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Free Convection Flow with Variable Viscosity Through Horizontal Channel Embedded in Porous Medium

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Abstract: Laminar free convection flow of viscous, incompressible fluid with variable viscosity and viscous dissipation through two horizontal parallel walls embedded in a porous medium is studied. Arrhenius model is used in which the variable viscosity is used to be decreasing exponentially with temperature. An approximation technique is defined to obtain the solution of the coupled non-linear equations of the velocity and the temperature fields. The expressions for wall shear stress and the rate of heat transfer are also derived. The effects of various parameters, entered into the momentum and energy equations, are studied on velocity field, temperature distribution, wall shear stress and rate of heat transfer, and are shown graphically.

INTRODUCTION

In nature and in engineering problems on the convective flow, viscosity of many fluids varies with temperature. Therefore, the results drawn from the flow of fluids with constant viscosity are not applicable for the fluid that flows with temperature dependent viscosity, particularly at high temperature. The fluids that flow with variable viscosity are useful in chemical, bio-chemical and process industries as well as in physics of fluid flows, wherein the flow of fluids is governed by different temperatures. A number of approximate solution techniques is available in the open literature for the convective flow in the vicinity of a vertical flat plate, embedded in porous medium under different conditions, as proposed by Raptis [1], Cheng and Pop [2], Ingham and Pop [3], Haq and Mulligan [4], and Pop and Herwig [5].

Rao and Pop [6] have studied free convection in a fluid saturated porous medium with temperature dependent viscosity. Recently, Singh et. al. [7] have studied free convection flow with temperature dependent viscosity in a fluid saturated porous medium, along a vertical isothermal and non-isothermal plate using Karman-Pohlhausen integral method. More recently, Kankane and Gokhale [8] have used Arrhenius model (commonly known as exponential model) to study fully developed flow through horizontal channel, wherein variable viscosity is decreasing exponentially with temperature. However, there are situations when the fluid with variable viscosity flows in porous medium. Important applications of such flows include geothermal energy utilization, thermal energy storage and recoverable systems, petroleum reservoirs, chemical catalytic convectors, storage of grain, fruits and vegetables, pollutant dispersions in aquifers, industrial and agricultural water distribution, combustions in situ in underground reservoirs for the enhancement of oil recovery, ceramic radiant porous burners used by industrial firms as efficient heat transfer devices, etc.

The object of the present work is to study free convection flow of a viscous fluid with variable viscosity and viscous dissipation through horizontal long channel, embedded in a porous medium using Arrhenius model. The coupled non-linear equations of momentum and energy are solved by approximation technique. The velocity field, temperature distribution, wall shear stress and rate of heat transfer are discussed through graphs for different numerical values of the parameters entered into the equations governing the flow. The results of this study are in well agreement with those of Kankane and Gokhale [8] when permeability of the medium is not taken into account.

FORMULATION OF THE PROBLEM

Consider fully developed laminar flow of a viscous incompressible fluid of temperature dependent viscosity and viscous dissipation between two long parallel, and horizontal walls, forming a channel embedded in a porous medium. The fluid flows under constant pressure gradient, i.e., $-\frac{\partial p'}{\partial x'} = -P'$. Since the walls are long enough, all the variables are functions of y' only. Let $2h$ be the width of the channel, T_1' and T_2' be the temperatures of the upper and lower walls, respectively. In two-dimensional Cartesian coordinate system (x', y'), the x'-axis is taken in the middle of the channel and y'-axis is taken normal to it. The viscosity of the fluid is assumed to be variable, decreasing exponentially with temperature and is given by (Arrhenius model, see Kankane & Gokhale [8]) :

$$\mu' = \mu_0 e^{-\beta'\left(T' - T_1'\right)}$$

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where, β' is the small positive constant, μ_0 is constant viscosity and μ' is variable viscosity.

Under the present configuration, the flow can be shown to be governed by the following system of coupled non-linear equations.

$$-\frac{\partial p'}{\partial x'}+\frac{d}{dy'}\left(\mu'\frac{du'}{dy'}\right)-\frac{\mu'}{K'}u'=0 \quad (1)$$

$$K_T\frac{d^2T'}{dy'^2}+\mu'\left(\frac{du'}{dy'}\right)^2=0 \quad (2)$$

where, the first term on the left hand side of equation (1) is the pressure gradient term, the second term is the momentum term with temperature dependent viscosity, and the third term is the Darcian bulk force term. In fact, the permeability term results from the modeling of the momentum exchange between the porous continuum and the fluid, according to the Darcy law. Darcy term is included as a volumetric force term in the momentum equation $\nabla.\sigma+f=0$ (inertia terms ignored). Also, from the physical condition of the problem and kinematic hypothesis $v_y=0$ and $v_x(y)$, which on applying the Newtonian flow rule, the momentum equation leads to $\frac{dp'}{dx'}=\text{constant}=P'$. Hence, the pressure gradient is necessarily constant. In Eq. (2), the first term on the left-hand side defines the thermal diffusion and the second term defines viscous heating effect.

The boundary conditions relevant to the problem are:

$$u'=0\ , \quad T'=T_2' \text{ at } \quad y'=-h$$

$$u'=0\ , \quad T'=T_1' \text{ at } \quad y'=h \quad (3)$$

where, u' is the velocity along the channel, K' is the permeability of the porous medium, T' is the fluid temperature, K_T is the thermal conductivity of the fluid, and other symbols are defined in the nomenclature.

We introduce the following non-dimensional quantities and parameters;

$$u=\frac{u'}{u_m},\ y=\frac{y'}{h},\ \mu=\frac{\mu'}{\mu_0},\ T=\frac{T'-T_1'}{T_2'-T_1'},\ P=\frac{P'h^2}{\mu_0 u_m}$$

$\beta=\beta'\left(T_2'-T_1'\right)$ (Viscosity parameter), $K=\frac{K'}{h^2}$ (Permeability parameter),

$Pr=\frac{\mu_0 C_p}{K_T}$ (Prandtl number),

$E_c=\frac{u_m^2}{C_p\left(T_2'-T_1'\right)}$ (Eckert number)

By introducing these non-dimensional quantities and parameters in the equations (1) and (2), we obtain:

$$-P+\frac{d}{dy}\left(\mu\frac{du}{dy}\right)-\frac{\mu}{K}u=0 \quad (4)$$

$$\frac{d^2T}{dy^2}+Pr\,Ec\,\mu\left(\frac{du}{dy}\right)^2=0 \quad (5)$$

The boundary conditions (3) reduce to:

$$u=0\ , \quad T=1 \quad \text{at} \quad y=-1$$

$$u=0\ , \quad T=0 \quad \text{at} \quad y=1 \quad (6)$$

SOLUTION OF THE PROBLEM

In order to solve the non-linear system of equations (4)-(5), we expand u and T in powers of Ec, under the assumption $Ec \ll 1$, which is valid for incompressible fluids (see Raptis et. al. [9]). Hence, we assume :

$$u=u_0+Ecu_1 \quad \text{and} \quad T=T_0+EcT_1 \quad (7)$$

By substituting (7) in (4) and (5), and equating the constant as well as the coefficients of Ec, neglecting the coefficients of $O\left(Ec^2\right)$, we obtain :

$$\frac{d}{dy}\left[e^{-\beta T_0}\frac{du_0}{dy}\right]-\frac{1}{K}e^{-\beta T_0}u_0=P \quad (8)$$

$$\frac{d}{dy}\left[e^{-\beta T_0}\frac{du_1}{dy}-\beta T_1 e^{-\beta T_0}\frac{du_0}{dy}\right]-\frac{1}{K}e^{-\beta T_0}\left(u_1-\beta u_0 T_1\right)=0 \quad (9)$$

$$\frac{d^2T_0}{dy^2}=0 \quad (10)$$

$$\frac{d^2T_1}{dy^2}+P_r e^{-\beta T_0}\left(\frac{du_0}{dy}\right)^2=0 \quad (11)$$

Also, the boundary conditions (6) are transformed to :

$$u_0=0,\ u_1=0,\ T_0=1,\ T_1=0 \quad \text{at} \quad y=-1$$

$$u_0=0,\ u_1=0,\ T_0=1,\ T_1=0 \quad \text{at} \quad y=1 \quad (12)$$

The solutions of equations (8) – (11), under the corresponding boundary conditions, (12) are obtained as follows :

$$T_0=\frac{1}{2}(1-y) \quad (13)$$

$$u_0 = C_1 e^{m_1 y} + C_2 e^{m_2 y} - K_1 e^{-(\beta/2)y} \tag{14}$$

$$T_1 = C_4 + C_3 y - A_1 e^{(2m_1+\beta/2)y} - A_2 e^{(2m_2+\beta/2)y} - A_3 e^{-(\beta/2)y}$$
$$- A_4 y^2 - A_5 e^{m_1 y} - A_6 e^{m_2 y} \tag{15}$$

$$u_1 = C_5 e^{m_1 y} + C_6 e^{m_2 y} + A_7 y e^{m_1 y} + A_8 y e^{m_2 y} - A_9 e^{(m_1-\beta/2)y}$$
$$- A_{10} e^{(m_2-\beta/2)y} - A_{11} e^{(3m_1+\beta/2)y} - A_{12} e^{(3m_2+\beta/2)y} - A_{13} e^{2m_1 y}$$
$$- A_{14} e^{2m_2 y} + A_{15} e^{-(\beta/2)y} + A_{16} e^{-\beta y} - A_{17} y^2 e^{m_1 y}$$
$$- A_{18} y^2 e^{m_2 y} + A_{19} y e^{-(\beta/2)y} + A_{20} y^2 e^{-(\beta/2)y} \tag{16}$$

The constants are given in the appendix.

WALL SHEAR STRESS AND RATE OF HEAT TRANSFER

The wall shear stress (τ) is given by :

$$\tau = \left.\frac{du}{dy}\right|_{y=-1} \tag{17}$$

$$= C_1 m_1 e^{-m_1} + C_2 m_2 e^{-m_2} - K_1 \frac{\beta}{2} e^{\beta/2}$$
$$+ Ec\Big[m_1 C_5 e^{-m_1} + m_2 C_6 e^{-m_2} + A_7 (1 - m_1) e^{-m_1}$$
$$+ A_8 (1 - m_2) e^{-m_2} - A_9 \left(m_1 - \frac{\beta}{2} \right) e^{-(m_1-\beta/2)}$$
$$- A_{10} \left(m_2 - \frac{\beta}{2} \right) e^{-(m_2-\beta/2)}$$
$$- A_{11} \left(3m_1 + \frac{\beta}{2} \right) e^{-(3m_1+\beta/2)}$$
$$- A_{12} \left(3m_2 + \frac{\beta}{2} \right) e^{-(3m_2+\beta/2)}$$
$$- 2m_1 A_{13} e^{-2m_1} - 2m_2 A_{14} e^{-2m_2} - \frac{\beta}{2} A_{15} e^{\beta/2} - \beta A_{16} e^{\beta}$$
$$- A_{17} (m_1 - 2) e^{-m_1} - A_{18} (m_2 - 2) e^{-m_2}$$
$$+ A_{19} \left(\frac{\beta}{2} + 1 \right) e^{-\beta/2} - A_{20} \left(\frac{\beta}{2} + 2 \right) e^{\beta/2} \tag{18}$$

The rate of heat transfer (Nu) is given by :

$$Nu = -\left.\frac{dT}{dy}\right|_{y=-1} \tag{19}$$

$$= \frac{1}{2} - Ec\Big[C_3 - \left(2m_1 + \frac{\beta}{2} \right) A_1 e^{-(2m_1+\beta/2)}$$
$$- \left(2m_2 + \frac{\beta}{2} \right) A_2 e^{-(2m_2+\beta/2)}$$
$$+ A_3 \frac{\beta}{2} e^{\beta/2} + 2A_4 - A_5 m_1 e^{-m_1} - A_6 m_2 e^{-m_2} \Big] \tag{20}$$

VERIFICATION OF THE RESULTS FOR SIMPLE CASES

1. In the limit, when $K \to \infty$, i.e., in the absence of porous medium, the results obtained are similar to those obtained by Kankane and Gokhale [8].
2. When viscosity parameter is zero, the results obtained are in agreement with those of Singh et. al. [10].

RESULTS AND DISCUSSION

Analytical solutions of the equations of momentum and energy (1)-(5) are obtained in (13)-(16). The equations governing the flow show that the fluid velocity is governed by viscosity parameter (β) and permeability parameter (*K*), while the temperature distribution is governed by Prandtl number (*Pr*), Eckert number (*Ec*), and viscosity parameter (β). In order to get physical depth of the problem and to establish the effects of various parameters on the flow and heat transfer processes, numerical calculations are performed and presented in the form of figures. The values of Prandtl number (*Pr*) are chosen to be 0.025, 1.0, 7.0, and 11.4, which respectively correspond to mercury, electrolyte solution, water at 20^0C and water at 4^0C and one atmospheric pressure, important fluids in energy and naval / aero-space technologies [4-5]. The numerical values of the remaining parameters are chosen arbitrarily, but do retain physical significance in real energy system applications [2, 3]. Besides, Eckert number (*Ec*) is included to add a dissipative effect in all the flow computations with the nominal value *Ec* = 0.01, 0.02. Observations of the variations in fluid velocity, temperature distribution, wall shear stress and rate of heat transfer are made with an aid of a number of graphical figures. The value of the constant pressure gradient is chosen to be unity, so in all the cases, a flow regime under constant pressure gradient is studied. Besides, when viscosity parameter (β) is zero, i.e., when the viscosity variation is independent of temperature, the solution converges to the known solution expressed in particular case of Singh *et al*. [10]. The software mathematica is used to draw the figures.

Fig. (**1**) shows the variations of the temperature distribution (*T*) versus non-dimensional *y* coordinate, for a set of four values of Prandtl number (*Pr*) and for fixed values of viscosity parameter (β); solid curves for $\beta = 0.5$ and dotted curve $\beta = 1.5$, with $Ec = 0.01$. As the Prandtl number increases, i.e., Pr rises from 0.025 (mercury) to 1.00 (electrolyte solution), 1.0 to 7.0 (water at 20^0C), and from 7.0 to 11.4 (water at 4^0C), i.e., curves I, II, III and IV, the temperature raised from *T* = 1 at the lower channel wall (*y* = −1) to approximately *T* = 1.15 at *y* = − 0.75. Thereafter it decreases and ultimately becomes zero at the upper wall (*y* = 1) of the

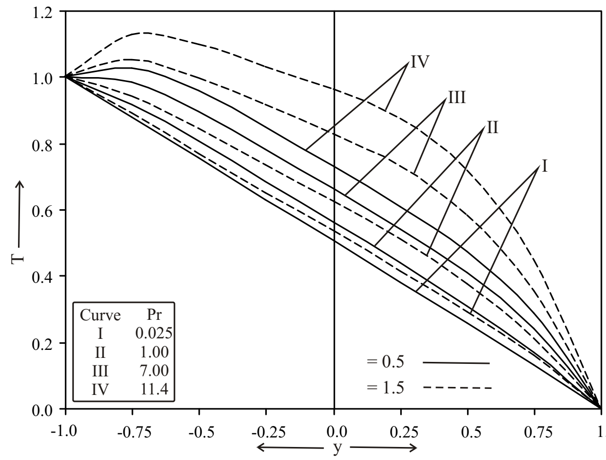

Fig. (1). Effect of Prandtl number on temperature profile at Ec = 0.01.

channel. As expected, the temperature is seen to decrease substantially, but in case of water at 4^0C (i.e., curve IV), it decreases drastically. In fact, the Prandtl number mathematically defines the rate of momentum diffusivity to the thermal diffusivity. Higher-*Pr* fluids transfer heat less effectively than do lower-*Pr* fluids; consequently, lower temperature is observed in profile I as compared with II, III and IV profiles. This general trend of decreasing temperature with increase in *Pr* is in agreement with [11-13]. Again, an increase in viscosity parameter (β) increases the temperature throughout the flow region between channel walls. Hence, the viscosity parameter exerts a positive influence on the temperature throughout the flow region of the channel ($-1.0 < y < 1.0$). In absence of the porous medium (i.e., when $K \to \infty$,) our results are in well agreement with those of Kankane and Gokhale [8]. Besides, the temperature is always constant at the channel walls, which satisfies and also verifies the relevant boundary conditions.

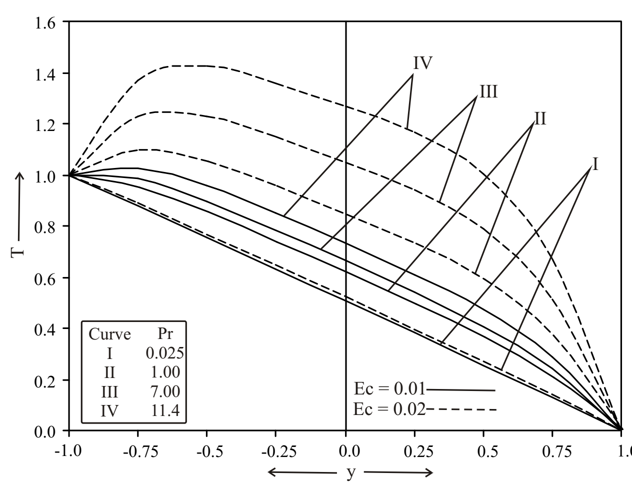

Fig. (2). Effect of Prandtl number on temperature profile at b = 0.5.

Fig. (**2**) represents the variations of the temperature distribution (*T*) versus *y* coordinate for a set of four values of Prandtl number (*Pr*) and for fixed values of Eckert number (*Ec*); solid curves for $Ec = 0.01$ and dotted curve $Ec = 0.02$, when $\beta = 0.5$. As the Prandtl number increases, i.e., *Pr* rises from 0.025 (mercury) to 1.00 (electrolyte solution), 1.0 to 7.0 (water at 20^0C), and from 7.0 to 11.4 (water at 4^0C), i.e., curves I, II, III and IV, the temperature raised from T = 1 at the lower channel wall ($y = -1$) to approximately $T = 1.18$ at $y = -0.75$ and thereafter it decreases and ultimately becomes zero at the upper wall (y = 1) of the channel. Again, the temperature is seen to decrease substantially, but in case of water at 4^0C (i.e., curve IV), it decreases drastically due to point of maximum density for water, this result is consistent with the behavior of higher-Prandtl-number industrial fluids. Again, an increase in Eckert number *Ec* increases the temperature throughout the flow region between channel walls.

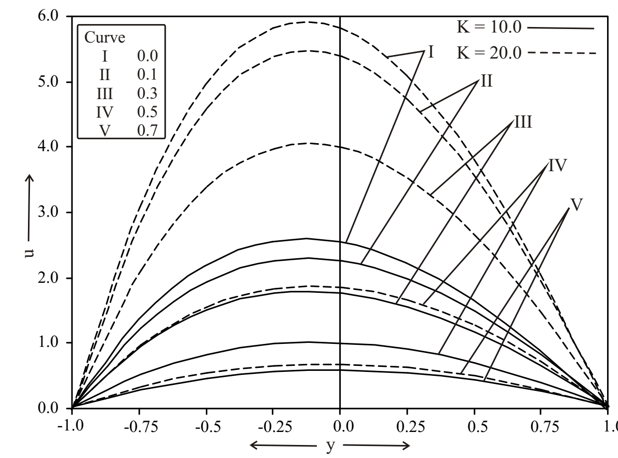

Fig. (3). Effect of viscosity parameter on velocity field at Pr = 1.0 and Ec = 0.01.

Fig. (**3**) shows the variations of the fluid velocity (u) versus y, for different values of viscosity parameter (β) with fixed values of Prandtl number (Pr = 1.0) and Eckert number ($Ec = 0.01$), when K = 10.0 the variations in velocity are shown by solid curves, while by dotted curves, when K = 20.0. The velocity (u) is maximum for $\beta = 0.0$ (indicated by I) and minimum for $\beta = 0.7$ (indicated by V). This indicates that the fluid with low value of viscosity parameter has a dominant effect in controlling the fluid velocity. Obviously, maximum magnitude is recorded for minimum magnitude of viscosity parameter. It is also noted that as permeability parameter, K, is increased from $K = 10.0$ (solid curves) to $K = 20.0$ (dotted curves), the fluid velocity, u, increased. The physical interpretation behind this property is the fact, that as K is increased, the bulk porous medium resistance is lowered. This in turn, increases the momentum development of the flow velocity. We note that in the fluid as $K \to \infty$, the Darcian term vanishes and the porous regime transforms to a pure fluid regime, and in this case our results are in good agreement to those of Kankane and Gokhale [8].

Fig. (**4**) shows variations of the fluid velocity (u) versus y, for values of viscosity parameter (β) with fixed values of Prandtl number (Pr = 1.0) and permeability parameter

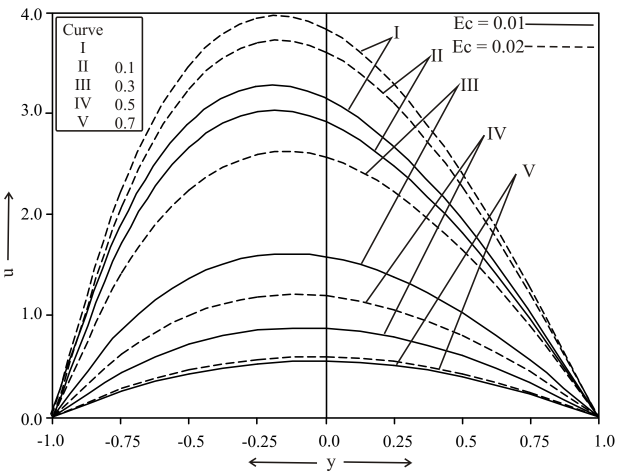

Fig. (4). Effect of viscosity parameter on velocity field at Pr = 1.0 and K = 10.0.

($K = 10.0$). When $Ec = 0.01$, the variations in velocity are shown by solid curves, while by dotted curves when $Ec = 0.02$. The velocity (u) is maximum for curve IV, when $\beta = 0.1$. Obviously, maximum magnitude of velocity is recorded for minimum value of viscosity parameter. This demonstrates that as viscosity parameter is increased, the velocity is decreased for fixed Prandtl number and permeability parameter; a result consistent with [14]. It is also noted that a rise in Eckert number (*Ec*) leads to an increase in the velocity.

In Fig. (**5**) we have plotted the wall shear stress $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=-1}$, i.e., the spatial velocity gradient at the lower wall, versus viscosity parameter (β) for different *Pr*-fluids. Again, two sets of profiles, four for *K* = 10.0 (solid curves) and four for *K* = 15.0 (dotted curves) are plotted. In these plots, we have varied the parameter *Pr* choosing $Ec = 0.01$. The *Ec* represents very weak viscous dissipation and realistically represents thermo-convection regimes in nuclear, geophysical and naval energy systems [15, 16]. As *Pr* increased from 0.025 (mercury) to 1.0 (electrolyte solution), 1.0 to 7.0 (water at 20^0C) and 7.0 to 11.4 (water at 4^0C), a clear rise in wall shear stress is witnessed. Thus, we note that the maximum wall shear stress occurs in higher *Pr*-fluid. This shows that maximum *Pr* value fluids boost the wall shear stress. Also, we observe that wall shear stress increases with an increase in permeability parameter and viscosity parameter. Physically, a rise in *K* implies lower Darcian bulk resistance, as the Darcian term, $-u/k$, in the momentum equation exists in denominator. This, therefore, accelerators the flow, increasing the wall shear stress with increase in viscosity parameter.

In Fig. (**6**) we have plotted the influence of Prandtl number (*Pr*) versus viscosity parameter (β). Heat transfer is embodied in Nusselt number $Nu = \left(\frac{\partial T}{\partial y}\right)_{y=-1}$, i.e., the surface temperature gradient at the lower wall ($y = -1$), as defined in equation (19). In all plots, we observe that *Nu* has maximum value for *K* = 15.0 and *Pr* = 11.4, and minimum value for *K* = 10.0 and *Pr* = 0.025 (mercury), which suggests and also confirms the utility and applications of mercury in energy systems and thermo-convection regimes [17, 18].

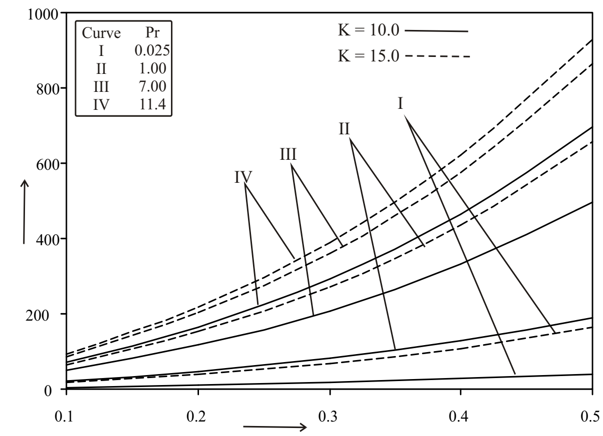

Fig. (5). Effect of Prandtl number on wall shear stress at Ec = 0.01.

CONCLUSIONS

In this work, a general criterion for free convection flow with variable viscosity through horizontal channel, embedded in porous medium using Arrhenius model, is presented in terms of parameters of engineering importance, which include viscosity parameter, permeability parameter, Prandtl number, and Eckert number. The criterion proposed in this study is more general than the previous criterion suggested by Kankane and Gokhale [8], because the present analysis is applicable when free convection is the dominant heat transfer mode in a porous medium; an important medium used is geophysical and aerospace systems technologies [2].

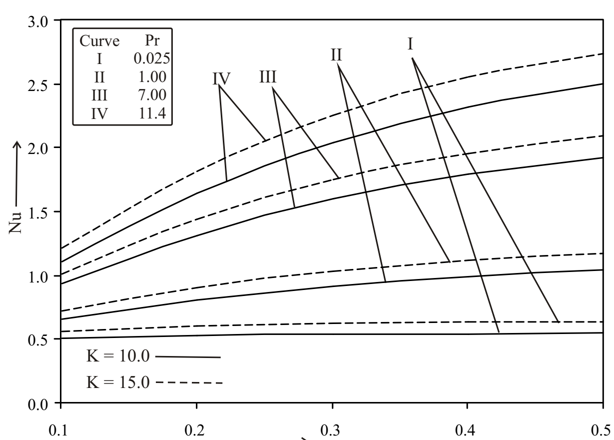

Fig. (6). Effect of Prandtl number on rate of heat transfer at Ec = 0.01.

In the present study, it is observed that the temperature dependent viscosity has a substantial effect on the drag and heat transfer characteristics within the boundary layer. Therefore, it can be concluded that when the viscosity of the working fluid is sensitive to temperature variation or the temperature differences between the channel walls and the ambient fluid is moderate, the variable viscosity effect has dominant role and be taken into consideration. Otherwise, in the prediction of wall shear stress and heat transfer rate, a considerable error may occur.

The conclusions of the study are as follows :

- An increase in Prandtl number (*Pr*) decreases temperature.
- An increase in viscosity parameter (β) increases the temperature between channel walls throughout the flow region.
- An increase in Eckert number (*Ec*) increases the temperature between channel walls throughout the flow region.
- An increase in permeability parameter or Eckert number increases the fluid velocity.
- An increase in viscosity parameter decreases the velocity.
- An increase in Prandtl number, permeability parameter or viscosity parameter increases the wall shear stress.
- An increase in Prandtl number or permeability parameter increases the rate of heat transfer.

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NOMENCLATURE

C_p	=	Specific heat at constant pressure
E_c	=	Eckert number
h	=	Width of the channel
K'	=	Permeability of the porous medium,
K_T	=	Thermal conductivity
K	=	Permeability parameter
Nu	=	Rate of heat transfer
P	=	Non-dimensional pressure
$P'\left(=-\dfrac{\partial p'}{\partial x'}\right)$	=	Constant pressure gradient
Pr	=	Prandtl number
T	=	Non-dimensional temperature
T'	=	Fluid temperature
T_1', T_2'	=	Temperatures of the upper and lower walls
u	=	Non-dimensional velocity
u'	=	Velocity along the channel
u_m	=	Mean velocity
x, y	=	Non-dimensional coordinates
x', y'	=	Coordinate system

Greek Symbols

β'	=	Small positive constant
β	=	Viscosity parameter
μ_0	=	Constant viscosity when $\beta' = 0$
μ'	=	Variable viscosity
τ	=	Wall shear stress

APPENDIX

$$m_1 = \frac{1}{4}\left[-\beta + \sqrt{\beta^2 + 16K^{-1}}\right],$$

$$m_2 = \frac{1}{4}\left[-\beta - \sqrt{\beta^2 + 16K^{-1}}\right], \qquad K_1 = KPe^{\beta/2},$$

$$C_1 = \frac{K_1\left[e^{(m_2+\beta/2)} + e^{-(m_2+\beta/2)}\right]}{\left(e^{m_2-m_1} - e^{m_1-m_2}\right)},$$

$$C_2 = \frac{-K_1\left[e^{(m_1+\beta/2)} - e^{-(m_1+\beta/2)}\right]}{\left(e^{m_2-m_1} - e^{m_1-m_2}\right)},$$

$$C_3 = \frac{1}{2}\left[A_1\left(e^{(2m_1+\beta/2)} - e^{-(2m_1+\beta/2)}\right) + A_2\left(e^{(2m_2+\beta/2)} - e^{-(2m_2+\beta/2)}\right) + A_3\left(e^{-\beta/2} - e^{\beta/2}\right) + A_5\left(e^{m_1} - e^{-m_1}\right) + A_6\left(e^{m_2} - e^{-m_2}\right)\right],$$

$$C_4 = \frac{1}{2}\left[A_1\left(e^{(2m_1+\beta/2)} + e^{-(2m_1+\beta/2)}\right) + A_2\left(e^{(2m_2+\beta/2)} + e^{-(2m_2+\beta/2)}\right) + A_3\left(e^{-\beta/2} + e^{\beta/2}\right) + 2A_4 + A_5\left(e^{m_1} + e^{-m_1}\right) + A_6\left(e^{m_2} + e^{-m_2}\right)\right],$$

$$C_5 = \frac{De^{-m_2} - Ce^{m_2}}{e^{m_1-m_2} - e^{-m_1+m_2}},$$

$$C_6 = \frac{De^{-m_1} - Ce^{m_1}}{e^{-m_1+m_2} - e^{m_1-m_2}},$$

$$\begin{aligned} C = &\left(A_7 + A_{17}\right)e^{-m_1} + \left(A_8 + A_{18}\right)e^{-m_2} \\ &+ A_9 e^{-\left(m_1-\beta/2\right)} + A_{10}e^{-\left(m_2-\beta/2\right)} \\ &+ A_{11}e^{-\left(3m_1+\beta/2\right)} \\ &+ A_{12}e^{-\left(3m_2+\beta/2\right)} + A_{13}e^{-2m_1} + A_{14}e^{-2m_2} \\ &-\left(A_{15} - A_{19} - A_{20}\right)e^{\beta/2} - A_{16}e^{\beta} \end{aligned},$$

$$\begin{aligned} D = &\left(A_{17} - A_7\right)e^{m_1} + \left(A_{18} - A_8\right)e^{m_2} \\ &+ A_9 e^{\left(m_1-\beta/2\right)} + A_{10}e^{\left(m_2-\beta/2\right)} \\ &+ A_{11}e^{\left(3m_1+\beta/2\right)} \\ &+ A_{12}e^{\left(3m_2+\beta/2\right)} + A_{13}e^{2m_1} + A_{14}e^{2m_2} \\ &-\left(A_{15} + A_{19} + A_{20}\right)e^{-\beta/2} - A_{16}e^{-\beta} \end{aligned},$$

$$A_1 = \frac{Pr\, e^{-\beta/2}C_1^2 m_1^2}{\left(2m_1 + \beta/2\right)^2},$$

$$A_2 = \frac{Pr\, e^{-\beta/2}C_2^2 m_2^2}{\left(2m_2 + \beta/2\right)^2},$$

$$A_3 = Pr\, K_1^2 e^{-\beta/2},$$

$$A_4 = Pr\, e^{-\beta/2}C_1 C_2 m_1 m_2,$$

$$A_5 = \frac{\beta K_1 C_1\, Pr\, e^{-\beta/2}}{m_1},$$

$$A_6 = \frac{\beta K_1 C_2\, Pr\, e^{-\beta/2}}{m_2},$$

$$A_7 = a_{44} + \frac{a_5}{\left(2m_1 + \beta/2\right)^2},$$

$$A_8 = a_{45} + \frac{a_{12}}{\left(2m_2 + \beta/2\right)^2},$$

$$A_9 = \frac{a_{37}}{m_1\beta},$$

$$A_{10} = \frac{a_{38}}{m_2\beta},$$

$$A_{11} = \frac{a_2}{9m_1^2 + 9m_1\left(\beta/2\right) + \left(\beta^2/2\right) - K^{-1}},$$

$$A_{12} = \frac{a_{10}}{9m_2^2 + 9m_2\left(\beta/2\right) + \left(\beta^2/2\right) - K^{-1}},$$

$$A_{13} = \frac{a_{39}}{4m_1^2 + m_1\beta - K^{-1}},$$

$$A_{14} = \frac{a_{40}}{4m_2^2 + m_2\beta - K^{-1}},$$

$$A_{15} = a_{46} - \frac{K^2\beta a_{43}}{2} + 2K^2 a_{34} + \frac{K^2\beta^2}{2},$$

$$A_{16} = \frac{a_{42}}{\left(\beta^2/2\right) - K^{-1}},$$

$$A_{17} = \frac{a_5}{2\left(2m_1 + \beta/2\right)},$$

$$A_{18} = \frac{a_{12}}{2\left(2m_2 + \beta/2\right)},$$

$$A_{19} = Ka_{43} - \beta K^2 a_{34},$$

$$A_{20} = Ka_{34},$$

$$a_1 = \beta C_1 C_3 m_1,$$

$$a_2 = \beta C_1 m_1 A_1\left(2m_1 + \beta/2\right),$$

$$a_3 = \beta C_1 m_1 A_2\left(2m_2 + \beta/2\right),$$

$$a_4 = \beta C_1 m_1 A_3\left(\beta/2\right),$$

$$a_5 = 2\beta C_1 A_4 m_1,$$

$$a_6 = \beta C_1 A_5 m_1^2,$$

$a_7 = \beta C_1 A_6 m_1 m_2$, $a_8 = \beta C_2 m_2 C_3$,

$a_9 = \beta C_2 m_2 A_1 \left(2m_1 + \beta / 2\right)$,

$a_{10} = \beta C_2 m_2 A_2 \left(2m_2 + \beta / 2\right)$,

$a_{11} = \beta C_2 m_2 A_3 \left(\beta / 2\right)$,

$a_{12} = \beta C_2 m_2 2 A_4$,

$a_{13} = \beta C_2 m_2 m_1 A_5$,

$a_{14} = \beta C_2 m_2^2 A_6$,

$a_{15} = \beta K_1 C_3 \left(\beta / 2\right)$,

$a_{16} = \beta K_1 A_1 \left(\beta / 2\right)\left(2m_1 + \beta / 2\right)$,

$a_{17} = \beta K_1 A_2 \left(\beta / 2\right)\left(2m_2 + \beta / 2\right)$,

$a_{18} = \beta K_1 A_3 \left(\beta / 2\right)^2$,

$a_{19} = K_1 A_4 \beta^2$,

$a_{20} = \beta K_1 A_5 m_1 \left(\beta / 2\right)$,

$a_{21} = \beta K_1 m_2 A_6 \left(\beta / 2\right)$,

$a_{22} = a_1 - a_9$,

$a_{23} = a_8 - a_3$,

$a_{24} = a_4 - a_{20}$,

$a_{25} = a_{11} - a_{21}$,

$a_{26} = a_6 + a_{16}$,

$a_{27} = a_{14} + a_{17}$,

$a_{28} = a_7 + a_{13} - a_{15}$,

$a_{29} = \beta P C_4 e^{\beta/2}$,

$a_{30} = \beta P C_3 e^{\beta/2}$,

$a_{31} = \beta P A_1 e^{\beta/2}$,

$a_{32} = \beta P A_2 e^{\beta/2}$,

$a_{33} = \beta P A_3 e^{\beta/2}$,

$a_{34} = \beta P A_4 e^{\beta/2}$,

$a_{35} = \beta P A_5 e^{\beta/2}$,

$a_{36} = \beta P A_6 e^{\beta/2}$,

$a_{37} = a_{24} - a_{35}$

$a_{38} = a_{25} - a_{36}$,

$a_{39} = a_{26} + a_{31}$,

$a_{40} = a_{27} + a_{32}$,

$a_{41} = a_{28} - a_{29}$,

$a_{42} = a_{18} - a_{33}$,

$a_{43} = a_{19} - a_{30}$,

$$a_{44} = \frac{a_{22}}{2m_1 + \beta / 2},$$

$$a_{45} = \frac{a_{23}}{2m_2 + \beta / 2}$$

and

$a_{46} = K a_{41}$.

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