	13.631	13.702	13.964	16.795	33.962
Clamped-Clamped	0.1	0.0	22.374	22.384	22.429	22.596	24.506	38.737
		1.0	22.082	22.093	22.138	22.307	24.238	38.552
		2.0	21.186	21.197	21.244	21.419	23.415	37.990
	0.3	0.0	22.374	22.384	22.429	22.596	24.506	38.737
		1.0	22.052	22.063	22.108	22.276	24.205	38.499
		2.0	21.068	21.080	21.126	21.300	23.285	37.781
	0.5	0.0	22.374	22.384	22.429	22.596	24.506	38.737
		1.0	22.022	22.033	22.078	22.246	24.172	38.447
		2.0	20.953	20.964	21.011	21.184	23.158	37.577

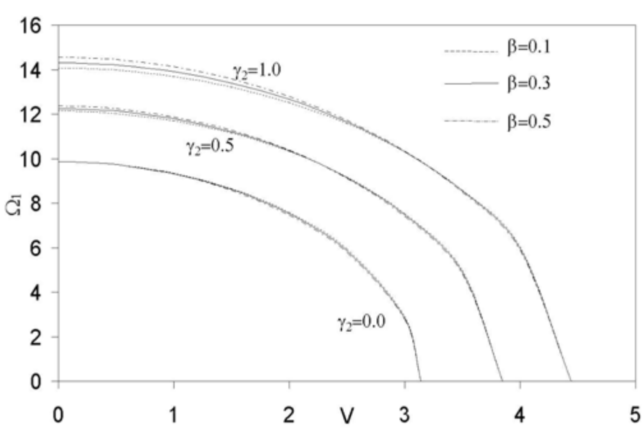

Fig. (9). Pinned-pinned pipe, γ_1 = 0: Variation of Ω_1 with V for various values of γ_2 and β.

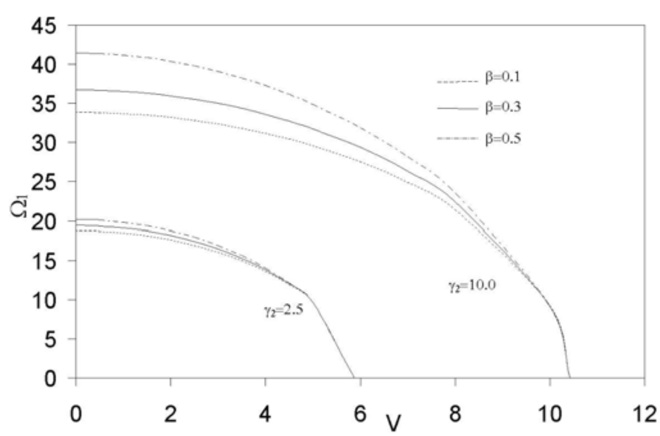

Fig. (10). Pinned-pinned pipe, γ_1 = 0: Variation of Ω_1 with V for various values of γ_2 and β.

Table 9. Fundamental Frequency Parameter Ω_1 for Pinned-Pinned Fluid Conveying Pipes for Various Values of γ_1 and γ_2, for $\beta = 0.1$ and $\beta = 0.5$

γ_2	V	γ_1									
		0.01		0.50		2.50		10.0		10^2	
		β =0.1	β =0.5	β =0.1	β =0.5	β =0.1	β = 0.5	β =0.1	β =0.5	β =0.1	β =0.5
0.01	1.0	9.40	9.37	9.42	9.39	9.53	9.50	9.91	9.88	13.71	13.67
	2.0	7.64	7.52	7.67	7.55	7.80	7.68	8.26	8.14	12.55	12.35
0.5	1.0	11.71	11.89	11.73	11.91	11.82	11.99	12.13	12.31	15.42	15.65
	2.0	10.33	10.37	10.36	10.40	10.45	10.49	10.80	10.85	14.38	14.44
1.0	1.0	13.70	14.13	13.71	14.15	13.79	14.23	14.06	14.51	17.01	17.54
	2.0	12.52	12.80	12.54	12.82	12.62	12.90	12.92	13.20	16.06	16.41
2.5	1.0	18.48	19.86	18.50	19.87	18.55	19.93	18.76	20.16	21.09	22.68
	2.0	17.61	18.78	17.63	18.80	17.68	18.86	17.90	19.09	20.32	21.69
10.0	1.0	33.69	41.16	33.70	41.17	33.73	41.22	33.85	41.38	35.24	43.36
	2.0	33.20	40.39	33.21	40.40	33.24	40.45	33.37	40.61	34.78	42.59

Table 10. Fundamental Frequency Parameter Ω_1 for Clamped-Pinned Fluid Conveying Pipes for Various Values of γ_1 and γ_2, for $\beta = 0.1$ and $\beta = 0.5$

γ_2	V	γ_1									
		0.01		0.50		2.50		10.0		10^2	
		β =0.1	β =0.5	β =0.1	β =0.5	β =0.1	β = 0.5	β =0.1	β =0.5	β =0.1	β =0.5
0.01	1.0	15.06	15.02	15.08	15.04	15.14	15.11	15.39	15.35	18.07	26.87
	2.0	13.83	13.66	13.85	13.68	13.92	13.75	14.19	14.01	17.05	25.90
0.5	1.0	16.87	17.08	16.88	17.09	16.94	17.15	17.16	17.37	19.62	19.87
	2.0	15.77	15.82	15.79	15.84	15.85	15.90	16.08	16.14	18.68	18.74
1.0	1.0	18.53	19.03	18.55	19.05	18.60	19.10	18.80	19.31	21.09	21.66
	2.0	17.54	17.86	17.55	17.87	17.61	17.93	17.82	18.15	20.21	20.58
2.5	1.0	22.84	24.35	22.85	24.37	22.89	24.41	23.06	24.59	24.99	26.66
	2.0	22.03	23.33	22.04	23.34	22.09	23.39	22.26	23.57	24.25	25.68
10.0	1.0	37.73	46.50	37.74	46.51	37.77	46.55	37.88	46.70	39.13	48.53
	2.0	37.24	45.63	37.25	45.64	37.28	45.68	37.39	45.83	38.65	47.65

CONCLUSIONS

In this work, natural frequencies of fluid-conveying pipes resting on two-parameter foundation have been computed. Three ideal boundary conditions, viz. pinned-pinned, clamped-pinned and clamped-clamped, were considered for analysis. A two-term Fourier series solution was adopted for the pinned-pinned case while a two-term Galerkin method was utilized to obtain solutions for the other two cases. Many earlier researchers have analysed the dynamics of fluid-conveying pipes either without foundation or with Winkler foundation. In this work, new results for a two-parameter foundation have been exhaustively given in the form of tables of numerical values, which could aid a designer of pipelines, as well as in the form of graphs, which are useful for showing the trend. Extensive results have been presented in tables and in figures for the following cases: no-flow condition, pipe with fluid flow without foundation, fluid-conveying pipe on Winkler foundation and finally, fluid-conveying pipe on two-parameter foundation. The results for the first three cases have been compared wherever possible and new results for the fourth case have been presented. From the results obtained, the following conclusions are drawn:

a. An attempt was made to validate the present formulation of the problem, by first considering the no-fluid, no-flow condition. The results obtained for this pipe on two- parameter foundation, which is nothing but a beam, have been compared with those of Chen, *et al.* [11]. There is very good agreement between the results and the maximum variation is 3.477% which is

Table 11. Fundamental Frequency Parameter Ω_1 for Clamped-Clamped Fluid Conveying Pipes for Various Values of γ_1 and γ_2, for $\beta = 0.1$ and $\beta = 0.5$

γ_2	V	γ_1									
		0.01		0.50		2.50		10.0		10^2	
		β =0.1	β =0.5	β =0.1	β =0.5	β =0.1	β = 0.5	β =0.1	β =0.5	β =0.1	β =0.5
0.01	1.0	22.11	22.06	22.12	22.07	22.16	22.11	22.33	22.28	24.26	24.20
	2.0	21.21	20.99	21.22	21.00	21.27	21.05	21.44	21.22	23.44	23.19
0.5	1.0	23.49	23.74	23.50	23.75	23.54	23.79	23.70	23.95	25.54	25.81
	2.0	22.64	22.70	22.65	22.71	22.69	22.76	22.86	22.92	24.75	24.82
1.0	1.0	24.82	25.41	24.83	25.42	24.88	25.46	25.03	25.62	26.78	27.41
	2.0	24.02	24.40	24.03	24.41	24.07	24.45	24.23	24.61	26.03	26.44
2.5	1.0	28.51	30.22	28.52	30.23	28.55	30.27	28.69	30.41	30.25	32.08
	2.0	27.80	29.26	27.80	29.27	27.84	29.31	27.98	29.46	29.58	31.15
10.0	1.0	42.72	53.22	42.73	53.23	42.75	53.27	42.85	53.41	43.96	55.13
	2.0	42.23	52.21	42.24	52.22	42.26	52.26	42.36	52.39	43.48	54.07

within acceptable engineering norms. The numerical values are shown in Tables **3** and **4**.

b. Further validation of the model was done by comparing the results obtained for a pipe conveying fluid without foundation, for three different mass ratios β and for all the three boundary conditions. Comparison of the results by visual inspection for pinned-pinned and clamped-clamped end conditions with those of Païdoussis & Issid [2], show good agreement.

c. For a pipe conveying fluid and resting on Winkler foundation, i.e. for $\gamma_2 = 0.0$, the results compare very well with those from Raghava Chary *et al*. [6] for all the three end conditions. Results have been presented for three values of β.

d. For a fluid-conveying pipe, resting on a two-parameter foundation, new results are presented in Tables **9**, **10** and **11**. The effect of the Pasternak foundation parameter on the fundamental frequency is clearly brought out in Figs. (**8**-**10**). The second foundation parameter γ_2 tends to increase the fundamental frequency as well as the critical flow velocity for the same Winkler constant γ_1. The effect of γ_2 may be interpreted in the following way: A pinned-pinned fluid conveying pipe, which is the weakest as far as stability is concerned, acquires the stability of a clamped-clamped pipe by increasing the shear parameter of the foundation. Also, it is found that as the flow velocity increases, the effect of the mass ratio β diminishes.

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