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Single Particle Schrödinger Fluid and Moments of Inertia of the Even-Even Uranium Isotopes

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Abstract: We have applied the concept of the single-particle Schrödinger fluid to calculate the reciprocal values of the rigid body-model, the cranking-model, and the equilibrium-model moments of inertia of the axially-symmetric deformed nuclei as functions of the deformation parameter β, and the non deformed oscillator parameter $\hbar\omega_0^0$. As examples for the application of this concept to the heavier deformed even-even nuclei the uranium isotopes ^{230}U, ^{232}U, ^{234}U, ^{236}U and ^{238}U have been chosen. The obtained results showed that the assumption that these nuclei have axes of symmetry is more reliable.

1. INTRODUCTION

It is well known that the nucleons inside the nucleus occupy approximately 1/50 of the volume of the nucleus. It is not surprising to find that nucleon properties are maintained inside the nucleus. In particular this situation is responsible for the fact that the magnetic moments of nucleons inside nuclei are the same as for free nucleons. In accordance with the above, we describe the motion of each nucleon individually in a common potential field. Because the surface of deformed nuclei is distorted at some moment, the potential felt by the nucleons is not spherically symmetric.

The basic ideas concerning non spherical nuclei have been most completely described by A. Bohr [1]. The most important distinction between non spherical and spherical nuclei is that the former can have rotational levels. The elongation of the nucleus is related to the interaction between the surface and the nucleons outside closed shells. Many of the light nuclei are spherical. This is due to the success of the shell model, which is based on states in a field of spherical symmetry. According to the basic ideas of quantum mechanics the concept of rotation in a spherically symmetric system is meaningless. However, in an elongated nucleus the concept of rotation is meaningful, and the nucleus can rotate about an axis perpendicular to the axis of symmetry.

An axially symmetric deformed nucleus is characterized by the moment of inertia about the axis perpendicular to the symmetry axis of the nucleus, its magnetic dipole moment and its electric quadrupole moment. In the present paper we are only interested in the nuclear moments of inertia. Accordingly, we have applied the concept of the single-particle Schrödinger fluid to calculate the reciprocal values of the rigid-body model, the cranking-model and the equilibrium - model moments of inertia of the uranium even-even isotopes ^{230}U, ^{232}U, ^{234}U, ^{236}U, and ^{238}U in their ground states as functions of the deformation parameter β and the non-deformed oscillator parameter $\hbar\omega_0^0$. Variations of these three reciprocal moments of inertia with the deformation parameter are also given.

2. THE SCHRÖDINGER FLUID

Consider a nucleon that is moving in a potential field, whose Hamiltonian operator is given by

$$H(\mathbf{r},\mathbf{p},\alpha(t)) = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},\alpha(t)). \qquad (2.1)$$

Here α represents some time-dependent collective parameter which in the case of rotation becomes the angle of rotation $\theta = \Lambda t$, where Λ is the angular velocity.

The single-particle time-dependent Schrödinger equation for this nucleus is

$$H(\mathbf{r},\mathbf{p},\alpha(t))\ \Psi_k(\mathbf{r},\alpha(t),t) = \mathbf{i}\hbar\frac{\partial}{\partial t}\Psi_k(\mathbf{r},\alpha(t),t) \qquad (2.2)$$

This equation has solutions of the form [2,3]

$$\Psi_k(\mathbf{r},\alpha(t),t) = \psi_k(\mathbf{r},\alpha(t))\exp\{-\frac{i}{\hbar}\int_0^t \varepsilon_k(\alpha(t'))\,dt'\}, \qquad (2.3)$$

where ε_k is the single-particle energy density, which depends on the time through the parameter $\alpha(t)$ The complex wave function $\psi_k(\mathbf{r},\alpha(t))$ can be written in the following polar form

$$\psi_k(\mathbf{r},\alpha(t)) = \Phi_k(\mathbf{r},\alpha(t))\exp\{-i\frac{m}{\hbar}S_k(\mathbf{r},\alpha(t)), \qquad (2.4)$$

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where Φ_k and S_k are assumed to be real functions and Φ_k is positive.

Substituting from equations (2.1), (2.3) and (2.4) into equation (2.2) and separating the real and the imaginary parts; a pair of coupled equations for Φ_k and S_k are obtained. The first is

$$\rho_k \nabla . v_k + v_k . \nabla \rho_k = -\frac{\partial \rho_k}{\partial t} , \tag{2.5}$$

which is the well-known equation of continuity in fluid mechanics. The density ρ_k is equal to the probability density of the particle distribution, $\rho_k = \Phi_k^2$, and the irrotational velocity field v_k is defined by

$$\mathbf{v}_k = - \nabla S_k , \tag{2.6}$$

$$S_k = \frac{i\hbar}{2m} Ln(\Psi_k / \Psi_k^*). \tag{2.7}$$

The second equation is

$$(H + V_{dyn}) \Phi_k = \varepsilon_k \Phi_k , \tag{2.8}$$

which is a modified Schrödinger equation through the modified dynamical potential

$$V_{dyn} = - m(\frac{\partial S_k}{\partial t} - \frac{1}{2} v_k^2). \tag{2.9}$$

In the adiabatic approximation where $\frac{d\alpha}{dt} \to 0$, the kth single-particle wave function ψ_k of equation (2.4) is approximated by [2]

$$\psi_k = \Phi_k \exp(-imS_k/\hbar) \cong u_k + i\mu_k , \tag{2.10}$$

where $u_k = u_k(\mathbf{r}, \alpha)$ is the quasi-static wave function satisfying the quasi-static Schrödinger equation

$$Hu_k = \varepsilon_k u_k = \varepsilon_k |k\rangle, \tag{2.11}$$

and μ_k is the first-order time-dependent perturbation correction, which for rotation about the x-axis is given by

$$\mu_k = \Lambda \sum_{j \neq k} \frac{\langle j|L_x|k\rangle}{\varepsilon_j - \varepsilon_k} | j\rangle. \tag{2.12}$$

The collective kinetic energy for a nucleon, in the adiabatic approximation, is given by [2]

$$T_k = \frac{m}{2} \int \rho_k \mathbf{v}_k . (\Lambda \times \mathbf{r}) d\tau , \tag{2.13}$$

and the collective kinetic energy of the nucleus is then given by

$$T = \frac{m}{2} \int \rho_T \mathbf{v}_T . (\Lambda \times \mathbf{r}) d\tau, \tag{2.14}$$

where ρ_T is the total density distribution of the nucleus and $\mathbf{v}_T$ is the total velocity field

$$\mathbf{v}_T = \frac{\sum_{k=occ} \rho_k \mathbf{v}_k}{\sum_{k=occ} \rho_k} . \tag{2.15}$$

The probability density of the nucleon distribution, in view of equation (2.10), assumes the form

$$\rho_k = u_k^2 + \mu_k^2 . \tag{2.16}$$

The occurrence of the two distinct velocity fields in equations (2.13) and (2.14), $\mathbf{v}$ and $\Lambda \times \mathbf{r}$ reflects the two essential aspects of the cranking motion: (i) the rotation of the potential well which can be described by the velocity field ($\Lambda \times \mathbf{r}$), and (ii) the response of the individual particle to that motion of the potential which is described by the velocity $\mathbf{v}$.

For the quasi-static motion of the nucleon we choosed the anisotropic harmonic oscillator potential to be the average potential field of the nucleus with oscillator frequencies given by [4]

$$\omega_x^2 = \omega_y^2 = \omega_0^2 \left(1 + \frac{2\delta}{3}\right), \tag{2.17}$$

$$\omega_z^2 = \omega_0^2 \left(1 - \frac{4\delta}{3}\right), \tag{2.18}$$

where δ is a deformation parameter related to the well-known deformation parameter β by

$$\delta = \frac{3}{2} \sqrt{\frac{5}{4\pi}} \beta . \tag{2.19}$$

In equations (2.17) and (2.18) the frequency ω_0 is given in terms of the non deformed frequency ω_0^0 by [4]

$$\omega_0 = \omega_0(\delta) = \omega_0^0 \left(1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right)^{-\frac{1}{6}}. \tag{2.20}$$

In the last equation ω_0^0 is the non-deformed frequency, which equals $\omega_0(\delta)$ when δ equals zero. The non-deformed oscillator parameter in our calculations is given in terms of the mass number A, the number of neutrons N, and the number of protons Z by [5]

$$\hbar\omega_0^0 = \frac{38.6 A^{-\frac{1}{3}}}{\left[1 + \frac{1.646}{A} - \frac{0.191(N - Z)}{A}\right]^2}. \tag{2.21}$$

The following expressions for the cranking-model and the rigid body-model moments of inertia can be easily obtained on the basis of the concept of the single-particle Schrödinger fluid [3]

$$\Im_{cr} = \frac{E}{\omega_o^2}\frac{1}{6+2\sigma}\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[\sigma^2(1+q)+\frac{1}{\sigma}(1-q)\right], \quad (2.22)$$

$$\Im_{rig} = \frac{E}{\omega_o^2}\frac{1}{6+2\sigma}\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[(1+q)+\sigma(1-q)\right], \quad (2.23)$$

where q is the anisotropy of the configuration, which is defined by

$$q = \frac{\sum_{occ}(n_y + \frac{1}{2})}{\sum_{occ}(n_z + \frac{1}{2})}, \quad (2.24)$$

and E is the total energy given by

$$E = \sum_{occ}\left[\hbar\omega_y(n_y + n_x + 1) + \hbar\omega_z(n_z + \frac{1}{2})\right]. \quad (2.25)$$

In equations (2.24) and (2.25) $n_x, n_y,$ and n_z are the state quantum numbers of the oscillator. In equations (2.22) and (2.23) σ is a measure of the deformation of the potential and is defined by

$$\sigma = \frac{\omega_y - \omega_z}{\omega_y + \omega_z}. \quad (2.26)$$

We note that the cranking-model and the rigid body-model moments of inertia are equal only when the harmonic oscillator is at the equilibrium deformation.

3. RESULTS AND CONCLUSIONS

The total energy E and the anisotropy of the configuration q are easily calculated for a given nucleus with mass

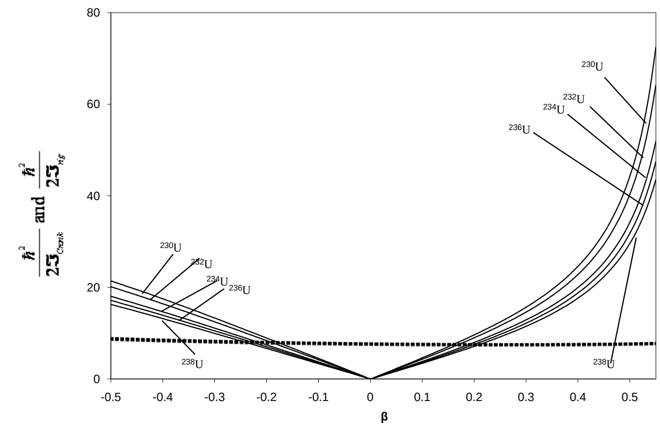

Fig. (1). Moments of inertia of the axially deformed nuclei ^{230}U, ^{232}U, ^{234}U, ^{236}U and ^{238}U. The solid curves represent the cranking-model moments of inertia. The dotted curve represents the rigid-body moment of inertia of the nucleus ^{234}U.

number A, number of neutrons N and number of protons Z. Accordingly, the cranking-model and the rigid-body model moments of inertia are obtained as functions of the deformation parameter β and the non deformed oscillator parameter $\hbar\omega_0^0$ by suitable filling of the single-particle states corresponding to the ground-state of the given nucleus.

In Fig. (**1**) we present the variations of the reciprocal values of the cranking-model moments of inertia of the uranium isotopes ^{230}U, ^{232}U, ^{234}U, ^{236}U and ^{238}U with respect to the deformation parameter β. Since the reciprocal values of the rigid-body moments of inertia of these isotopes are slowly varying with respect to β, we present only in Fig. (**1**) the variation of the reciprocal values of the rigid-body model moment of inertia of the nucleus U^{234} with respect to β.

In Table **1** we present the calculated values of the reciprocal moments of inertia for the uranium isotopes:

Table 1. Rigid-Body and Cranking-Model Reciprocal Moments of Inertia, in KeV, of the Uranium Isotopes

Nucleus	β	$\hbar\omega_0^0$ (MeV)	$\frac{\hbar^2}{2\Im_{rig}}$	$\frac{\hbar^2}{2\Im_{cr}}$	$\frac{\hbar^2}{2\Im_{exp}}$
^{230}U	0.18	6.69	7.75	8.63	
	-0.19	6.69	8.23	8.74	8.68
^{232}U	0.19	6.69	7.60	8.27	
	-0.20	6.69	8.09	8.32	8.28
^{234}U	0.19	6.69	7.43	7.31	
	-0.20	6.69	7.88	7.25	7.29
^{236}U	0.20	6.69	7.33	7.55	
	-0.21	6.69	7.80	7.49	7.57
^{238}U	0.19	6.68	7.29	7.80	
	-0.21	6.68	7.76	7.84	7.82

Table 2. Reciprocal Equilibrium-Deformation Moments of Inertia of the Uranium Isotopes, in KeV

Nucleus	β	$\hbar\omega_0^{\ 0}$ (MeV)	$\frac{\hbar^2}{2\Im_{equ}}$	$\frac{\hbar^2}{2\Im_{exp}}$
^{230}U	0.160	6.69	7.76	
	-0.185	6.69	8.20	8.68
^{232}U	0.170	6.69	7.62	
	-0.197	6.69	8.08	8.28
^{234}U	0.194	6.69	7.43	
	-0.220	6.69	7.92	7.29
^{236}U	0.195	6.69	7.35	
	-0.218	6.69	7.83	7.57
^{238}U	0.182	6.68	7.29	
	-0.205	6.68	7.75	7.82

^{230}U, ^{232}U, ^{234}U, ^{236}U and ^{238}U for the rigid-body model and the cranking-model together with the corresponding experimental values [6]. The values of the deformation parameter β and the non deformed oscillator parameter $\hbar\omega_0^0$ are also given in this table.

In Table **2** we present the calculated values of the reciprocal moments of inertia for the five nuclei in the case of the equilibrium deformation, where the rigid and the cranking reciprocal moments are equal. The values of the deformation parameter β and the non deformed oscillator parameter $\hbar\omega_0^0$ are also given in this table.

It is seen from the tables that the obtained results of the reciprocal cranking-model moments of inertia are in good agreement with the corresponding experimental values for the five isotopes while the values of the reciprocal moments of inertia by using the other two models are not in good agreement with the corresponding experimental values, a result which has been also occurred for the even-even nuclei in the s-d shell [7,8] and showed that the pairing correlation [9] is very important to improve the results in these two cases.

In Appendix-**1** we present the protons oscillator quantum numbers for the ground-state wave function of the nucleus $^{230}_{92}U$, the corresponding quantum numbers for neutrons are similar. The classifications of the protons and neutrons in the ground-state wave functions of the other isotopes are also similar.

Appendix 1. Protons Oscillator Quantum Numbers for the Ground State Wave Function of the Nucleus $^{230}_{92}U$

shell qu. no.	n_z	n_y	n_x	shell qu. no.	n_z	n_y	n_x
1s, *N*=0	0	0	0	1d, *N*=2	0	0	2
	0	0	0		0	0	2
1p, *N*=1	1	0	0		2	0	0
	1	0	0		2	0	0
	0	1	0		1	1	0
	0	1	0		1	1	0
	0	0	1		1	0	1
	0	0	1		1	0	1
2s, *N*=2	0	2	0		0	1	1
	0	2	0		0	1	1

(Appendix 1). Contd…..

Shell qu. no.	n_x	n_y	n_z	Shell qu. no.	n_x	n_y	n_z
1f, *N*=3	2	1	0	1g, *N*=4	2	1	1
	2	1	0		2	1	1
	1	2	0	2d, *N*=4	1	2	1
	1	2	0		1	2	1
	3	0	0		0	2	2
	3	0	0		0	2	2
	0	3	0		0	1	3
	0	3	0		0	1	3
	1	0	2		1	0	3
	1	0	2		1	0	3
	0	1	2		1	1	2
	0	1	2		1	1	2
	0	2	1	3s, *N*=4	0	0	4
	0	2	1		0	0	4
2p, *N*=3	2	0	1	1h, *N*=5	5	0	0
	2	0	1		5	0	0
	1	1	1		0	5	0
	1	1	1		0	5	0
	0	0	3		0	0	5
	0	0	3		0	0	5
1g, *N*=4	4	0	0		4	1	0
	4	0	0		4	1	0
	0	4	0		4	0	1
	0	4	0		4	0	1
	3	1	0		0	4	1
	3	1	0		0	4	1
	1	3	0		0	1	4
	1	3	0		0	1	4
	2	2	0		1	4	0
	2	2	0		1	4	0
	2	0	2		1	0	4
	2	0	2		1	0	4
	0	3	1		3	2	0
	0	3	1		3	2	0
	3	0	1		3	0	2
	3	0	1		3	0	2

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