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MHD Unsteady Flow of a Second Order Fluid Through Porous Region Bounded by Rotating Infinite Plate

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Abstract: The aim of the paper is to study non Newtonian fluid flow of second order flowing through porous medium bounded by an impermeable infinite plate rotating with acceleration. We have studied all the special cases regarding the effect of permeability of the medium and the micro rotation of the fluid above the infinite plate under the influence of magnetic field. Various situations of the flow with Newtonian case are derived as special cases. Several investigations are made at length. The effect of porous medium increase the flow of the fluid with a decrease in permeability K.

The presence of magnetic parameter decrease the flow pattern as its values increase. The effects of magnetic parameter and permeability will decrease the boundary layer and the results of steady non Newtonian Fluid flow will also be deduced as $k \to 0$.

Mathematics Subject Classification: 76D, 76S.

Keywords: Rotating acceleration permeability, porous medium.

1. INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse fields of Science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering, and also in the inter disciplinary fields such as bio medical engineering. The lung alveolar is an example that finds application in an animal body. The classical Darcy's law Musakat [1, 2] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as

$\vec{v} = -\left(\frac{k}{\mu}\right)\nabla p$ with usual notation.

The classical Darcy's law gives good result in the situations where the flow is unidirectional or at low speed. In general, specific discharge in the medium need not be always low. As the specific discharge increase the convective forces developed and the internal stress generates in the fluid due to viscous nature and produces distortions in the velocity field. Modifications in the classical Darcy's law are considered by Beavers and Joseph [3] Saffman [4] and others. A generalized Darcy's law is proposed by Brinkman [5].

$$\rho\frac{d\bar{v}}{dt} = Div\ S_{ij} - \left(\frac{\mu}{k}\right)\vec{v}$$

where S_{ij} is the Stress Tensar of the fluid, ρ is the density, $\bar{v}$ is velocity of the fluid, k is the permeability coefficient.

Yamamoto. and Iwamura [6], V. Narasimhacharyulu [7, 8], Narasimhacharyulu and Pattabi Ramacharyulu [9] and Narasimhacharyulu and Sunder Ram [10, 11], and several other investigators adopted the generalized law proposed by Brinkman.

The visco elastic fluid of second order is one of the popular models of non-Newtonian fluid. The Rivlin Erickson constitutive equations for the fluid is given by

$$T = -PT + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2$$

with

$$A_1 = (\nabla\bar{v}) + (\nabla\bar{v})^T \tag{1.1}$$

$$A_2 = \frac{dA_1}{dt} + (\nabla\bar{v})^T A_1 + A_1(\nabla v)$$

A_1, A_2 are called Rivilin Ericson tensors denoting the rate of strain and the acceleration, p is the pressure, T is the Cauchy stress Tensor, μ, α_1, α_2 are the material constants representing viscosity, lasticity, and cross viscosity, $\frac{d}{dt}$ are the material derivatives. Fosdick and Raja Gopal [12] discussed about the Rivlin Erickson tensors and a comprehensive discussion of the material constants is available in the work of Dunn and Rajagopal [13]. The signs of the material moduli α_1 and α_2 are the subject of controversy. The experiments on the non-Newtonian fluids do not confirm to the restrictions of α_1 and α_2. Thus, it may

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be stated that the fluids that have been tested are not second grade fluids and are characterized by different constitutive structure. Second grade fluids are dilute polymeric solutions, such as poly-iso-butylene, methyl-methacrylate in the n-butyl acetate, poly ethylene oxide in water etc. Rajagopal and Gupta [14] have studied the Thermo Dynamics in the form of Dissipation Helmholtz Inequality, and conversely accepted that the specific free energy should be minimum inequilibrium. It is assumed from thermodynamic consideration that $\mu \geq 0, \alpha_1 > 0, \alpha_1 + \alpha_2 = 0$. The Colemann and Noll [15] derived the constitution equation (1.1) for simple fluids assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in distant past.

A number of investigators Ting [16], Erdogen [17], Light hill [18] Stuart [19] have studied the unsteady flow of second order fluids. The behavior of visco elastic fluid in Laminar flow through Porous Medium has been the subject of many investigators including Pillitis and Beires [20], H. Pascal and F. Pascal [21], Jones and Walters [22], Vafai and Kim [23] etc. and V.A. Krysko *et al*. have discussed about the non classical thermo elastic problems in non linear dynamics of shells [24]. The published results related to the paper can be found in their monograph.

An unsteady flow of visco elastic second order fluids under the influence of magnetic field has many applications such as Electro Magnetic Propulsions and the flow of nuclear fuel slurries etc.

In the present problem, the unsteady flow of a visco elastic second order fluid through porous medium past an infinite plate is considered under the influence of magnetic field. The effect of magnetic permeability and the permeability of the porous medium both are examined through the flow of the fluid. Different situations are considered and deduced as special cases. The graphical representation is given to illustrate the effect of the parameters on the flow.

2. FORMULATION AND SOLUTION OF THE PROBLEM:

The coordinate system is taken so that y-axis is perpendicular to the plate and x-axis in the plane along the direction of the fluid motion. The system containing the fluid and porous medium is put into rotation by the impermeable boundary infinite plate.

The equation of continuity is satisfied by (u (y, t), v (y, t), 0) velocity of the fluid.

$$\text{div} = \bar{\text{V}} = 0 \tag{2.1}$$

The velocity components are taken so that u and v are the functions of y, t only.

$$\frac{\partial \vec{v}}{\partial t} + (v.\nabla)\vec{v} + 2\Omega_k \times \vec{v} = \frac{-1}{\rho}\nabla p - \left(\frac{v}{k} + \sigma\beta_o^2\right)\vec{v} + \left(v + \alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 \vec{v}}{\partial y^2} \tag{2.2}$$

where $\alpha = \dfrac{\alpha_1}{\rho}, \nu = \dfrac{\mu}{\rho}$

Let u(t) be the free stream velocity parallel to the plate. The velocity components should be independent of x axis as the plate is infinite. The velocity and other physical quantities depend on y and t only.

$$\frac{\partial u}{\partial t} - 2\Omega u = \left(v + \alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \left(\frac{v}{k} + \sigma\beta_o^2\right)u \tag{2.3}$$

$$\frac{\partial u}{\partial t} + 2\Omega v = \left(v + \alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial y^2} - \left(\frac{v}{k} + \sigma\beta_o^2\right)v \tag{2.4}$$

By writing $\bar{\text{V}} = u + iv$

Rewriting the above equations into one equation we get

$$\frac{\partial \bar{v}}{\partial t} + i2\Omega\bar{v} = \left(v + \alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 \bar{v}}{\partial y^2} - m^2\bar{v}$$

where

$$m^2 = v\left(\frac{1}{k} + \sigma\beta_0^2\right) \tag{2.5}$$

With the Boundary conditions

$$\begin{aligned} &\bar{v}(y,t) = 0 \quad \text{for all } \ y, t \leq 0 \\ &\bar{v}(y,t) = u(t)e^{i\omega t}, \ u = u_0 \quad \text{for } \ y = 0, t > 0 \end{aligned} \tag{2.6}$$

where u(t) is a constant denoting uniform velocity of the fluid.

By using suitable non-dimensional quantities the non-dimensional equation of motion is given by

$$\frac{\partial^2 \bar{\upsilon}}{\partial y^2} - \frac{1}{2}\left(\frac{iE + E + \frac{1}{K}}{1 + s\sigma}\right)\frac{\partial \bar{v}}{\partial t} = 0, E = \frac{\Omega\upsilon}{u^2} \tag{2.7}$$

By applying Laplace transform

$$v(y,t) = \int_0^\infty e^{-st}\,\bar{v}(t)\,dt \tag{2.8}$$

where s is the parameter of Laplace. Transform with

$$\left.\begin{aligned} &\bar{v} = 0 \ \text{ for all } \ y,t \leq 0 \\ &\bar{v} = \frac{1}{s - i\sigma} \ \text{ at } \ y = 0, t > 0 \end{aligned}\right\} \tag{2.9}$$

$v \to 0$ is finite as $y \to \infty$ and $y \to 0$

The solution of $\bar{v}$ is given by

$$\begin{aligned} &v = \exp[i\sigma t - y(A + iB)] \\ &v = e^{-Ay}\sin(\sigma t - yB) \end{aligned} \tag{2.10}$$

$$v = e^{-Ay}\sin(\sigma t - yB) \tag{2.11}$$

where

$$\left.\begin{aligned} &A=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2+\ell_1\ell_2} \\ &B=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2-\ell_1\ell_2} \\ &\ell_1=\frac{2\alpha^2-ES(2+\sigma)}{2(1+\sigma^2 s^2)} \\ &\ell_2=\frac{E(2+\sigma)+2\alpha^2\sigma s}{2(1+\sigma^2 s^2)} \\ &\text{and } \alpha^2=\frac{1}{k}+\frac{\alpha_1\beta^2\upsilon}{\rho v^2} \end{aligned}\right\} \tag{2.12}$$

3. SPECIAL CASES

The solution

At y = 0, t = 0 u and v are assumed zero.

The flow will be exponential type for t = ∞, the velocity v is zero for both Newtonian and non Newtonian cases and if y is ∞, total velocity Q_v will be zero

Case-1: At t = 0 and y = 0 the real velocity u = 0 and v = 0 i.e. initial velocity of the flow of the fluid is zero

$$\left.\begin{aligned} u&=e^{-Ay}\cos(\sigma t-yB) \\ v&=e^{-Ay}\sin(\sigma t-yB) \end{aligned}\right\} \tag{2.13}$$

u and v are velocities in Horizontal and perpendicular to the plane yx.

At y = 0 u and v will be sinusoidal.

Given by
$$\left.\begin{aligned} u&=\cos\sigma t \\ v&=\sin\sigma t \end{aligned}\right\} \tag{2.14}$$

Case-2: The case of non Newtonian fluid through porous medium bounded by infinite and impermeable plate rotating with angular velocity will become

$$\upsilon=\exp\left[i\sigma t-y(A+iB)\right] \tag{2.15}$$

with $\alpha^2=\dfrac{\sigma\beta_0^2\upsilon}{\rho\mu^2}$

and

$$\left.\begin{aligned} &u=e^{Ay}\cos(\sigma t-yB) \\ &v=e^{Ay}\sin(\sigma t-yB) \\ &\text{Where } A=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2+\ell_1\ell_2} \\ &B=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2-\ell_1\ell_2} \\ &\text{and } \ell_1=\frac{\frac{2\sigma\beta_0^2\upsilon}{\rho u^2}-ES(2+\sigma)}{2(1+\sigma^2 s^2)} \\ &\ell_2=\frac{E(2+\sigma)+\frac{2\sigma^2\beta_0^2\upsilon}{\rho u^2}}{2(1+\sigma^2 s^2)} \end{aligned}\right\} \tag{2.16}$$

Case-3: The flow of non Newtonian fluid through porous medium not under the influence of magnetic field which is rotating about y axis is obtained by replacing

The velocity components are given by

$$\bar{\upsilon}=\exp\left[i\sigma t-y(A+iB)\right] \text{ with } \alpha^2=\frac{1}{k} \tag{2.17}$$

where

$$\left.\begin{aligned} &u=e^{-Ay}\cos(\sigma t-yB) \\ &v=e^{-Ay}\sin(\sigma t-yB) \\ &A=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2+\ell_1\ell_2} \\ &B=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2-\ell_1\ell_2} \\ &\ell_1=\frac{2\frac{1}{k}-ES(2+\sigma)}{2\left(1+\sigma^2 s^2\right)} \\ &\ell_2=\frac{E(2+\sigma)+2\frac{\sigma}{k}s}{2\left(1+\sigma^2 s^2\right)} \end{aligned}\right\} \tag{2.18}$$

Case-4: The flow of the non Newtonian fluid $\alpha^2 = 0$ i.e. the flow is independent of magnetic field and the permeability K is infinity i.e., there is no resistance from the medium. In this case

$$v=\exp\left[i\sigma t-y(A+iB)\right] \tag{2.19}$$

$$\left.\begin{aligned} &u=e^{-Ay}\cos(\sigma t-yB) \\ &v=e^{-Ay}\sin(\sigma t-yB) \\ &A=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2+\ell_1\ell_2} \\ &B=\frac{1}{\sqrt{2}}\cdot\sqrt{\ell_1^2+\ell_2^2-\ell_1\ell_2} \\ &\ell_1=\frac{-ES(2+\sigma)}{2\left(1+\sigma^2 s^2\right)} \\ &\ell_2=\frac{E(2+\sigma)}{2\left(1+\sigma^2 s^2\right)} \end{aligned}\right\} \tag{2.20}$$

There results are relating to the results obtained by Komal Kumar and Varshney [25].

Case-5 : The flow of the Newtonian fluid when the plate is not rotating and k is ∞ i.e. the medium is clear in this case the flow will be non Newtonian through clear medium under magnetic field and without rotation.

a) If s = 0 the solution will coincide with the results given by Thornely [26], and for the case of non rotating infinite plate with magnetic field i.e. σ = 0, k = ∞ the results agree with Gupta [27].

b) Taking k→∞ and s = 0 we can deduce all the results of Debnath and Mukerjee [28], Kishore [29].

CONCLUSIONS

We have discussed the flow of non Newtonian fluid through porous medium bounded by an impermeable infinite rotating plane with magnetic field and all the results of Newtonian fluids are derived as s = 0. The rotations of non Newtonian fluid through porous medium under magnetic field are also derived. The results of the flow of fluid in Newtonian and non Newtonian cases are graphically represented (Figs. **1-12**). The observations are that the flow of Newtonian Fluid and non Newtonian Fluids are same with small difference of the flow pattern.

The effect of porous medium increase the flow of the fluid with a decrease in permeability K (Figs. **1**, **3** **7**, **9**).

The presence of magnetic parameter decrease the flow pattern its values increase (Figs. **2**, **4**, **8**, **10**). The effect of magnetic parameter and permeability will decrease the boundary layer the results of steady non Newtonian Fluid flow will be deduced as k→0. Figs. (**5**, **11**) shows the variation of velocity for different Non Newtonian parameters as the non Newtonian parameter is increasing the velocity is decreasing. The variation of velocity for different time periods is illustrated in (Figs. **6**, **12**) as the time t is increasing the velocity is decreasing.

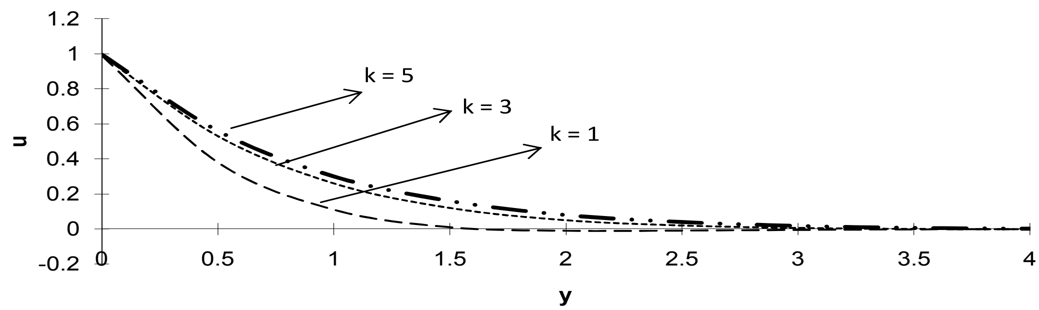

Fig. (1). Variation of velocity u with different permeabilities.

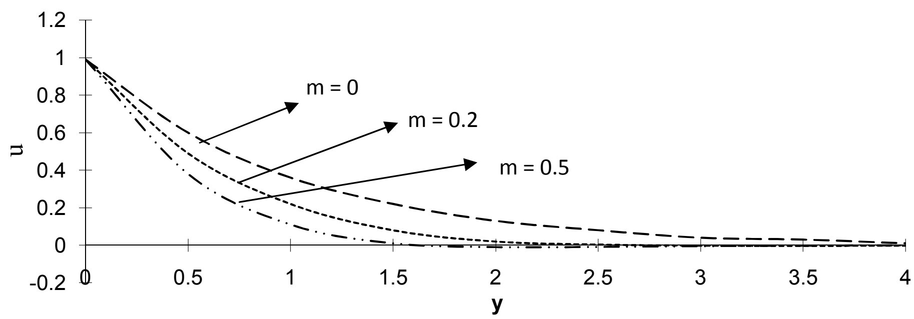

Fig. (2). Variation of velocity u with different magnetic parameters.

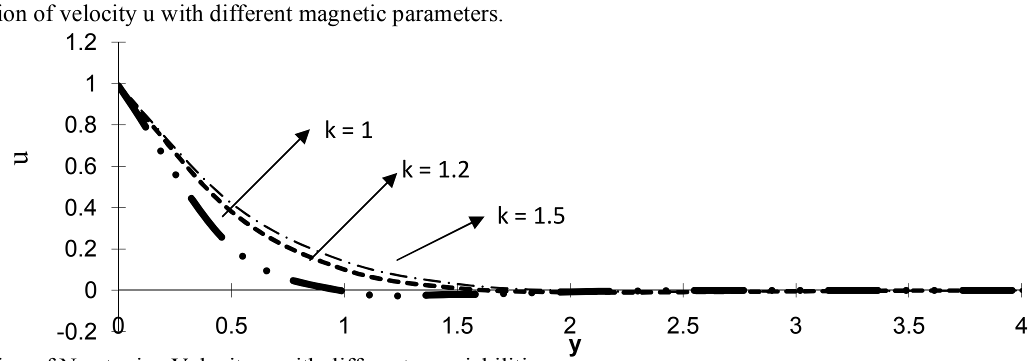

Fig. (3). Variation of Newtonian Velocity u with different permiabilities.

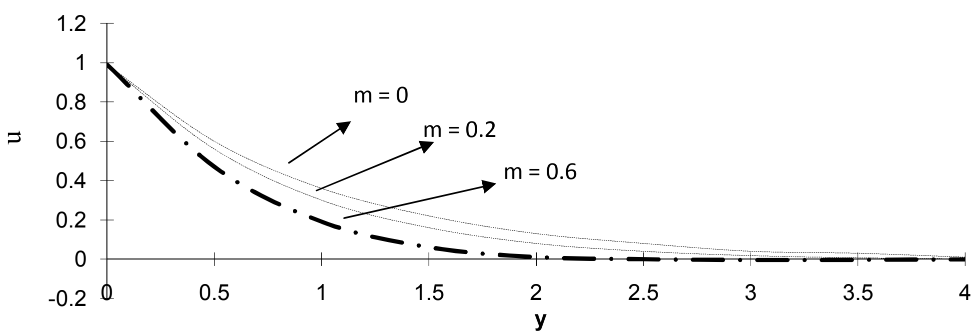

Fig. (4). Variation of Newtonian Velocity u with different magnetic parameters.

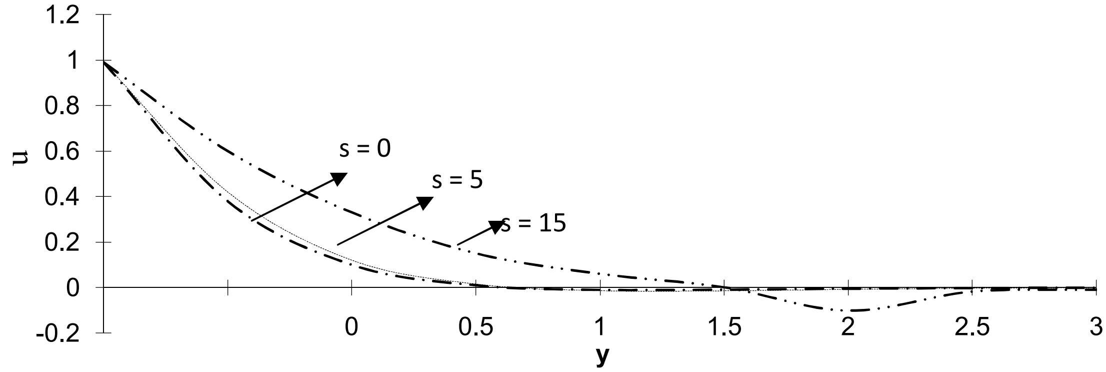

Fig. (5). Variation of Velocity u for different Non-Newtonian parameters.

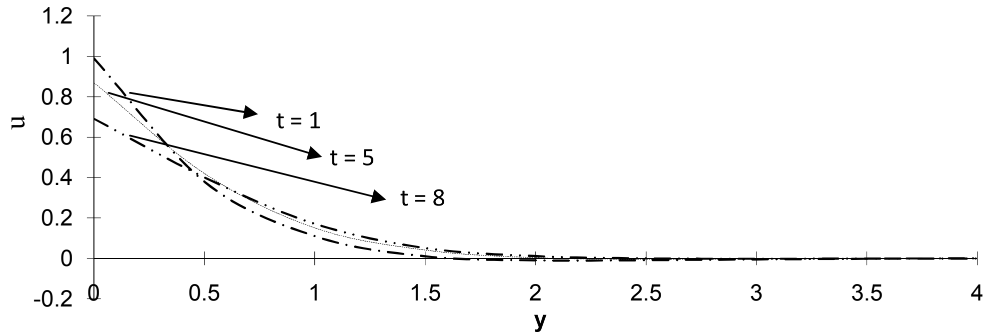

Fig. (6). Variation of Velocity u for different time periods.

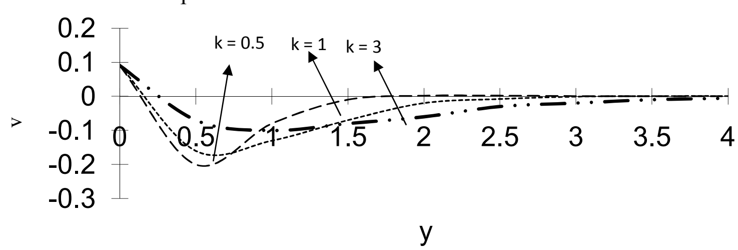

Fig. (7). Variation of Velocity v for different permeabilities.

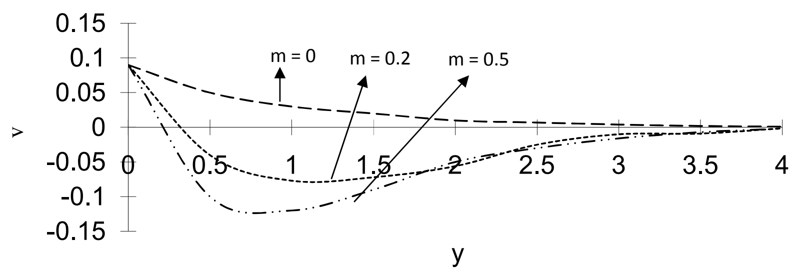

Fig. (8). Variation of Velocity v for different magnetic parameters.

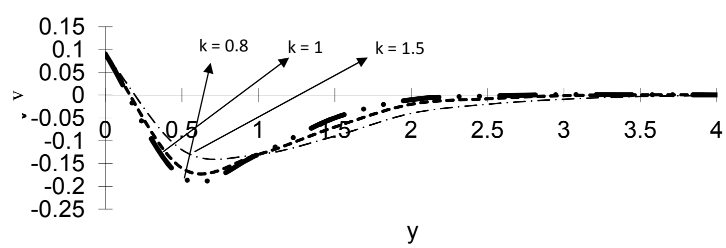

Fig. (9). Variation of Newtonian Velocity v for different permeabilities.

GRAPHICAL REPRESENTATION

$\bar{v} = u + iv$ is the real velocity, v is the imaginary velocity.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

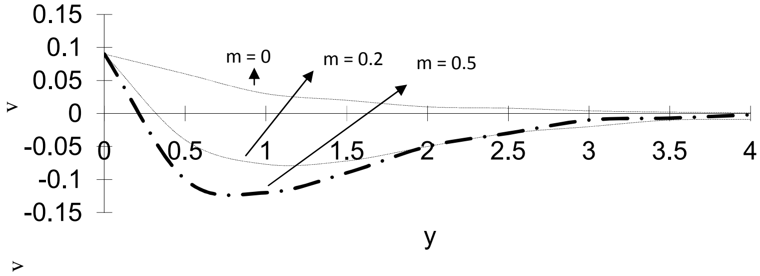

Fig. (10). Variation of Newtonian Velocity v for different magnetic parameters.

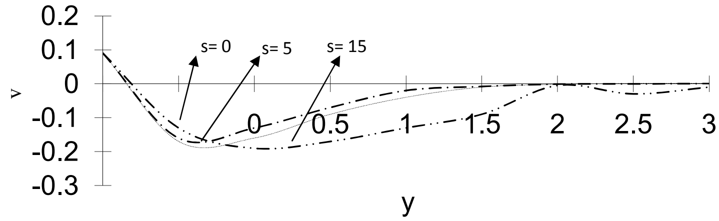

Fig. (11). Variation of Velocity v for different Non-Newtonian parameters.

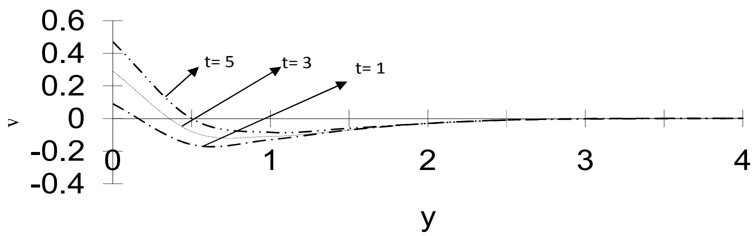

Fig. (12). Variation of Velocity v for different time periods.

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