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Thermal Mechanisms of Stable Macroscopic Penetration of Applied Currents in High Temperature Superconductors and their Instability Conditions

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Abstract: Macroscopic mechanisms of applied current penetration in high-T_c superconductors are discussed to understand the basic physical trends, which are characteristic for the stable and unstable formation of the thermo-electrodynamics states of high-T_c superconductors placed in DC external magnetic field. The performed analysis shows that the definition of the current stability conditions of high-T_c superconductors must take into consideration the development of the interconnected thermal and electrodynamics states when the temperature of superconductor may stably rise before instability. As a result, the thermal degradation effect of the current-carrying capacity of superconductor exists. The boundary values of the electric field and the current above which the applied current is unstable are defined taking into account the size effect, cooling conditions, non-linear temperature dependences of the critical current density of superconductor. It is proved that the allowable stable values of electric field and current can be both below and above those determined by a priori chosen critical values of the electric field and current of the superconductor. The violation features of the stable current distribution in high-T_c superconductors cooled by liquid coolant are studied. The necessary criteria allowing one to determine the influence of the properties of superconductor and coolant on the current instability onset are written.

Keywords: High-T_c superconductors, I-V characteristics, macroscopic behavior, critical current, current instability.

1. INTRODUCTION

The electrodynamics of superconductors is one of the most fundamental issues of superconductivity. Such investigations made in the macroscopic approximation are of importance for both understanding the basic mechanisms underlying the stable superconducting state formation and many applications of superconductivity, in particular, when the practical use of superconductors requires the analysis of stability conditions of superconducting state. As known, they depend on the macroscopic phenomena occurring in the superconductors and may lead to the so-called magnetic or current instabilities [1, 2].

The numerous previous investigations of the magnetic instabilities in high-T_c superconductors have been made [3-12]. In the meantime, basic features of the stable charging of applied current into high-T_c superconductors, especially, influence of thermal mechanisms on the electrodynamics state formation have still not discussed. This is partly explained by the fact that the limiting current-carrying capacity of low-T_c superconductors, which have steep voltage-current characteristics, is described by their critical currents [1, 2]. This concept is also widely used to determine the current-carrying capacity of high-T_c superconductors as their basic property. However, the finite voltage induced in high-T_c superconductors by applied current appears long before the current instability onset. In this case, the macroscopic electrodynamics of superconductor is a consequence of the non-local dynamics of vortex lattice that is gradually coming to the motion [13]. The concept of the resistive states significantly expands the range of permissible stable operating mode of high-T_c superconducting materials [14-23]. However, they are characterized by non-trivial temperature change of superconductor, which depends on the electrodynamics processes in superconductor. For the first time, this feature, which takes place even in low-T_c superconductors, was formulated in [24, 25].

The heating issues of high-T_c superconductors are also important for the c-axis intrinsic Josephson junction geometries. There is a big discussion on the pseudogap in high-T_c superconductors inferred from the V-I curves during the intrinsic Josephson junction tunneling spectroscopy [26, 27]. The intrinsic tunneling spectroscopy of bismuth cuprate superconductors has received considerable attention. This technique utilizes intrinsic Josephson junctions, which are unique in probing bulk electronic properties, among other important methods such as scanning tunneling spectroscopy and the break-junction technique. However, intrinsic tunneling spectra can be seriously affected by self-heating due to poor thermal conductivity of the cuprate materials and relatively large current densities required to reach the gap voltage. Therefore, experimentally, intrinsic tunneling spectra are usually different from those obtained in scanning tunneling spectroscopy and break-junction experiments. As a result, it is important to eliminate the heating in order to obtain the genuine intrinsic tunneling spectra.

Another important stability problem concerns thermally activated phase slips and quantum phase slips in a long channel or thin superconducting wire and films. They are treated as so-called one dimensional (1D) system that has

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1874-1835/12

width smaller than the coherence length. During a phase slip, the superconducting order parameter fluctuates to zero at some point along the superconductor resulting in a voltage pulse. The sum of these pulses gives the resistive voltage. By applying a current, the 1D superconductor can transform from the superconducting state to normal. One of the most striking features of this transition is the multiple steps in the voltage-current curve induced by phase slip. These resistive states are nonstationary and nonequilibrium (see, for example, [28, 29] and references therein). The stability of their formation can be investigated using numerical integration of the time-dependent Ginzburg - Landau equations. However, the discussion of the instability of these metastable microscopic states is beyond the scope of given paper because our main purpose is the macroscopic phenomena in high-T_c superconductors when magnetic penetration depth is not microscopic and dynamics of which can be described by models based on the Maxwell equations.

Thus, in this paper, the role of the heat transfer mechanisms in the formation of the macroscopic electrodynamics states of bulk high-T_c superconductors during current charging is discussed in detail. The models proposed permit:

- to investigate the existence of the non-isothermal $E(J)$ characteristics of a high-T_c superconductor that may take place during experiments;
- to study the thermal peculiarities of the electrodynamics state formation of high-T_c superconductor accounting for non-uniform temperature distribution in its cross section and to get the appropriate current instability conditions defining the maximum stable value of applied current;
- to consider thermal features of violation of the stable current mode of high-T_c superconductors under different cooling regimes, and formulate the criteria defining the possible mechanisms of current instability in high-T_c superconductors.

Accordingly, the results obtained allow to formulate the macroscopic features of the thermo-electrodynamics processes in high-T_c superconductors, which are important for the analysis of applied current experiments.

2. GOVERNING THERMO-ELECTRODYNAMICS MODELS

To investigate the basic features of the thermo-electrodynamics phenomena in superconductors, let us consider simple approximation believing that a superconductor with a slab geometry ($-a<x<a$, $-\infty<y<\infty$, $-b<z<b$, $b>>a$) placed in a constant external magnetic field parallel to its surface in the Z-direction that is penetrated over its cross section ($S=4ab$). Suppose that the applied current is charged in the Y-direction increasing linearly from zero at the constant sweep rate dI/dt and its self magnetic field is negligibly lower than the external magnetic field [2]. Let us describe a constitutive relation $E(J)$ of superconductor by a power law and approximate the dependence of the critical current on the temperature by the linear relationship. As known, power law corresponds to a logarithmic current dependence of the potential barrier. This dependence is observed in many experiments that deal with both low- and high-temperature superconductors [30-33]. Assume also that the superconductor has the transverse size in the X-direction, which does not lead to the magnetic instability. As it was shown in [34, 35], the flux penetration phenomena during creep are characterized by a finite velocity. Therefore, taking into consideration the existence of the moving boundary of the current penetration region, the transient equations describing the evolution of the temperature and electric field inside the superconducting slab is independent of z and y coordinates and may be written as follows

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + \begin{cases} 0, & 0 < x < x_p \\ EJ, & x_p < x < a \end{cases} \quad (1)$$

$$\mu_0 \frac{\partial J}{\partial t} = \frac{\partial^2 E}{\partial x^2}, \quad t > 0, \quad 0 \le x_p < x < a \quad (2)$$

Here, the electric field $E(x,t)$, current density $J(x,t)$ and critical current density $J_c(T,B)$ conform the following relationships

$$E = E_c\left[J / J_c(T,B)\right]^n \quad (3)$$

$$J_c(T,B) = J_{c0}(B)\left(T_{cB}(B) - T\right) / \left(T_{cB}(B) - T_0\right) \quad (4)$$

For the problem under consideration, the initial and boundary thermo-electrodynamics conditions are given by

$$T(x,0) = T_0, \quad E(x,0) = 0 \quad (5)$$

$$\frac{\partial T}{\partial x}(0,t) = 0, \ \lambda\frac{\partial T}{\partial x}(a,t) + h\left[T(a,t) - T_0\right] = 0 \quad (6)$$

$$\begin{aligned} &E(x_p,t) = 0, \quad x_p > 0 \\ &\frac{\partial E}{\partial x}(0,t) = 0, \quad x_p = 0 \\ &\frac{\partial E}{\partial x}(a,t) = \frac{\mu_0}{4b}\frac{dI}{dt} \end{aligned} \quad (7)$$

Here, C and λ are the specific heat capacity and thermal conductivity of the superconductor, respectively; h is the heat transfer coefficient; T_0 is the cooling bath temperature; n is the power law exponent of the E-J relation; E_c is the voltage criterion defining the critical current density of the superconductor; J_{c0} and T_{cB} are the known critical parameters of superconductor at the given external magnetic field B; x_p is the moving boundary of the current penetration area following from the integral relation

$$4b\int_{x_p}^{a} J(x,t)dx = \frac{dI}{dt}t \quad (8)$$

defining the conservation law of charged current [34, 35].

Note that approach (4) is reasonable for operating states when the temperature variation of a superconductor is not noticeable. However, the huge flux creep of high-T_c superconductors leads to the strong nonlinear temperature dependence of their critical currents in the high magnetic field. Therefore, the relationship (4) is not universal. In these cases, the various approximations are used [36-42] to describe the temperature-dependent critical current of high-

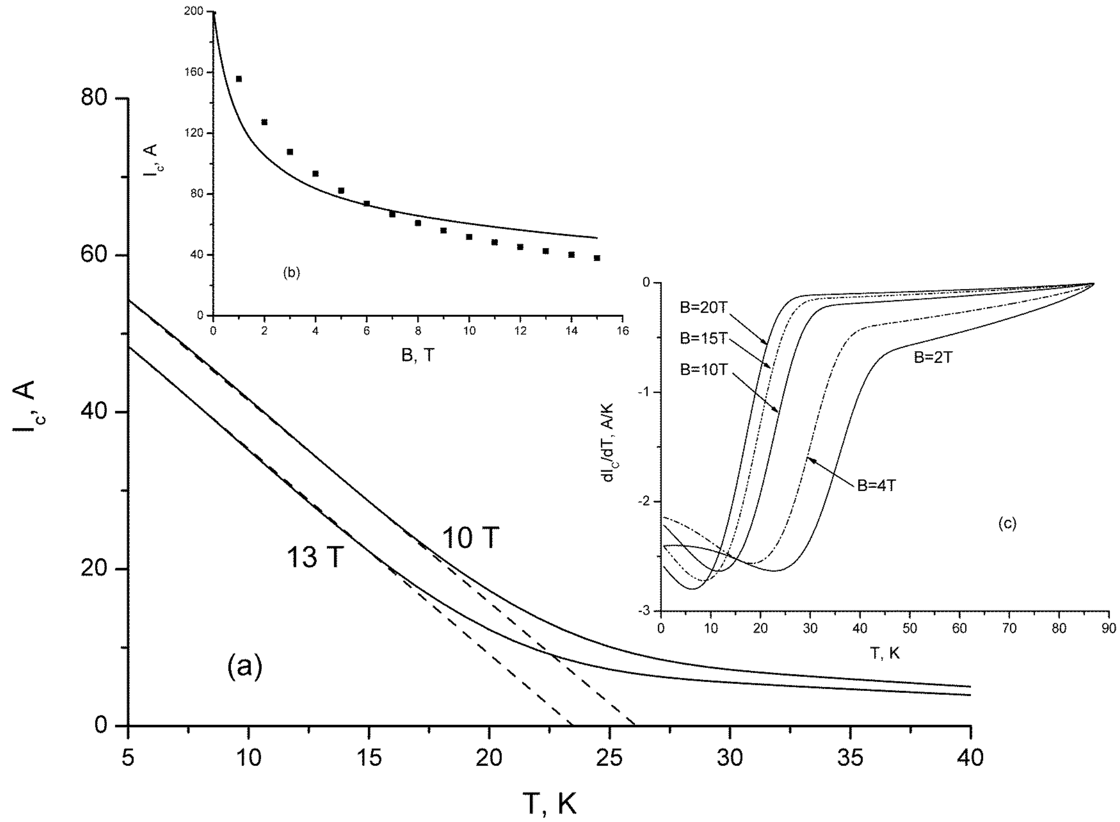

Fig. (1). Critical current of Ag/Bi2212 versus temperature (**a**), magnetic induction (**b**) and temperature dependence of dI_c/dT (**c**): (▪) - after [40], (——) - calculations according to (9), (- - - -) - linear approximation.

T_c superconductors. In particular, this quantity may be described by equation

$$J_c(T,B) = J_0\left(1-\frac{T}{T_c}\right)^{\gamma} \left[(1-\chi)\frac{B_0}{B_0+B} + \chi\exp\left(-\frac{\beta B}{B_{c0}\exp(-\alpha T/T_c)}\right)\right] \quad (9)$$

proposed in [42]. It is good approximation for Bi-based superconductors. Fig. (**1**) shows the possible temperature variation of the critical current $I_c=\eta J_c S$ scaled for the $Bi_2Sr_2CaCu_2O_8$ (Bi2212) composite (η is the volume fraction of a superconductor in a composite). The calculations presented were made at T=4.2 K using the following constants

$$T_c=87.1 \text{ K}, B_{c0}=465.5 \text{ T}, \alpha=10.33, \beta=6.76, \gamma=1.73, \chi=0.27, B_0=1\text{T}, J_0=5.9\times10^4 \text{ A/cm}^2 \quad (10)$$

which were reported in [42] and were also adopted according to the measured data published in [43] for the Ag/Bi2212 superconductor having S=0.01862 cm^2 and η=0.2.

The diffusion one-dimensional model defined by equations (1) - (8) may be simplified in the cases when the temperature distribution inside the superconductor is practically uniform. Under the considered slab geometry, the uniform temperature distribution exists when the condition $ha/\lambda \ll 1$ takes place (Below this condition will be strictly proved). Integrating equation (1) with respect to x from 0 to a and considering the boundary conditions (6), it is easy to get the following transient zero-dimensional heat equation

$$C(T)\frac{dT}{dt} = -\frac{h}{a}(T-T_0) + E(t)J(t) \quad (11)$$

This equation with the relations (3) and (4) describes the uniform time variation of the temperature and electric field as a function of the applied current $I(t)=J(t)S=dI/dt\times t$.

The limiting transition $dI/dt\to 0$, applying to the model described by equation (11), leads to the static zero-dimensional heat balance equation. In this case, i. e. at $ha/\lambda \ll 1$, uniform static thermal state of the superconducting slab is the solution of the following equation

$$EJ = h(T-T_0)/a \quad (12)$$

The models formulated allow one to investigate the influence of the thermal mechanisms on the macroscopic formation of the electrodynamics states of high-T_c superconductors during current charging and find the corresponding stability criteria.

3. POTENTIAL THERMO-ELECTRODYNAMICS STATES OF HIGH - T_C SUPERCONDUCTORS

To appreciate the temperature influence on the electric field dynamics inside superconductor, let us reduced the set of equations (1) - (3), eliminating the current density. Omitting the intermediate algebra, it is easy to get the following equation

$$\frac{\mu_0 J_c}{nE}\left(\frac{E}{E_c}\right)^{1/n}\frac{\partial E}{\partial t}=\frac{\partial^2 E}{\partial x^2}-\frac{\mu_0}{C(T)}\frac{dJ_c}{dT}\left(\frac{E}{E_c}\right)^{1/n}\left[EJ_c\left(\frac{E}{E_c}\right)^{1/n}+\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)\right]$$

which is valid in the current penetration region. To simplify the analysis, the existence only the temperature dependence of J_c was assumed during this transformation.

The term in left part and the first term in right part of this equation describe the electric field evolution in the isothermal approximation. In this case, the inequality $\partial^2E/\partial x^2 > 0$ always takes place in accordance with equation (2) when $\partial J_c/\partial B$ is negligible. Besides, as it follows from [34, 35], the isothermal sweep dynamics of the electric field on the surface of superconductor satisfies the estimator $E(a,t) \sim t^{n/(n+1)}$. Therefore, the next inequalities $\partial E(a,t)/\partial t > 0$ and $\partial^2 E(a,t)/\partial t^2 < 0$ will exist in the current penetration region when electric field is small ($E << E_c$) and temperature distribution is uniform. However, the temperature rise of the superconductor changes these regularities. This effect depends on the third term in right part of equation written. Using equation (3), it is easy to prove that the finite value of dJ_c/dT, which is negative for practicable superconductors, will lead to other relations when the temperature of superconductor induced by applied current become differ from its initial value. Namely, one can obtain $\partial E(a,t)/\partial t > 0$ and $\partial^2 E(a,t)/\partial t^2 > 0$. In other words, the dynamics of the electric field will become more intensive when temperature increases. Physically, this behavior takes place because the applied current diffusion is accompanied with the energy dissipation. Indeed, the temperature rise increases the electric field and changes the induced current density of the applied current, which penetrates more deeply into the superconductor. In turn, this will lead to new dissipation producing the corresponding rise of temperature and so on. As a result, the sharp rise of the electric field and temperature may occur when the current penetration front exceeds some character boundary. The latter defines the stability margin of electrodynamics states [1, 2].

To prove the existence of the stability margin, let us rewrite the latter equation in the form

$$\frac{1}{D_m}\frac{\partial E}{\partial t}=\frac{\partial^2 E}{\partial x^2}+\gamma E+q \tag{13}$$

where

$$D_m=\frac{nE}{\mu_0 J_c(T)}\left(\frac{E_c}{E}\right)^{1/n},\quad \gamma=\frac{\beta(T)}{a^2}\left(\frac{E}{E_c}\right)^{2/n},$$

$$q=\Lambda(T)\tau(T)\left(\frac{E}{E_c}\right)^{1/n}$$

Here, D_m is the magnetic diffusion coefficient,

$$\beta(T)=\frac{\mu_0 J_c(T)a^2}{C(T)}\left|\frac{\partial J_c}{\partial T}\right|$$

is the so-called magnetic stability parameter [1, 2],

$$\Lambda(T)=\mu_0\frac{\lambda(T)}{C(T)}\left|\frac{\partial J_c}{\partial T}\right|,\ \tau(T)=\frac{1}{\lambda(T)}\frac{d\lambda}{dT}\left(\frac{\partial T}{\partial x}\right)^2+\frac{\partial^2 T}{\partial x^2}.$$

Equation (13) is diffusion type equation describing the fission-chain-reaction phenomena, in which the quantity γ is the like-fission coefficient and q is the volume source. In the case under consideration, the coefficient γ is positive due to the negative value of dJ_c/dT for practically used superconductors. Thus, the evolution of the thermo-electrodynamics state of superconductor induced by applied current has the fission-chain-reaction nature. As known, such processes become an irreversible when γ exceeds some character value γ_q in accordance with the fission quality obeying the time-dependent law depending on $[exp(\gamma-\gamma_q)t]$ - terms. As a result, the electric field evolution in superconductor has the instability nature leading to the avalanche modes. The latter happens as the spontaneous rise of the electric field and temperature.

Note that the quantity D_m increases with increasing n-value. Therefore, the instability develops more intensively when $E(J)$ relation of superconductor is steeper. Consequently, it has sharp character in low-T_c superconductors and gets more smoothed nature in high-T_c superconductors. This feature shows experiments. However, subsequent temperature rise may lead to the character peculiarities that underlie the basis of the macroscopic phenomena in superconductors.

The quantity γ has essential temperature dependence. First of all, it varies in inverse proportion to the heat capacity of a superconductor. So, the fission-chain evolution of the induced electric field in high-T_c superconductors may decay after a certain temperature increase. In other words, high-T_c superconductors have the thermal self-stabilization modes even after instability onset. In particular, the swept electric field may come close to the steady modes without the transition of the high-T_c superconductor to the normal state due to its large critical temperature.

There exist other reasons of thermal effect on the electric field evolution. They are due to the temperature dependence of the critical current density and the value dJ_c/dT. Indeed, Fig. (**1**) shows that not only J_c may be small but the dJ_c/dT-values become very small in accordance with the huge flux-creep degradation in high temperature range, which, however, is not close to the critical temperature. Thereby, the thermal effect on the electric field evolution depends on the quantity dJ_c/dT after relatively high temperature rise of superconductor. Besides, the effect of the temperature on the current diffusion depends also on the value and sign of $d\lambda/dT(\partial T/\partial x)^2$ - term. The latter may be both positive and negative over the cross section of a superconductor. This depends on the cooling conditions, cross section of superconductor, its thermal conductivity coefficient and critical parameters.

Thus, the evolution of thermo-electrodynamics states of high-T_c superconductors is the phenomenon having the fission-chain-reaction character. There exist the characteristic thermal effects according to which stable electrodynamics state formation of high-T_c superconductors depends on the joint non-trivial temperature variation of the temperature dependences of superconductor's properties, which are $C(T)$, $\lambda(T)$, $J_c(T)$ and dJ_c/dT. The importance of this conclusion should be emphasized. To understand the stability mechanism of high-T_c superconductors, one usually allows for only the heat capacity of high-T_c superconductor believing that it just decreases the probability of the instability onset. However, macroscopic electrodynamics phenomena in high-T_c superconductors depend on the thermal interconnection between above-mentioned properties of high-T_c superconductors. This feature plays essential role in the macroscopic electrodynamics of high-T_c superconductors, as it will be shown below.

4. ZERO-DIMENSIONAL APPROXIMATION OF APPLIED CURRENT INSTABILITY

The boundary after which the unstable electrodynamics states exist may be determined, first of all, from the equilibrium analysis of the static states. Within the framework of the mathematical models under consideration, the simplified study may be made using static zero-dimensional model described by equations (3), (4) and (12), as the first step allowing to get the criteria of instability. Accordingly, the following relations

$$T = T_0 + \frac{aJ_{c0}E}{h\left(1+\dfrac{J_{c0}}{E_c^{1/n}}\dfrac{a}{h(T_{cB}-T_0)}E^{\frac{n+1}{n}}\right)}\left(\frac{E}{E_c}\right)^{1/n}, \qquad (14)$$

$$J = \frac{J_{c0}}{1+\dfrac{J_{c0}}{E_c^{1/n}}\dfrac{a}{h(T_{cB}-T_0)}E^{\frac{n+1}{n}}}\left(\frac{E}{E_c}\right)^{1/n} \qquad (15)$$

describe the $T(E)$ and $J(E)$ characteristics of a superconducting slab. These formulae demonstrate the features, which are rooted in a formation of the thermo-electrodynamics states of superconductors. Namely, the term

$$\frac{J_{c0}}{E_c^{1/n}}\frac{a}{h(T_{cB}-T_0)}E^{(n+1)/n}$$

describes the influence of temperature on the electrodynamics state. In particular, the limiting case $h\to\infty$ (perfect cooling) corresponds to the isothermal states ($T=T_0$). In general, the nearly isothermal electrodynamics states are defined by the condition

$$E << \left[\frac{h(T_{cB}-T_0)E_c^{1/n}}{aJ_{c0}}\right]^{n/(n+1)} \qquad (16)$$

If this condition does not satisfy, then the thermal dissipation occurs in a superconductor and its overheating influences on the electrodynamics states. In this case, the temperature change of superconductor depends not only on the cross section and heat transfer coefficient but its critical parameters, as follows from (16). In particular, the higher the critical current, the higher the temperature influence.

Formulae (14) and (15) are convenient for the validity analysis of the different electric field criteria used for the description of the critical current measurements. Indeed, using (14) one can obtain the temperature of the sample during an experiment and estimate of the temperature influence on the stable penetration of an applied current. However, formulae (14) and (15) do not describe the boundary of the stable states. To define it, let us analyze the differential resistivity of a superconductor, which determines the slope of its $E(J)$ characteristic. According to (15), it is written as

$$\frac{\partial E}{\partial J} = \frac{\left[1+\dfrac{aJ_{c0}}{h(T_{cB}-T_0)E_c^{1/n}}E^{(n+1)/n}\right]^2}{\dfrac{1}{n}\dfrac{J_{c0}}{E_c^{1/n}}E^{(1-n)/n}-\left(\dfrac{J_{c0}}{E_c^{1/n}}\right)^2\dfrac{a}{h(T_{cB}-T_0)}E^{2/n}} \qquad (17)$$

This formula indicates that the differential resistivity of the superconductor may have both positive ($\partial E/\partial J>0$) and negative ($\partial E/\partial J<0$) branches. The positive derivative of $\partial E/\partial J$ corresponds to the stable states in the case under consideration. Correspondingly, the negative value determines unstable states. According to these peculiarities, the slope of $\partial T/\partial J$ has a similar character of the change. Therefore, analyzing $\partial E/\partial J$ (or $\partial T/\partial J$) it is easy to find the stable and unstable operating thermo-electrodynamics states under the applied conditions. Accordingly, the boundary between them is defined by condition

$$\partial E/\partial J\to\infty \qquad (18)$$

For the first time, this condition was formulated in [14] for low-T_c superconductors cooled by a coolant with the constant coefficient of heat transfer. This cooling mode is characteristic of the conduction-cooled or gas-cooled superconductors.

Under the equality (17) and condition (18), the boundary of the instability is determined by the so-called quench values of the electric field $E_{q,0}$, current $I_{q,0}$ and temperature $T_{q,0}$. They are equal to

$$E_{q,0} = \left[\frac{E_c^{1/n}}{nJ_{c0}}\frac{h}{a}\left(T_{cB}-T_0\right)\right]^{n/(n+1)}, \qquad (19)$$

$$I_{q,0} = \frac{n}{n+1}S\left[\frac{J_{c0}^n}{nE_c}\frac{h}{a}\left(T_{cB}-T_0\right)\right]^{1/(n+1)} \qquad (20)$$

$$T_{q,0} = T_0 + E_{q,0}J_{q,0}a/h \qquad (21)$$

in the static zero-dimensional approximation under consideration.

These formulae demonstrate the following peculiarities of the stable electrodynamics state formation of high-T_c superconductors with the power $E(J)$-characteristic.

First, it is seen that the stability parameters are invariant to the coupled quantities $\{J_{c0}, E_c\}$ when J_{c0} and E_c correspond to the values defined in the isothermal states. Indeed, in these cases, the ratio $J_{c0}/E_c^{1/n}$ is constant for the given value n. Therefore, the quench values do not depend on the ratio $J_{c0}/E_c^{1/n}$ within the framework of the linear temperature dependence of the critical current density described by equation (4). Consequently, the quantities J_{c0} and E_c may be arbitrarily chosen on the isothermal part of the $E(J)$-characteristic of a superconductor. This transformation does not change the stability boundary defined by $\{E_{q,0}, I_{q,0}, T_{q,0}\}$.

Second, many experiments show [see, for example, 21 - 23] that the stable over-critical regimes exist when applied current may be higher than the critical current determined by arbitrarily chosen value E_c ($E > E_c$). This feature is the result of an a priori set of coupled isothermal values $\{J_{c0}, E_c\}$. The conditions of the existence of over-critical regimes are formulated below.

Besides the condition (18), one can use another method of the determination of current instability onset analyzing the removal efficiency of the Joule heating into the coolant [21]. Accordingly, the stable regimes must satisfy the following conditions

$$G(T)=W(T),\ \partial G(T)/\partial T=\partial W(T)/\partial T \tag{22}$$

in the static homogeneous states. Here, G is the power of the Joule heating, W is the power of the heat removal. In the case under consideration, $G=E(T)J(T)a$ and $W=h(T-T_0)$. Then using stability condition (22) it is easy to find that the quench values are also described by relations (19) - (21).

The results presented allow one to investigate the physical features of the formation of stable thermo-electrodynamics states of high-T_c superconductors cooled by a cooler with the constant heat transfer coefficient, i.e. without the infringement of the heat exchange conditions with a coolant on a surface of a superconductor. Let us find the instability boundary of steady current modes of superconductors cooled by coolant when the heat transfer coefficient depends on temperature. The latter exist, for example, when liquid coolant is used.

As above, let us use zero-dimensional approximation. Then the static uniform distribution of the electric field and temperature in superconducting slab for a given value of the current density may be defined solving the following system of equations

$$E = E_c\left[J / J_c(T)\right]^{n(T)}, \tag{23}$$

$$EJ = W(T) \tag{24}$$

Here, $W(T)=q(T)/a$ and $q(T)$ is the heat flux to the coolant. Assume that it is defined as follows

$$q(T) = h(T)(T - T_0)^{\nu} \tag{25}$$

In particular, this formula may be used to describe the heat transfer conditions during nucleate and film boiling regimes [1]. Note also that the temperature dependence of n-value as well as arbitrary dependence $J_c(T)$ are taken into account within the model formulated.

To find the temperature of the superconductor before the current instability let use the inequality

$$\partial J / \partial T < 0 \tag{26}$$

that is a consequence of the fact that the condition (18) divides the temperature-current characteristic of a superconductor into two areas.

Excepting the electric field from equations (23) and (24), one gets relation between the current density and temperature. The latter may be written as

$$J(T) = \left[W(T) J_c^n(T) / E_c\right]^{1/(n+1)} \tag{27}$$

Then considering that the critical current density of real superconductor and its n-value decrease with temperature, the current distribution in the superconductor is unstable, if the inequality

$$\frac{n}{J_c}\left|\frac{dJ_c}{dT}\right| + \left|\frac{dn}{dT}\right| \ln\frac{J_c}{J} > \frac{1}{W}\frac{dW}{dT} \tag{28}$$

is realized. Accordingly, the stable states are determined by the boundary values, which follow from the solution of equation

$$\frac{n}{J_c}\left|\frac{dJ_c}{dT}\right| + \frac{1}{n+1}\left|\frac{dn}{dT}\right| \ln\frac{E_c J_c}{W} = \frac{1}{W}\frac{dW}{dT} \tag{29}$$

Criteria (28) or (29) allow one to determine directly the temperature of superconductor T_q before applied current instability. In particular, let us find this value assuming for simplicity that the critical current density is described by the linear relationship (4). Then the value of T_q satisfies the solution of equation

$$\frac{1}{n(T_q)+1}\left|\frac{dn}{dT}\right|_{T=T_q} \ln\frac{aE_c J_c\left(T_q\right)}{h\left(T_q - T_0\right)^{\nu}} = \frac{\nu}{T_q - T_0} - \frac{n}{T_{cB} - T_q}$$

In the simplest approximation n=const, it equals

$$T_q = T_0 + \frac{T_{cB} - T_0}{1 + n/\nu} \tag{30}$$

The written above analytical expressions make it possible to define the instability boundary in the close form. They are convenient to evaluate the stability conditions of many practically important modes without the large volume of computations taking into considerations different mechanisms of the instability onset.

5. THE INSTABILITY CONDITIONS OF APPLIED CURRENT IN ONE-DIMENSIONAL APPROXIMATION

As it was shown above, the current after which unstable states exist can be determined by the analysis of the static modes of a superconductor and can be obtained from the conditions (18), (22) or (29). However, these criteria were formulated under the assumption that the distributions of the temperature, electric field and current inside superconductor are uniform. As follows from the heat conduction theory [44], such thermal modes take place at $ha/\lambda \ll 1$. Under this condition, the uniform temperature distribution in the cross section of superconductor is the most probable in the cases of low thickness, non-intensive cooling conditions or high thermal conductivity. For example, operating regimes with uniform temperature distribution may be observed during current charging into a superconducting composite (superconductor with a high conductivity matrix). At the same time, the thermal conductivity of superconducting materials having no stabilizing matrix is low. Therefore, the temperature distribution inside the superconductor is not, strictly speaking, uniform [1, 2, 45]. In addition, the criterion of the spatial heterogeneity of the thermal states, i.e. condition $ha/\lambda \gg 1$, does not describe the influence of the critical properties of superconductor on the spatial features of the development of stable thermo-electrodynamics states. Therefore, let us find the conditions describing the instability boundary of the electrodynamics state of high-T_c superconductor accounting for non-uniform temperature distribution in its cross section.

Let us investigate the static heterogeneity operating mode of the slab, which takes place in the fully penetrated mode at $dI/dt \to 0$, using the following set of equations

$$\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + EJ = 0, \quad (31)$$

$$\frac{\partial^2 E}{\partial x^2} = 0 \quad (32)$$

under the boundary conditions

$$\frac{\partial T}{\partial x}(0,t) = 0,\ \lambda\frac{\partial T}{\partial x}(a,t) + h\left[T(a,t) - T_0\right] = 0, \quad (33)$$

$$\frac{\partial E}{\partial x}(0,t) = 0,\ \frac{\partial E}{\partial x}(a,t) = 0 \quad (34)$$

taking into account the relationships (3) and (4).

As follows from (32) and (34), E=const at $dI/dt \to 0$. The existence of states when the distribution of the electric field induced by the applied current is practically uniform at a small rate will be shown below.

In general, the solution of the problem described by equation (31) and condition (33) requires the use of numerical methods. However, assuming that the thermal conductivity coefficient is constant (λ=λ_0=const), it can be solved analytically. Indeed, let us introduce the dimensionless variables

$$\theta = \left(T_{cB} - T\right)/\left(T_{cB} - T_0\right), \quad X = x/a$$

Then the problem (3), (4), (31) and (33) can be rewritten as follows

$$\frac{d^2\theta}{dX^2} - \gamma\theta = 0, \quad \frac{d\theta}{dX}(0) = 0, \quad \frac{d\theta}{dX}(1) + H\theta(1) = 0 \quad (35)$$

Here, H=ha/λ_0 is the thermal resistance of superconductor, $\gamma = \dfrac{J_{c0}E_c a^2}{\lambda_0\left(T_{cB} - T_0\right)}\left(\dfrac{E}{E_c}\right)^{(n+1)/n}$ is the dimensionless parameter that describes the influence of the material properties of the superconductor on the heterogeneous character of the thermo-electrodynamics state formation. This parameter can be rewritten in the form of $\gamma = \dfrac{JEa}{\lambda_0\left(T_{cB} - T\right)/a}$. Therefore, the physical meaning of the parameter γ is equal to the ratio of the power of the Joule heating in the superconductor to the power of the heat flux transferred by thermal conductivity. Thus, it describes directly the influence of the conductive heat transfer mechanism on the electrodynamics state formation in superconductor. In particular, the boundary problem (35) leads to a model with a uniform temperature distribution in the superconductor at γ<< 1, i.e., at $JEa^2 << \lambda_0\left(T_{cB} - T\right)$.

Parameter γ also allows one to estimate the legitimacy of the usage of the zero-dimensional model taking into consideration the critical properties of superconductor. In particular, it is seen that the influence of the non-uniform distribution of temperature on the electrodynamics state of superconductor will be more noticeable when its critical current density is higher or the so-called temperature margin of superconductor, which is equal to (T_{cB} - T_0), is lower with other parameters being equal. Their influence the more essential, the more intensive the cooling.

The analytical solution of the boundary problem (35) is given by

$$\theta(X) = \theta_0\cosh\left(\sqrt{\gamma}X\right), \quad \theta_0 = \frac{H}{\sqrt{\gamma}\sinh\sqrt{\gamma} + H\cosh\sqrt{\gamma}}$$

Then, in the dimensional form, the static non-uniform distribution of temperature in the superconducting slab is described by the expression

$$T(x) = T_{cB} - \left(T_{cB} - T_0\right)\theta_0\cosh\left(\sqrt{\gamma}\frac{x}{a}\right) \quad (36)$$

during fully penetrated mode. In this case, the relationship between the current per width and electric field determined as $I^*(E) = I/(2b) = 2\int_0^a J(E)dx$ is written in the form of

$$I^* = 2J_{c0}a\left(\frac{E}{E_c}\right)^{1/n}\frac{H}{\gamma(E) + H\sqrt{\gamma(E)}/\tanh\sqrt{\gamma(E)}} \quad (37)$$

according to the symmetry of the examined problem. This formula strongly proves that the boundary of the current instability in the static non-uniform approximation follows from the condition

$$\partial E/\partial I \to \infty \quad (38)$$

Its formal difference from the condition (18) shows that the current instability condition depends on the character of the current redistribution in the superconducting slab when the temperature distribution in the superconductor is non-uniform. Moreover, it will be shown below that the currents in the internal part of superconductor are more unstable in comparison with the ones flowing on its surface due to the corresponding non-uniform temperature distribution. As a result, the condition (18) loses its physical meaning.

Accordingly, the non-uniform distribution of the temperature in the superconductor also changes the formulation of the stability criterion (22). In order to formulate this condition taking into account the non-uniform distribution of temperature, let us integrate the heat equation (31) with respect to x from 0 to a. Then taking into account the boundary conditions at x=0 and x=a it is not difficult to obtain the following relationship

$$\int_0^a EJ\,dx = h\left(T - T_0\right)\Big|_{x=a} \quad (39)$$

This relation describes the first equality of the conditions (22) in the integral form. Obviously, the second equality is modified according to the given formulation of the G and W.

Using the current-voltage characteristic (37) and the condition (38), it is easy to find the quench electric field and quench current per width of the slab defining the current instability onset in the non-uniform distribution of temperature in the slab. Accordingly, in terms of the dimensionless variables used, the value E_q is determined by the solution of equation

$$H = \frac{2n}{n+1}\frac{\sqrt{\gamma_q}\tanh\sqrt{\gamma_q}}{\dfrac{2\sqrt{\gamma_q}}{\sinh\left(2\sqrt{\gamma_q}\right)} - \dfrac{n-1}{n+1}} \qquad (40)$$

Then the value of I^*_q is equal to

$$I_q^* = 2J_{c0}a\left(\frac{E_q}{E_c}\right)^{1/n}\frac{H}{\gamma_q + H\sqrt{\gamma_q}/\tanh\sqrt{\gamma_q}} \qquad (41)$$

Here,

$$\gamma_q = \frac{J_{c0}E_c a^2}{\lambda_0\left(T_{cB}-T_0\right)}\left(\frac{E_q}{E_c}\right)^{1+\frac{1}{n}}$$

Note that the dependence $\gamma_q(H)$ in the framework of the zero-dimensional approximation is calculated by formula

$$\gamma_{q,0} = H/n \qquad (42)$$

which follows from (19) or can be obtained from (40) under the assumption that $\gamma_q << 1$.

The calculations presented below show that the value γ_q does not exceed unit within a wide range of the thermal resistance variations. This conclusion allows one to find the approximate solution of equation (40). After some transformation, it is easy to get

$$\gamma_q \approx \frac{H}{n+H(n-1)/3} \qquad (43)$$

In the dimensional variables, the equality (43) is written as follows

$$E_q \approx E_c\left[\frac{1}{n+\left(n-1\right)H/3}\frac{h\left(T_{cB}-T_0\right)}{J_{c0}E_c a}\right]^{n/(n+1)} \qquad (44)$$

In this case, the limiting current stably flowing in superconductor is equal to

$$I_q \approx I_c\frac{\left[\dfrac{1}{n+H\left(n-1\right)/3}\dfrac{h\left(T_{cB}-T_0\right)}{J_{c0}E_c a}\right]^{1/(n+1)}}{\dfrac{1}{n+H(n-1)/3}+\sqrt{\dfrac{H}{n+H(n-1)/3}}\Big/\tanh\sqrt{\dfrac{H}{n+H(n-1)/3}}} \qquad (45)$$

Here, I_c is the critical current of a superconductor ($I_c = J_{c0}S$).

Comparing this value of quench electric field with the value described by the formula (19), it is not difficult to find difference between the estimated boundary values of the electric field calculated in the framework of the zero-dimensional and one-dimensional approximations. The corresponding relationship can be written as

$$\frac{E_{q,0}}{E_q} = \left(1+\frac{n-1}{3n}H\right)^{n/(n+1)} \qquad (46)$$

Therefore, difference between the corresponding values of the current of instability is defined as

$$\frac{I^*_{q,0}}{I^*_q} = \frac{n}{n+1}\left(\frac{1}{n+H(n-1)/3}+\frac{\sqrt{\dfrac{H}{n+H(n-1)/3}}}{\tanh\sqrt{\dfrac{H}{n+H(n-1)/3}}}\right)\left(1+\frac{n-1}{3n}H\right)^{1/(n+1)} \qquad (47)$$

These expressions allow one to estimate quantitatively the influence of the spatial heterogeneity of thermo-electrodynamics states on the current stability conditions. In particular, the formulae (46) and (47) show that difference between the quench values of electric field and current, which follow from the zero-dimensional and one-dimensional calculations, always takes place for any finite value of γ. Moreover, difference increases with increasing γ. As a result, the one-dimensional approximation correctly describing the current stability conditions at intensive cooling conditions ($H >> 1$) allows one to find the limiting value $\gamma_{q,i}$ at $H\to\infty$. According to (40), it is the solution of equation

$$\frac{2\sqrt{\gamma_{q,i}}}{\sinh\left(2\sqrt{\gamma_{q,i}}\right)} = \frac{n-1}{n+1}$$

It is essential that the value $\gamma_{q,i}$ cannot be found in terms of the zero-dimensional model because the spatially heterogeneous character of the thermo-electrodynamics processes is critically dependent on its thermal resistance at $H > 1$. In these states, the increase in the limiting values of the electric field and current becomes significantly non-linear with increasing H and the following estimate

$$I_q^* \approx I^*_{q,0}\frac{n+1}{n}\frac{\tanh\sqrt{\gamma_q}}{\sqrt{\gamma_q}\left(1+\dfrac{n-1}{3n}H\right)^{1/(n+1)}}$$

may be used according to (47). This relation shows that the current of instability will not increase proportionally to an increase in the thickness or the critical current density of superconductor in the intensive cooling conditions unlike the corresponding proportional change of the critical current of the superconductor with increasing quantities a and J_{c0}. In other words, the thermal size effect will lead to unavoidable thermal degradation of the current-carrying capacity of superconductor. In this case, the increase in the quantities a

and J_{c0} leading to increase in the critical current of the superconductor is not accompanied by a corresponding increase in the currents of instability. This thermal degradation effect, which occurs due to stable temperature rise of superconductor, will be discussed below.

6. EFFECT OF HEAT CAPACITY ON TRANSIENT ELECTRODYNAMICS STATES OF HIGH-T_C SUPERCONDUCTORS

Using the results obtained above, let us discuss the physical features of stable and unstable formation of thermo-electrodynamics states in the current charging into a Bi2212-superconductor under non-intensive cooling conditions, which take place in conduction-cooled magnets [46-48]. Under this mode, the temperature distribution in a superconductor is practically uniform if $ha/\lambda << 1$.

The typical temperature and voltage dependences of high-T_c superconductors as a function of a current are shown in Fig. (**2**). The results presented were based on the numerical solution of the problem defined by equations (1) - (8). For convenience of the performed analysis, it was done using the current normalized by the slab width ($I^*=0.5I/b$). In this case, normalized current sweep rate $dI^*/dt=0.5b^{-1}dI/dt$ was single variable quantity. The simulation was made for a $Bi_2Sr_2CaCu_2O_8$ superconducting slab ($a=5\times10^{-2}$cm) for two values of dI^*/dt initially cooled at $B=10$ T and $T_0=4.2$ K. The following constants $h=10^{-3}$W/(cm$^2\times$K), $n=10$, $E_c=10^{-6}$V/cm were used. According to the linear fitting presented in Fig. (**1**), the critical parameters of the superconductor in the framework of approximation (4) are equal to

$$T_{cB}=26.1\text{ K},\ J_{c0}=1.52\times10^4\text{ A/cm}^2 \quad (48)$$

The specific heat capacity and thermal conductivity of the superconductor were defined as follows

$$C\left[\frac{J}{cm^3\cdot K}\right] = 10^{-6}\times\begin{cases}58.5T+22T^3,\ T\le10K\\ -10.54\times10^4+1.28\times10^4T,\ T>10K\end{cases} \quad (49)$$

$$\lambda(T)\left[\frac{W}{cm\cdot K}\right]= (-1.234\times10^{-5}+1.654\times10^{-4}T +4.608\times10^{-6}T^2-1.127\times10^{-7}T^3+6.061\times10^{-10}T^4) \quad (50)$$

according to [49, 50], respectively.

As it follows from Fig. (**2**), the temperature variation of the superconductor under consideration has a minor effect on the dynamics of electrodynamics states in the stage of partially penetrated mode. Therefore, it is easy to find the electric field E_f on the surface of the superconductor when its cross section is completely filled with current. According to the scaling approximation formulated in [34], this value is equal to $E_f=\frac{a\mu_0}{4b}\frac{dI}{dt}$. The corresponding values are shown in Fig. (**2**). Using it, let us estimate the influence of the current charging rate on the non-uniform redistribution of the electric field inside superconductor during initial stage of the fully penetrated mode when $I > I_f$. Here,

$$I_f=\frac{n}{n+1}\frac{\mu_0J_{c0}}{E_c^{1/n}}a^{\frac{n+1}{n}}\left(\frac{\mu_0}{4b}\frac{dI}{dt}\right)^{\frac{1-n}{n}}\frac{dI}{dt}.$$

Curves 2, 2', 3 and 3' in Fig. (**2**) depict the evolution of electric field and temperature in the superconductor during both partially and fully penetrated states on the surface of a slab (2, 3, 4) and in the center (2', 3'). Curve 1 corresponds to a static uniform distribution induced in the slab by infinitely slow current charging and follows from approximation (14) and (15). As a whole, the curves in Fig. (**2**) indicate the existence of the characteristic dynamics of the fully penetrated states that are proper for the stable and unstable current charging modes of high-T_c superconductors and depend on *dI/dt* and must be considered during experimental definition of the critical current of superconductors.

First, comparing curves 1-3 in the sub-critical electric fields ($E < E_c$), it is easy to understand that the electrodynamics state formation will be practically uniform in the

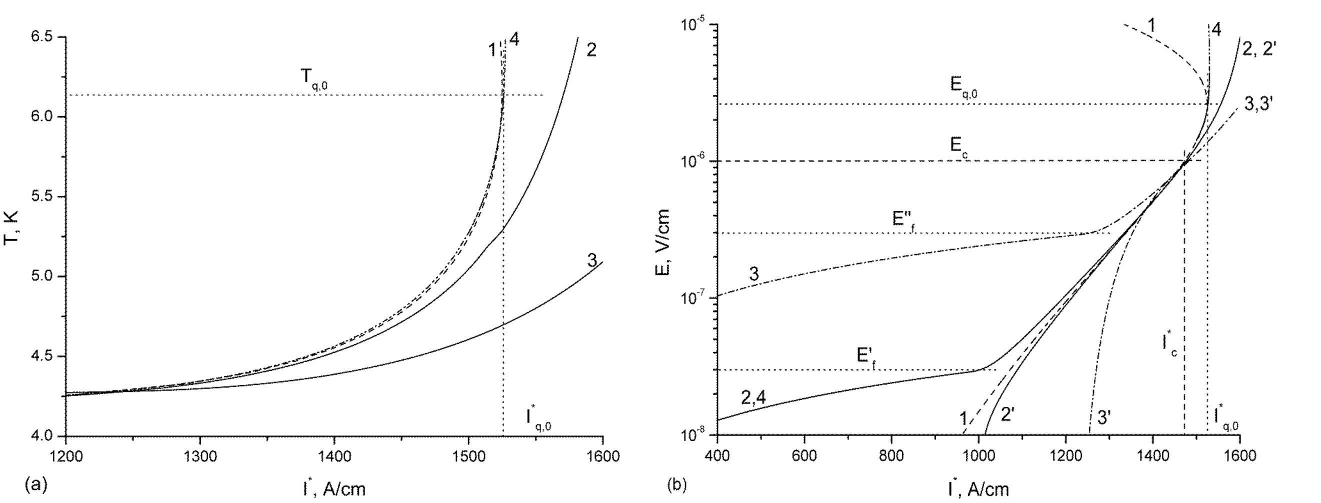

Fig. (2). Sweep rate dependence of the temperature-current (**a**) and voltage-current (**b**) characteristics of a high-T$_c$ superconductor: 1 - $dI^*/dt\to0$; 2, 2' - $dI^*/dt=10^2$A/(s$\times$cm); 3, 3' - $dI^*/dt=10^3$A/(s$\times$cm), 4 - $dI^*/dt=10^2$A/(s$\times$cm), $C=C(T_0)$.

fully penetrated mode when $E_f << E_c$, for example, at $dI/dt << 4bE_c/\left(a\mu_0\right)$.

Second, there exists the transient period after which the fully penetrated electric fields on the surface of the slab and in its centre become practically equal. To estimate the influence of the charging rate on the time during which the non-uniform distribution of the electric field exists, let us use the estimator

$$\partial E/\partial t \sim \left(E/a^2\right)\partial E/\partial J \tag{51}$$

following from equation (2). It shows that the redistribution of the electric field in the cross-section of the slab becomes more intensive with increasing differential resistivity of a superconductor $\partial E/\partial J$ and value of the induced electric field. Taking into consideration this conclusion and evident fact that induced electric field increases with increasing *dI/dt* (Fig. **2**), it is easy to understand that the time window of the electric field redistribution between non-uniform and practically uniform fully penetrated states will decrease with increasing sweep rate, as it is depicted in Fig. (**2**).

Third, Fig. (**2**) also reveals that in the over-critical electric field region ($E>E_c$) the fully penetrated dependences $E(I^*, dI^*/dt)$ and $T(I^*, dI^*/dt)$, which follows from transient model, do not identically with the corresponding static dependences defined by the static model. Namely, the transient differential resistivity of the superconductor not only decreases with increasing dI^*/dt but has only positive values. Consequently, the transient voltage-current characteristic of a high-T_c superconductor does not allow one to find the boundary of the current instability onset in the continuous current charging experiments. Really, the limiting current-carrying capacity is defined by the condition (18) in the static zero-dimensional approximation. The corresponding quench values $E_{q,0}$ and $I^*_{q,0}$ defined according to the (19) - (21) are depicted in Fig. (**2**) by the dotted lines. Note that the calculated boundary of stability is located in the over-critical electric field region for the superconductor under consideration. The conditions describing the existence of the stable over-critical static states are formulated below.

To explain why the condition (18) is not observed in the transient thermo-electrodynamics states and the transient $E(I^*)$ relation has only positive slope, let us use the transient zero-dimensional approximation. Assuming that the critical current of a superconductor does not depend on the magnetic field, it is easy to get

$$\frac{dE}{dJ} = \frac{1 + \dfrac{dT}{dJ}\left|\dfrac{dJ_c}{dT}\right|\left(\dfrac{E}{E_c}\right)^{1/n}}{\dfrac{J_c(T)}{nE}\left(\dfrac{E}{E_c}\right)^{1/n}} \tag{52}$$

using equation (3). To find the term *dT/dJ*, let us utilize equation (12) taking into consideration that

$$\frac{dT}{dt} = \frac{1}{S}\frac{dT}{dJ}\frac{dI}{dt}$$

Then dependence *dT/dJ* is given by

$$\frac{dT}{dJ} = \frac{EJ - (T - T_0)h/a}{C(T)\,{dI}/{dt}}S \tag{53}$$

As *dT/dt*>0 during current charging then *dT/dJ*>0. Therefore, as follows from equations (52) and (53), the differential resistivity of a superconductor is always positive in the monotonously current charging and the slope of the transient $E(I^*, dI^*/dt)$ and $T(I^*, dI^*/dt)$ dependences will become smaller when the heat capacity of a superconductor is higher, i.e., will decrease with increasing temperature of a superconductor during both stable and unstable states. At the same time, as follows from Fig. (**2**), there exists stable temperature rise of a superconductor before current instability. This unavoidable temperature rise of a superconductor will change its heat capacity and the effect of heat capacity of superconductor on the electrodynamics state is inevitable during current charging. To illustrate this feature, Fig. (**2**) also presents the dependencies $E(I^*)$ and $T(I^*)$ obtained under the assumption that the heat capacity of the superconductor calculated at the temperature of the coolant does not change in the temperature (curve 4).

Fig. (**2**) clearly depicts that the temperature of the superconductor before the current instability onset is not equal to the temperature of the coolant. In particular, allowable temperature rise of a superconductor before current instability onset depends on the current charging rate: the higher the *dI/dt*, the higher the stable overheating of a superconductor, as shown in the inset of Fig. (**2**). In these cases, the effect of the heat capacity of a superconductor on the electrodynamics states of a superconductor becomes more noticeable.

The results obtained demonstrate the reason according to which the transient voltage-current characteristics of high-T_c superconductor do not allow to find the boundary of the current instability onset. To avoid this complexity, the current charging with break (method of the fixed current) is used in the experiments [16-18, 21-23]. According to this method, there are two final states: temperature and voltage either are stabilized or begin to grow spontaneously. In the first case, the stable current distribution is a direct conesquence of the existence of stable static states that underlie the formation of a stable branch of the voltage-current characteristic of a superconductor. However, if the charged current exceeds the corresponding quench current $I_{q,0}$ then the formed states are unstably even in spite of the fact that the corresponding values of the electric field and the temperature may be below than the boundary values $E_{q,0}$ and $T_{q,0}$ on the transient voltage-current or temperature-current characteristics of a superconductor. Therefore, all currents in Fig. (**2**) exceeding the relevant value I^*_q correspond to unstable currents. Note that this method of the current instability determination was proposed for the first time in [24, 25] analyzing the conditions of the current instability onset of the low-T_c superconductors.

To prove these peculiarities, the transient states of the superconductor that will be observed near the instability current boundary are shown in Fig. (**3**). The calculations were carried out both in the continuous current charging and during current charging to a fixed value I^*_0 after which *dI/dt* is equal to zero. Here, the corresponding values of $E_{q,0}$ and

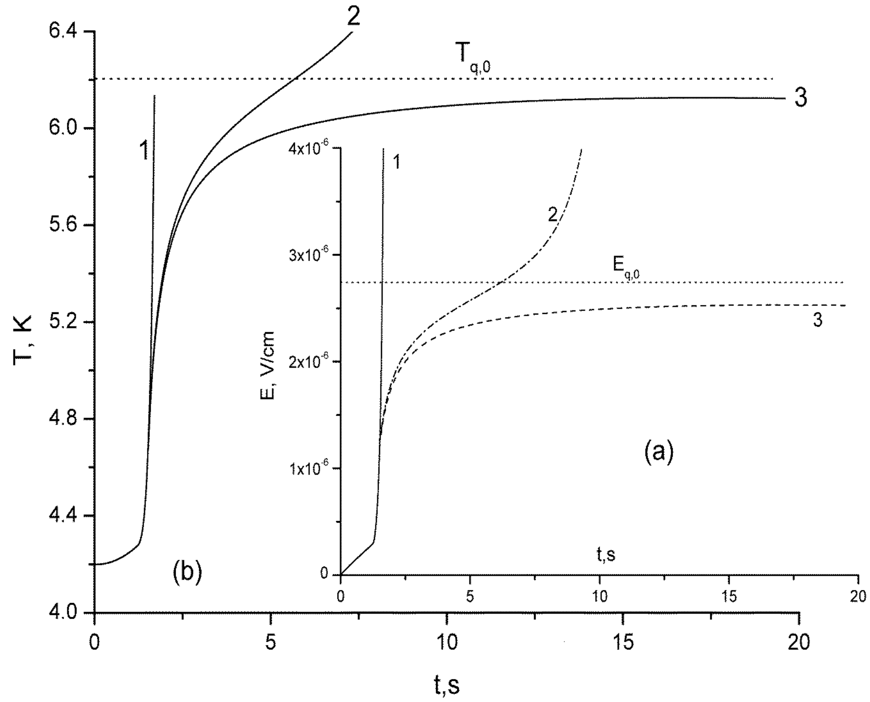

Fig. (3). Time variation of the electric field (**a**) and temperature (**b**) on the surface of superconductor during continuous charging at dI^*/dt=10^3A/(s×cm) (curve 1) and current charging with break (curve 2 - I^*_0=1524 A/cm; curve 3 - I^*_0=1523 A/cm).

$T_{q,0}$ that are the same as in Fig. (**2**) are also given. It is seen that the values $E_{q,0}$ and $T_{q,0}$ are the boundary values between transient stable and unstable states during fully penetrated mode when the condition ha/λ <<1 takes place regardless of the finite value of current charging rate and the temperature dependence of heat capacity of a superconductor.

Note important feature that is characteristic of modern high-T_c superconductors. The results presented show that the current instability occurs during fully penetrated modes. In this case, the instability onset does not depend on the current charging rate and temperature dependence of heat capacity. As known, the current instability onset in the low-T_c superconductors can also occur in the partially penetrated modes [2, 24, 25]. This fact is due to differences in critical properties of low-T_c and high-T_c superconductors. Let us write the condition under which the current instability onset in superconductors will occur only in the fully penetrated mode. This would be in the case when $E_{q,0} > E_f$, i.e. at

$$\left[\frac{E_c^{1/n}}{nJ_{c0}}\frac{h}{a}\left(T_{cB}-T_0\right)\right]^{\frac{n}{n+1}} > \frac{a\mu_0}{4b}\frac{dI}{dt} \tag{54}$$

If this condition is not valid then limiting charged current will depend on the sweep rate and temperature dependence of heat capacity [2, 24, 25]. In this case, the permissible overheating of the superconductor before the instability may essentially differ from $T_{q,0}$ and increases with increasing dI/dt [24, 25]. The condition (54) shows that, in general, the current instability onset depends on dJ_c/dT as $dJ_c/dT \sim J_{c0}/(T_{cB}-T_0)$. Therefore, the probability of the current instability onset in the partially penetrated mode will increase with increasing critical current density of high-T_c superconductors. In other words, there may be such current charging modes when the current-carrying properties of the high-T_c superconductor with high critical current density will be not fully used due to the instability of the charged currents that may occur in the partially penetrated mode. In addition, the partially or fully penetrated regimes of instability onset depend on the n-value of $E(J)$ relation of a superconductor. In particular, the probability of instability onset decreases with decreasing n.

Thus, the heat capacity of high-T_c superconductor may play essential role in the stable and unstable formation of its electrodynamics states in the over-critical modes. It appears that it is impossible to see in an experiment a steep transition from the stable state to the unstable one even at helium temperature level. Strictly speaking, this is possible only at very low current charging rates. Note that the heat capacity of low-T_c superconductors also influences on the formation of their voltage-current and temperature-current characteristics. However, its role is insignificant because of the small temperature rise in the stable states. Curves 4 in Fig. (**2**) clearly show the characteristic type of voltage-current and temperature-current characteristics of superconductors with low heat capacity.

The role of the heat capacity will increase when the operating temperature increases. Fig. (**4**) shows the influence of the current charging rate on the thermo-electrodynamics states of the superconducting slab under consideration cooled at operating temperature T_0 = 20 K. The numerical simulation was performed under the assumption of the non-intensive cooling condition (h=10^{-3} W/(cm^2×K) at B=10 T and the critical current density is described by equation (9) with parameters (10). This allows one to use the transient and static zero-dimensional models (11) and (12). Accordingly, solid lines 2-4 describe the transient thermo-electrody-

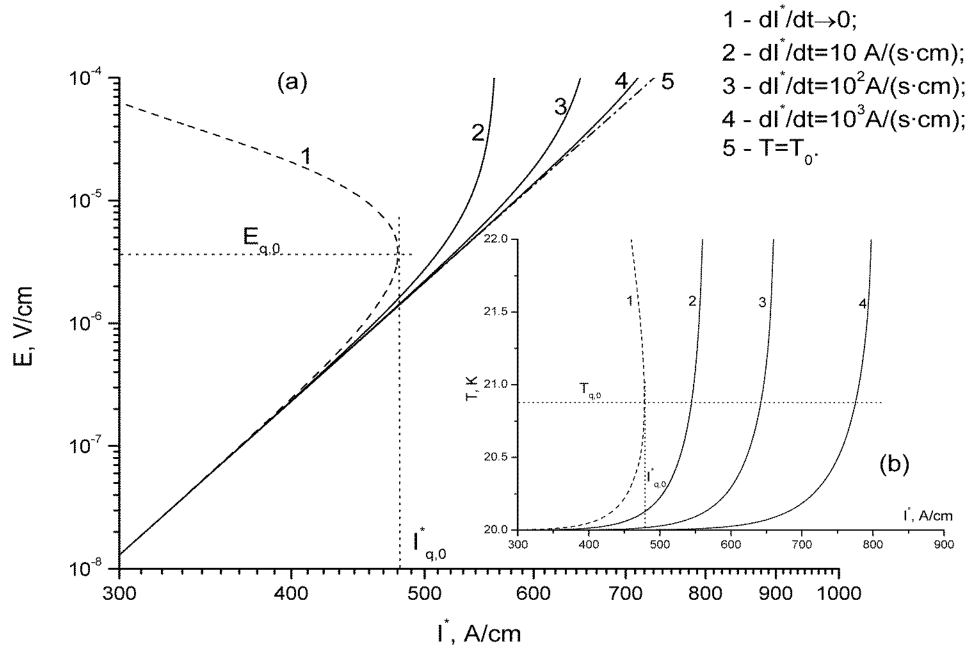

Fig. (4). Electric field (**a**) and temperature (**b**) *versus* applied current at T_0=20 K and various current charging rate.

namics states of the superconductor and line 1 shows the static state. Dash-dotted curve 5 corresponds to the isothermal $E(J)$ characteristic calculated at $T = 20$ K.

Comparing results shown in Fig. (**2**) and Fig. (**4**), it is easy to see the following operating temperature effect: the higher the temperature of the coolant or the current charging rate, the more close the non-isothermal dependences to the corresponding isothermal ones in the high electric field range. This peculiarity follows from the relationships (52) and (53) at the limiting transition $dT/dJ \to 0$ if $C(T)dI/dt \to \infty$. It allows one to make unexpected conclusion that is important for practical applications: an isothermal voltage-current characteristics in the over-critical electric field range can be obtained charging the current into the non-intensive cooled superconductor at high value of the current charging rate. It is obvious that in this case the condition $ha/\lambda<<1$ must take place.

Thus, in the continuous current charging modes, transient voltage-current and temperature-current characteristics of high-T_c superconductors during fully penetrated states have only one branch with the positive slope, which depends on the temperature dependence of heat capacity. That is why their slope decreases when the value dI/dt increases. This thermal effect exists both in the stable and unstable thermo-electrodynamics states. It is due to the finite temperature rise of a high-T_c superconductor both before and after instability onset that increases the heat capacity of a superconductor. Its role is the most significant in the high electric fields or high operating temperature. Therefore, the transient $E(J)$ characteristics of high-T_c superconductors do not allow to find the current instability onset in the experiments and the possible stable increase in temperature of high-T_c superconductors should be taken into account for the correct determination of their critical currents.

7. THERMAL PECULIARITIES OF ELECTRODYNAMICS STATE FORMATION IN THE NON-UNIFORM TEMPERATURE DISTRIBUTION

Let us investigate the formation peculiarities of the electrodynamics states of high-T_c superconductor accounting for non-uniform temperature distribution in its cross section using the appropriate current instability conditions formulated above. To discuss the basic physical features, let us use, first of all, the one-dimensional diffusion model described by equations (1) - (8) considering the non-isothermal penetration of applied current into examined $Bi_2Sr_2CaCu_2O_8$ superconductor at B=10 T and T_0=4.2 K under parameters (48) - (50) assuming that intensive cooling condition takes place (h=0.1W/(cm^2×K)). As above, the performed analysis will be done using the current and current sweep rate normalized by the slab width ($I^*=0.5I/b$, $dI^*/dt=0.5b^{-1}dI/dt$).

Fig. (**5**) shows the electric field and the temperature as a function of applied current, which will exist on the surface and in the center of the superconducting slab at dI^*/dt=10^{2}A/(s×cm) at different current sweep rate. To understand the peculiarities of the non-isothermal development of electrodynamics states in the superconductor that take place both in stable and unstable regimes, the isothermal $E(I^*)$ characteristic of a superconductor (curve 1) calculated according to the formulae (3) and (4) at T=4.2 K is also presented in Fig. (**5**). Besides, the curve 2 describes the non-isothermal static $E(I^*)$ characteristic determined under the assumption of the uniform temperature and current distributions that takes place over the whole range of the transport current charging. In this case, the static thermo-electrodynamics states of the superconductor are described by equations (3), (4) and (12) in which the density of the transport current is equal to $J=I^*/(2a)$. Fig. (**5a**) also depicts the electric field $E_{q,0}$ and the current $I^*_{q,0}$ defining the stability boundary of spatially

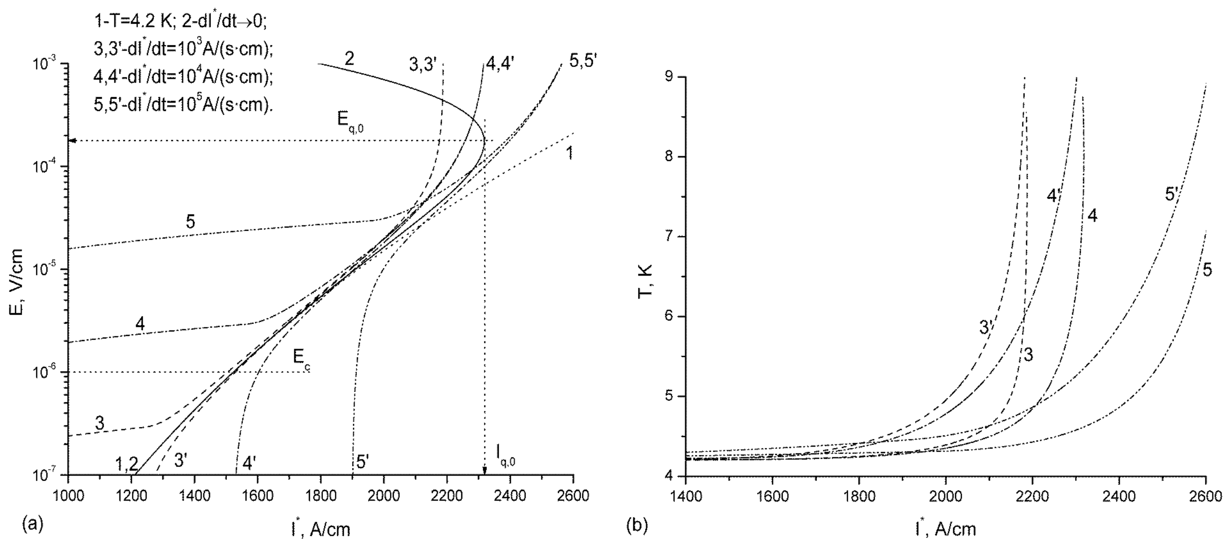

Fig. (5). Dependences $E(J)$ (**a**) and $T(J)$ (**b**) on the surface (curves 3, 4, 5) and in the center (curves 3', 4', 5') of Bi2212 in the various operating modes.

homogeneous states, which follow from the relationships (19) and (20).

Fig. (**5**) allows one to formulate the characteristic peculiarities of the temperature and electric field formation in superconductors in the continuous current charging allowing for size effect that has to be taken into account during experiments.

First, as shown above, the temperature distribution in a superconductor ceases to be uniform even at a relatively low current charging rate while the distribution of the electric field becomes almost uniform after some transient period. This trend is kept also at higher current charging rates, as can be seen from Fig. (**5**). Nevertheless, it should be noted that the uniform distribution of the electric field in a superconductor would occur in the fully penetrated mode when the current charging rate satisfies the above-formulated condition $dI/dt << 4bE_c/(\mu_0 a)$.

Second, during intensive cooling condition the transient states of the high-T_c superconductor (curves 3, 3' - 5, 5' in Fig. (**5**)) quantitatively and qualitatively differ from the dependences $E(I^*)$ calculated not only in the isothermal approximation (curve 1) but also in terms of the static non-isothermal zero-dimensional model (curve 2). In the high electric field ($E > E_c$), this feature is observed because of the thermal influence of the heat capacity of the superconductor on the transient electrodynamics states in the continuous current penetration, as it was discussed above. Therefore, the slope of curves 3, 3' - 5, 5' is always positive and decreases with the increasing current charging rate. Due to this peculiarity, the non-stationary $E(I^*)$ characteristic of superconductor during fully penetrated regime should be below the corresponding static $E(I^*)$ characteristic before the current instability, if the formation of the electric field in the superconductor depends only on its heat capacity. However, Fig. (**5**) depicts that the transient $E(I^*)$ characteristics of a superconductor at a relatively low current charging rate (curves 3, 3' and 4, 4') may be higher than the static $E(I^*)$ characteristic calculated according to the non-isothermal zero-dimensional model in the area of the electric fields preceding the current instability. As a result, this rise in the electric field will change the range of the stable currents calculated in the framework of the non-uniform distribution of temperature in comparison with a similar range obtained by the zero-dimensional model. This formation feature taking place at low values of dI/dt cannot be explained by the influence of superconductor's heat capacity on the electrodynamics phenomena occurring in superconductors. In order to understand the physical reason of this peculiarity, let us use above-formulated static one-dimensional model considering the thermal heterogeneity of the temperature distribution in the slab under consideration, which takes place in the fully penetrated mode at $dI/dt \to 0$.

Fig. (**6**) shows the static $E(I^*)$ characteristics determined in the assumption of the uniform and non-uniform distribution of temperature in the slab. In the first case, curve 2 was defined in accordance with formula

$$I^* = 2J_{c0}a\left(\frac{E}{E_c}\right)^{1/n}\frac{H}{\gamma + H} \qquad (55)$$

as follows from the zero-dimensional model defined by relations (3), (4) and (12). For heterogeneous states, the performed calculations were based on the formula (37) under assumption that the thermal conductivity of the superconductor is defined as $\lambda_0=\lambda(T^*)$ at different T^* according to (50) (curves 3 and 4), as well as on the numerical solution of the problem (3), (4), (31), (33) taking into account the dependence (50) (curve 5).

These results show clearly that, in the theoretical analysis of current instability conditions, the non-uniform temperature distribution in the cross-section of superconductor leads to a reduction in the stable currents relative to the corresponding values determined in the framework of the zero-dimensional model. To understand the physical meaning of this feature, let us find the average temperature of the superconductor both in the case of one-dimensional and

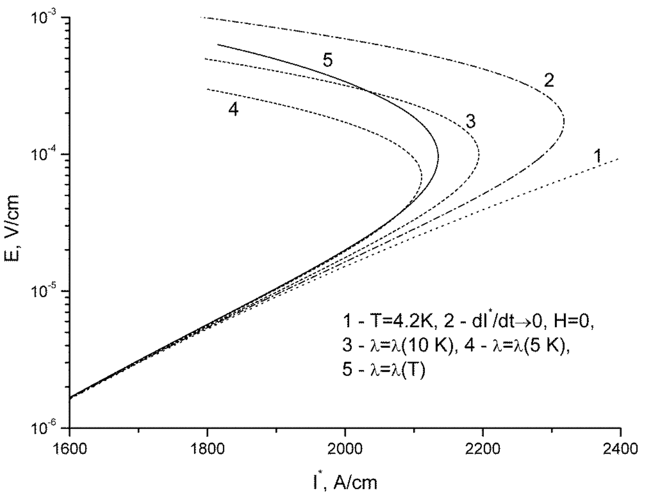

Fig. (6). Influence of the thermal conductivity coefficient of superconductor on the $E(J)$ characteristic. Here, $\lambda(T)$ is defined by (50).

zero-dimensional approximations. In accordance with one-dimensional model and formula (36) its value is equal to

$$T_v = \frac{1}{a}\int_0^a T(x)\,dx = T_0 + \left(T_{cB} - T_0\right)\left(1 - \frac{H}{\gamma + H\sqrt{\gamma}/\tanh\sqrt{\gamma}}\right) \tag{56}$$

At the same time, according to the zero-dimensional model, the temperature of superconductor is defined as

$$T_{v,0} = T_0 + \left(T_{cB} - T_0\right)\frac{\gamma}{\gamma + H} \tag{57}$$

in terms of the variables used. Comparing the values of T_v and $T_{v,0}$ it is easy to see that $T_v > T_{v,0}$ because the condition $\sqrt{\gamma} > \tanh\sqrt{\gamma}$ is always observed. So, the average temperature of superconductor always exceeds the corresponding values calculated on the assumption of existence of homogeneous states at any finite γ. The difference the higher, the higher the parameter γ due to the corresponding increase in the term $\sqrt{\gamma}/\tanh\sqrt{\gamma}$. The latter has the same influence on the $E(I^*)$ characteristic of a superconductor. In other words, there is a thermal size effect, which is based on spatial peculiarities of the temperature distribution in the superconductor and depends on the value of γ. In accordance with this effect, the current stability conditions will be modified.

Note that in the cases discussed, the change in the stable current range is associated with the influence of the temperature-dependent thermal conductivity coefficient on the electrodynamics processes in a superconductor. As a result, difference between the $E(J)$ characteristics calculated according to the size effect and without it, becomes more noticeable when the thermal conductivity coefficient of the superconductor is reduced. Therefore, the influence of spatial heterogeneity of temperature distribution in the superconductor on the formation of its electrodynamics states is more noticeable, if the operating temperature of coolant is lower with other parameters being equal.

As it was formulated above, the condition (38) shows that the current instability condition depends on the character of the current redistribution in the superconductors when the temperature distribution in the superconductor is non-uniform. To show the contravention of the condition (18) more strictly, Fig. (**7**) depicts the $E(J)$ dependences, which take place on the surface of the superconducting slab and in its center during current charging. The simulation was made using numerical solution of the problem (1) - (8) for different current charging regimes. Curves 1 and 1' describe the growth of the electric field in the continuous current charging. Curves 2, 2' and 3, 3' were calculated in the current charging with a break when the value dI^*/dt was taken zero if the current is equal to some value I^*_0. According to the results shown in Fig. (**7**), curves 2 and 2' describe stable dynamics of the electric field, and curves 3 and 3' correspond to unstable states. Fig. (**7**) shows that the currents in the internal part of superconductor are always more unstable in comparison with the ones flowing on its surface due to the corresponding non-uniform temperature distribution. As a result, the condition (18) loses its physical meaning, as discussed above. Consequently, to determine in a static approximation the boundary of stable currents taking into account the heterogeneity of the thermal states, it is necessary to use the condition (38).

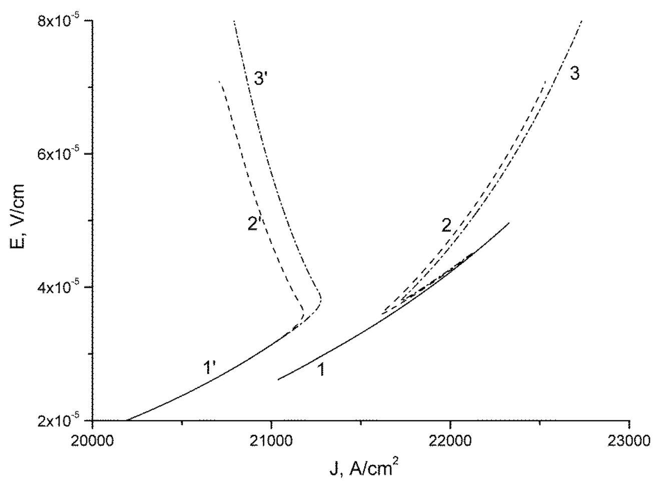

Fig. (7). Electric field on the surface (1, 2, 3) and in the centre (1', 2', 3') of superconductor as a function of current density at $dI^*/dt=10^5$A/(s×cm): 1, 1' - continuous current charging, 2, 2' - I^*_0=2130A/cm, 3, 3' - I^*_0=2140 A/cm.

As it was formulated above, the current stability conditions are described by parameter γ_q in the non-uniform temperature distribution. The quantity γ_q as a function of thermal resistance H is presented in Fig. (**8**) for various values of the exponent of the E-J curve. The calculations were made both for the case of spatially homogeneous thermo-electrodynamics states (curves 1, 2, 3) and taking into account their heterogeneity (curves 1', 2', 3'). It shows that the value γ_q calculated for thermally heterogeneous states does not exceed unit within a wide range of the thermal resistance variations. At the same time, the thermal size effect is not observed at $H \ll 1$, as expected. However, it is a rough estimate. According to Fig. (**8**), variation of H

will not be accompanied by a noticeable influence of the spatial heterogeneity on the thermo-electrodynamics states and γ_q at $H < 0.1$. In this case, the boundary values of the electric field and the current are described by the relations (19) and (20) with a good degree of accuracy. At the same time, zero-dimensional approximation does not correctly describe the current stability conditions at intensive cooling conditions, namely, at $H > 10$. In these cases, the thermal mechanism formation of the electrodynamics states leads to essentially non-uniform thermal modes of a superconductor that may cause the thermal degradation of the current-carrying capacity during intensive cooling conditions discussed below.

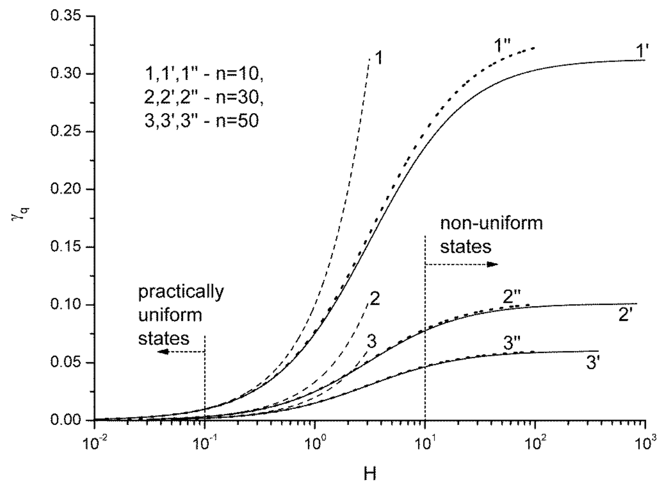

Fig. (8). Influence of thermal resistance of the superconductor on the γ_q calculated in the framework of zero-dimensional (1, 2, 3) and one-dimensional models (1', 2', 3'). Here, the curves (1', 2', 3') and (1'', 2'', 3'') are obtained according to (40) and (43), respectively.

8. STABILITY OF ELECTRODYNAMICS STATES OF HIGH - T_C SUPERCONDUCTORS COOLED BY LIQUID COOLANT

Let us discuss the features of the static electrodynamics state formation of superconductors cooled by liquid coolants when the nucleate and film heat transfer regimes exist considering the liquid helium, hydrogen and nitrogen as coolants. In these cases, the values of $q(T)$ during these boiling regimes were fitted by the expression

$$q(T)\left[W/cm^2\right] = \begin{cases} h_1(T-T_0)^{\nu_1}, & T \le T_0 + \Delta T_{cr} \\ h_2(T-T_0)^{\nu_2}, & T > T_0 + \Delta T_{cr} \end{cases} \qquad (58)$$

Here, ΔT_{cr} is the critical overheating depending on the coolant, beyond which the transition from the nucleate boiling regime to the film boiling one occurs. Values h_1, h_2, ν_1, ν_2, T_0 and ΔT_{cr} estimated according to the results presented in [51] are given in Table **1** for each type of coolant.

As it was shown above, the current instability in the superconductors may occur if the condition (18) takes place. However, the current instability in superconducting materials cooled by liquid coolant may also occur during the transition from the nucleate boiling regime to the film boiling one. This feature raises the question about the mechanism of the instability onset in superconductors cooled by liquid coolant.

Table 1. Coolant Properties

Coolant	T_0, K	ΔT_{cr}, K	h_1	h_2	ν_1	ν_2
Liquid helium	4.2	0.6	2.15	0.06	1.5	0.82
Liquid hydrogen	20	3	0.66	0.024	2.6	1.1
Liquid nitrogen	77.3	10	0.04	0.036	2.4	0.76

Using above-written solution, let us find the condition under which the current instability occurs because of the conditions (18) rather than the thermal transition from the nucleate to the film boiling regimes. It takes place at $T_0+\Delta T_{cr} > T_q$, i.e. at

$$\frac{n}{\nu_1} > \frac{T_{cB}-T_0}{\Delta T_{cr}} - 1 \qquad (59)$$

according to (30). Thus, the parameters of the superconductor and the liquid coolant are connected by the non-trivial relationship leading to different mechanisms of the current instability onset. In particular, they depend on difference between the rise rates of $E(J)$ characteristic of superconductor (n-value) and the heat flux to the coolant during nucleate boiling regime (ν_1-value). As a whole, the condition (59) indicates the following characteristic peculiarities of the current instability onset in high-T_c superconductor cooled by liquid coolant.

First, as follows from the condition (59), characteristic n-value exists. It equals

$$n_\nu = \left(\frac{T_{cB}-T_0}{\Delta T_{cr}} - 1\right)\nu_1$$

If $n>n_\nu$, then the current instability during nucleate boiling mode will have electrodynamics nature. As a result, the current instability will take place before the violation of the nucleate boiling mode at the lower n-value, if the operating temperature T_0 or the critical overheating ΔT_{cr} are higher while the quantity $T_{c\,B}$ is smaller. As a consequence, in the limiting case, satisfying the condition

$$\left(\frac{T_{cB}-T_0}{\Delta T_{cr}} - 1\right)\nu_1 < 1$$

the current instability will always occur before the crisis of the coolant boiling for all $n > 1$.

Second, the electrodynamics mechanism of the current instability, i.e. the criterion (18), is the most probable when the temperature margin of the superconductor that equals $\Delta T=T_{c\,B}-T_0$ is higher at the fixed value of the quantity ΔT_{cr}. Let us use the condition (59) and estimate the potential values of ΔT at which the current instability follows from the conditions (18) rather than the boiling crisis. Relevant values of ΔT as a function of n-value are presented in Table **2** for

Table 2. Potential Temperature Margin

n-value	10	20	30	40	50
ΔT, K liquid helium	4.6	8.6	12.6	16.6	20.6
ΔT, K liquid hydrogen	14.54	26.08	37.62	49.15	60.69
ΔT, K liquid nitrogen	51.67	93.33	135	177	218

different types of coolant. They were obtained according to the data given in Table **1**.

Table **2** shows that the current instability may satisfy the condition (18) even when high-T_c superconductor is cooled by liquid helium. However, these unstable regimes can be observed when high-T_c superconductor will be placed in very high external magnetic field. At the same time, in the cases of high-T_c superconductor cooled by liquid nitrogen the electromagnetic mechanism of the destruction of the stable current distribution described by the condition (18) will be realized in a wide range of variations of the external magnetic field.

Figs. (**9-11**) illustrate the peculiarities discussed.

Fig. (**9**) depicts the static dependences of the temperature and electric field as a function of applied current density that take place in the $Bi_2Sr_2CaCu_2O_8$ slab (a=10^{-2} cm) placed in the external magnetic field B=10 T and cooled by liquid helium. Two types of simulations were made. They were performed both for the nucleate boiling regime and for the cooling conditions that could exist if only the film boiling mode takes place. The temperature boundary between these regimes is indicated by the dotted horizontal curve. The critical current density of the $Bi_2Sr_2CaCu_2O_8$ was determined by the formula (9). The n-value is equal to 10. The results

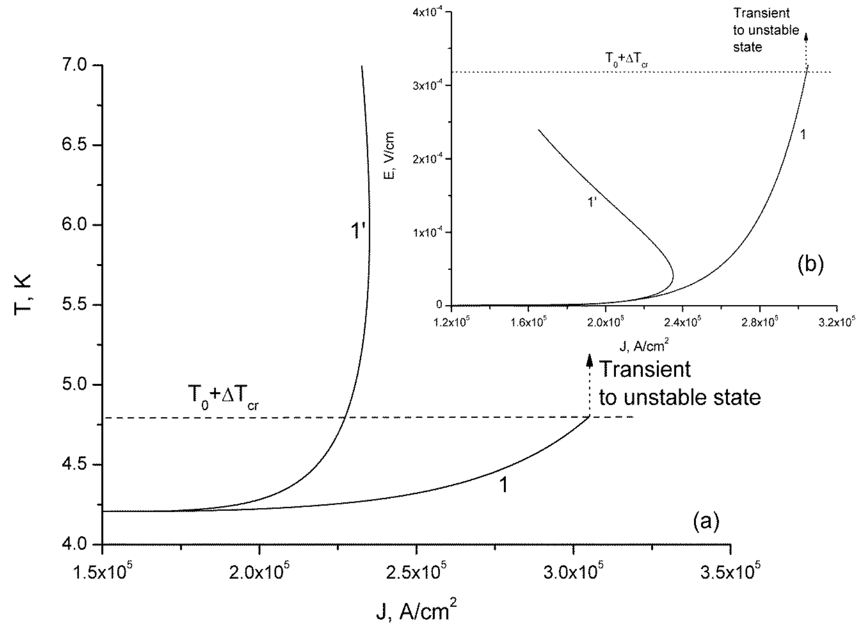

Fig. (9). Temperature-current (**a**) and voltage-current (**b**) characteristics of the $Bi_2Sr_2CaCu_2O_8$ cooled by liquid helium: 1 - the nucleate boiling regime, 1' - the film boiling regime.

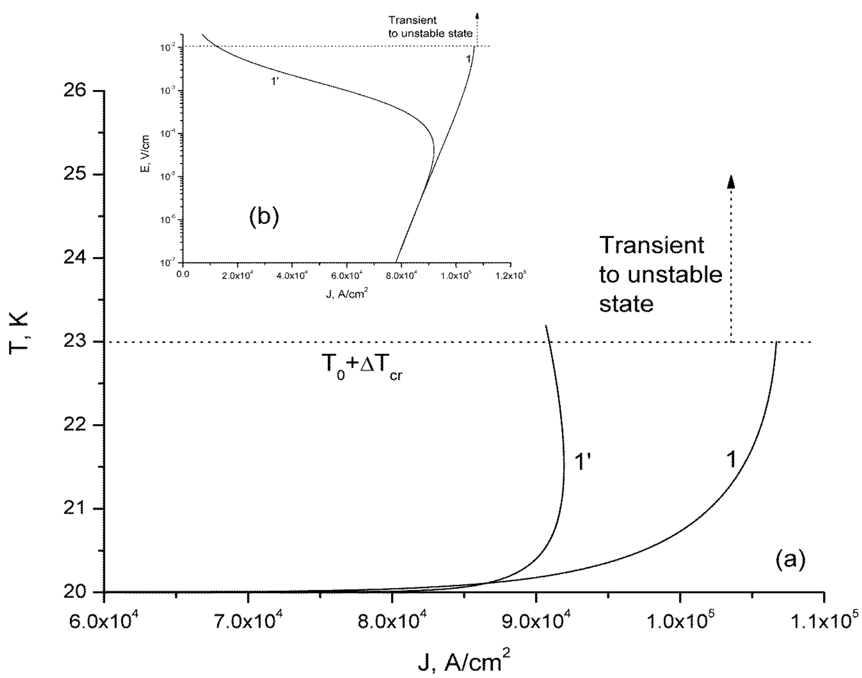

Fig. (10). Temperature-current (**a**) and voltage-current (**b**) characteristics of the $Bi_2Sr_2Ca_2Cu_3O_8$ cooled by liquid hydrogen: 1- the nucleate boiling regime, 1' - the film boiling regime.

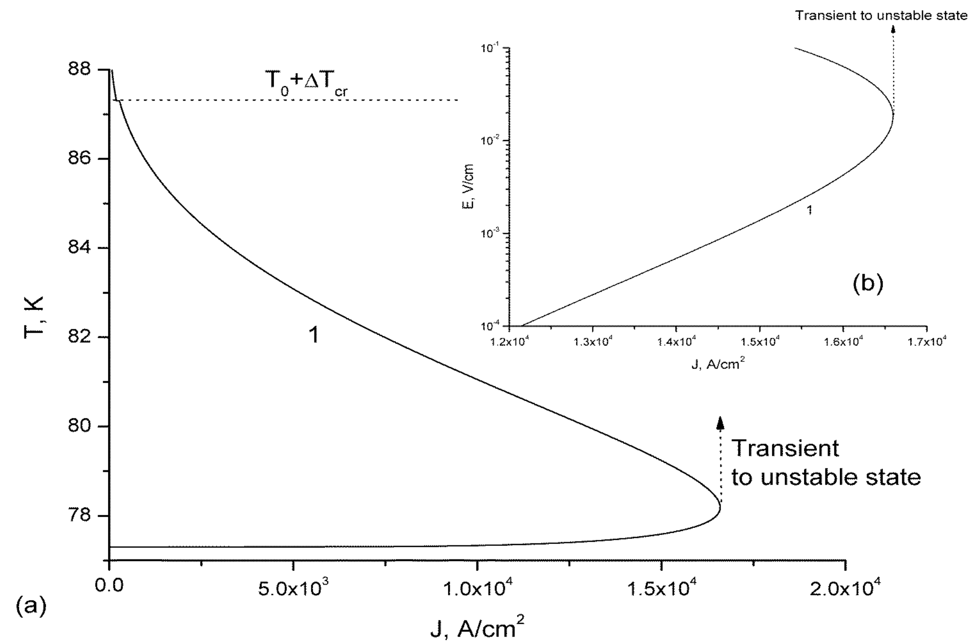

Fig. (11). Temperature-current (**a**) and voltage-current (**b**) characteristics of the $YBa_2Cu_3O_7$ cooled by liquid nitrogen: 1 - the nucleate boiling regime, 1' - the film boiling regime.

presented show that the current instability in the examined superconductor is a consequence of its trivial overheating exceeding the quantity ΔT_{cr}. This overheating initiates the thermal transition from the nucleate to the film boiling modes. In this case, the slopes of $E(J)$ and $T(J)$ are positive before the current instability onset. Therefore, the current instability starts at their steady growth.

The results presented in Figs. (**10** and **11**) were calculated for the $Bi_2Sr_2Ca_2Cu_3O_8$ slab (a=10^{-2} cm), which is in the external magnetic field B=1 T, and for the $YBa_2Cu_3O_7$ film (a=10^{-5} cm) placed in the external magnetic field B=5 T. The calculations were performed both for the nucleate and for the film boiling modes (curves 1 and 1') when the $Bi_2Sr_2Ca_2Cu_3O_8$ slab is cooled by liquid hydrogen and the $YBa_2Cu_3O_7$ film is cooled by liquid nitrogen. In this case, the n-values are assumed to be equal to n=30 for the $Bi_2Sr_2Ca_2Cu_3O_8$ and n=10 for the $YBa_2Cu_3O_7$, respectively. The temperature dependences of the critical current density of the superconductors under consideration were set as follows. The critical current density of the $Bi_2Sr_2Ca_2Cu_3O_8$ is determined by the formula (4) for which the values J_{c0} and T_{cB} were adopted from the linear interpolation of the experimental data given in [40]. Therefore, they were equal to J_{c0}=8.42×10^4 A/cm^2 and T_{cB}=62.72 K at B=1 T, T_0 = 20 K, E_c=10^{-6} V/cm. For the $YBa_2Cu_3O_7$ the critical current density is calculated from the formula

$$J_c(T,B) = J_0\left(1 - T/T_c\right)^{1.5} B^{-0.62}\left(1 - B/B_{irr}\right)^{2.27},$$
$$B_{irr} = 376.235\exp(-0.048T)$$

at E_c=2×10^{-6} V/cm, J_0=2.8×10^6 A/cm^2, T_c=90 K in accordance with the results presented in [52, 53].

As can be seen from Fig. (**10**), the current instability in the $Bi_2Sr_2Ca_2Cu_3O_8$ slab occurs when the nucleate boiling mode of the cooling is destabilized. Nevertheless, the value of T_q=23.4 K calculated for the superconductor under consideration using the formula (30) at n=30 is close to the corresponding value of ΔT_{cr} that equals 23 K. That is why the curves 1 depicted the dependences $E(J)$ and $T(J)$ in Fig. (**10**) have sharply increasing character before the current instability unlike similar curves 1 shown in Fig. (**9**). Consequently, the mechanism of the current instability onset in the $Bi_2Sr_2Ca_2Cu_3O_8$ slab may be modified, for example, due to the corresponding decrease in the quantity $T_{c\,B}$ that may take place increasing the magnetic field.

The current instability in the $YBa_2Cu_3O_7$ film under consideration occurs before a violation of the nucleate boiling regime (Fig. **11**) according to the criteria (28). Let us estimate the value of n when the violation of the stable state of current charged into the high-T_c superconductors cooled by liquid nitrogen will occur due to the unlimited growth of their differential resistivity. Assuming $T_{c\,B}$-T_0~20 K, it is easy to find from (59) that the current instability in the high-T_c superconductors cooled by liquid nitrogen will occur before the onset of the boiling crisis of the coolant at n>2.4. In the meantime, experiments show [40, 41] that the n-values of high-T_c superconductors cooled by liquid nitrogen may change in the wide range becoming close to unit with increasing temperature of the superconductor. Therefore, as follows from the comparison of the results presented in Figs. (**9-11**), it is necessary to take into account the temperature-dependent nature of the values n when liquid nitrogen is used. This is because its value ΔT_{cr} is higher than the corresponding values of liquid helium or hydrogen.

The results and criteria discussed were obtained under the assumption that there exists a spatial homogeneity of the thermo-electrodynamics states during current charging, i.e. at ha/λ << 1. However, this condition may be valid at high value of h. Therefore, the current stability problem should be solved taking into account the size effect, i.e. considering the heterogeneous distribution of the temperature in the cross section of the superconductor. Let us discuss its possible influence on the current instability conditions of the above-discussed bismuth-based superconductors. (It is easy to find that the size effect is insignificant for the $YBa_2Cu_3O_7$ film considered because of its small thickness).

Let us determine the limiting current stably flowing in the superconductor using the solution of the one-dimensional stationary equation (31) with the boundary conditions

$$\frac{\partial T}{\partial x}(0,t) = 0,\ \lambda\frac{\partial T}{\partial x}(a,t) = -q(T)$$

In this case, describing the *E*(*J*) characteristic of superconductor and its temperature dependence on the critical current density and *n*-value as it was made above, we will use the relationship (50) for the $Bi_2Sr_2CaCu_2O_8$ and

$$\lambda(T) = 2\times10^{-4}T\left[W/(cm\times K)\right]$$

to define thermal conductivity of $Bi_2Sr_2Ca_2Cu_3O_8$ according to [54].

Figs. (**12** and **13**) depict the temperature-current characteristics of the $Bi_2Sr_2CaCu_2O_8$ and the $Bi_2Sr_2Ca_2Cu_3O_8$ numerically calculated at n=10 and n=30, respectively. These curves were determined for the nucleate boiling regime both excluding the size effect (curve 1) and taking it into account (curves 2, 3).

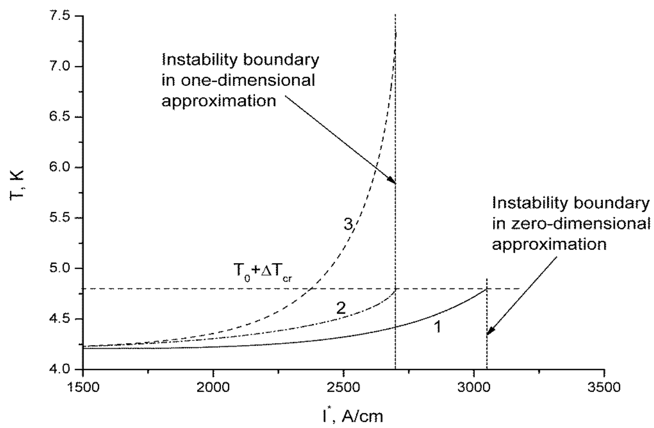

Fig. (12). Temperature of the $Bi_2Sr_2CaCu_2O_8$ ($a=10^{-2}$ cm) cooled by liquid helium *versus* normalized current ($I^*=J\times a$): 1 - zero-dimensional approximation, 2, 3 - one-dimensional approximation (2 - $T(a)$, 3 - $T(0)$).

The results presented in Fig. (**12**) show that in the case of the $Bi_2Sr_2CaCu_2O_8$ the current instability occurs because of the violation of the nucleate boiling regime even in spite of the significantly heterogeneous temperature distribution in the cross section of the superconductor. However, the attention should be paid to the difference in the growth character of temperature-current curves calculated in the framework of zero- and one-dimensional approximations. It is seen that the slope of the curve 1 obtained in terms of the zero-dimensional model is less smooth than the slope of the curves 2 and 3, which were calculated taking into account the heterogeneous distribution of temperature in the cross section of the superconductor. This feature is a consequence of the size effect discussed: the formation of a spatially heterogeneous temperature is accompanied by its sharper rise in comparison with the corresponding change in the uniform temperature during current charging. This feature becomes more evident before the onset of the boiling crisis when the curves 2 and 3, unlike the curve 1, start to grow quickly. Obviously, this difference between the zero-dimensional and one-dimensional approximations also takes place in the formation of the electric field. Therefore, the current instability will be a result of the electrodynamics mechanism described by the condition (18) rather than by the thermal mechanism based on the transition from the nucleate to film boiling mode when the subsequent size effect takes place. Such an influence depends, in particular, on the type of coolant. This follows from Fig. (**13**). Thereby, if, in the framework of the zero-dimensional model, the current instability in $Bi_2Sr_2Ca_2Cu_3O_8$ cooled by liquid hydrogen is the result of its thermal overheating exceeding the quantity $T_0+\Delta T_{cr}$, then according to the one-dimensional approximation the current instability occurs before the violation of the nucleate boiling regime. In this case, the temperature-current characteristics of the $Bi_2Sr_2Ca_2Cu_3O_8$ calculated both on the surface of the superconductor and inside its central part have two characteristic branches: stable ($\partial T/\partial J>0$) and unstable ($\partial T/\partial J<0$). Therefore, an adequate description of the current instability mechanisms in the high-T_c superconductors cooled by liquid coolant at the intermediate operating temperatures also depends on the spatial homogeneity of the temperature field induced in the superconductor due to current charging. As a result, the size effect leads to a steady reduction in the stable current range of the liquid-cooled superconductors. The discussion of this peculiarity was made above.

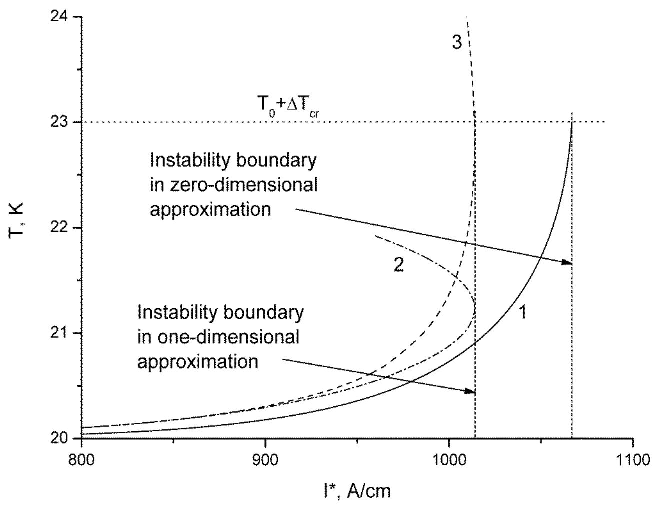

Fig. (13). Temperature of the $Bi_2Sr_2Ca_2Cu_3O_8$ ($a=10^{-2}$ cm) cooled by liquid hydrogen *versus* normalized current ($I^*=J\times a$): 1 - zero-dimensional approximation, 2, 3 - one-dimensional approximation (2 - $T(a)$, 3 - $T(0)$).

Thus, the electrodynamics state formation in high-T_c superconductors cooled by liquid coolant has intrinsic peculiarity defined by the noticeable stable variation of their temperature before the onset of instability. As a result, the non-trivial correlation between properties of superconductor and coolant leads to different mechanisms of the current instability onset. The latter also depends on the character of the temperature distribution in the cross section of the superconductor. Consequently, the instability onset may be caused by both the trivial overheating of the superconductor's surface above the temperature transition from the nucleate boiling regime to the film boiling one and by the appearance of an unstable branch in the voltage-current characteristic even under the nucleate boiling mode of cooling. The criteria formulated allow one to define each of the mentioned mechanisms. They show that the thermal mechanism of the current instability onset is the most typical instability mechanism for high-T_c superconductors cooled by a liquid coolant with low operating temperature, for example, when they are cooled by liquid helium. At the same time, the boiling crisis will lead to unstable current states of high-T_c superconductors cooled by liquid nitrogen only when the n-value is reduced to possible values closed to unit. Moreover, the stability analysis of the thermo-electrodynamics states of superconductors cooled by liquid nitrogen should take into account the decrease in the temperature of the n-value.

These peculiarities make the current instability onset in the high-T_c superconductors cooled by liquid hydrogen or nitrogen different compared with the superconductors cooled by liquid helium. Therefore, the electrodynamics study of high-T_c superconductors must consider this peculiarity, particularly, when the liquid nitrogen is used.

9. THERMAL EFFECTS AND DEGRADATION OF CURRENT-CARRYING CAPACITY OF SUPERCONDUCTOR

As follows from the above-discussed results, there exist three thermo-electrodynamics modes of applied current penetration: partially penetrated mode with negligible overheating, fully penetrated mode with inconsiderable but finite overheating and fully penetrated mode with noticeable overheating. Let us find the characteristic boundary temperature of superconductor that it will have in the fully penetrated stable state at $E=E_c$ and $dI/dt \to 0$. This value is maximum at $x=0$ in accordance with (36) and equals

$$T(0)\big|_{E=E_c} = T_{\max} = T_{cB} - \left(T_{cB} - T_0\right)\frac{H}{\sqrt{\gamma_c}\sinh\sqrt{\gamma_c} + H\cosh\sqrt{\gamma_c}},$$

$$\gamma_c = \gamma(E_c) = \frac{J_{c0}E_c a^2}{\lambda_0\left(T_{cB} - T_0\right)}$$

Therefore, the overheating $\Delta T_c = T_{\max} - T_0$ of superconductor at $E=E_c$ is equal to

$$\Delta T_c = \left(T_{cB} - T_0\right)\left[1 - \frac{1}{\left(\sqrt{\gamma_c}/H\right)\sinh\sqrt{\gamma_c} + \cosh\sqrt{\gamma_c}}\right]$$

Since γ_c does not exceed unit (Fig. **8**) then the approximate value of ΔT_c may be written as follows

$$\Delta T_c \approx \left(T_{cB} - T_0\right) / \left(1 + \frac{2H}{2+H}\frac{\lambda_0\left(T_{cB} - T_0\right)}{J_{c0}E_c a^2}\right)$$

after some algebra. This value may be calculated as

$$\Delta T_c \approx \left(T_{cB} - T_0\right) / \left[1 + \frac{h\left(T_{cB} - T_0\right)}{aJ_{c0}E_c}\right]$$

when $H << 1$.

Thereby, the overheating of a superconductor is considerable at $E=E_c$ and $dI/dt \to 0$, if

$$\frac{\lambda_0\left(T_{cB} - T_0\right)}{J_{c0}E_c a^2} << \frac{2+H}{2H}$$

or

$$\frac{h\left(T_{cB} - T_0\right)}{aJ_{c0}E_c} << 1$$

at $H << 1$. The latter condition also follows from (14) or (15).

The written estimates allow one to investigate the influence of the finite temperature overheating of superconductor on limiting current-capacity of superconductors. Note that it is not defined by the critical current because instability current depends on the temperature of superconductor before instability onset. Let us investigate this thermal feature in detail.

Rewrite the relationship (37) in the form of

$$I = I_c\frac{\left(E/E_c\right)^{1/n}}{\sqrt{\left(E/E_\lambda\right)\left(E/E_c\right)^{1/n}} / \tanh\sqrt{\left(E/E_\lambda\right)\left(E/E_c\right)^{1/n}} + \left(E/E_h\right)\left(E/E_c\right)^{1/n}} \qquad (60)$$

Here,

$$E_h = \frac{h\left(T_{cB} - T_0\right)}{J_{c0}a}, \quad E_\lambda = \frac{\lambda_0\left(T_{cB} - T_0\right)}{J_{c0}a^2}$$

This equation of the current-voltage relation of the superconductor shows that there exist two characteristic values E_h and E_λ above which the intensity of growth of $E(J)$ relation changes during current charging. The quantities E_h and E_λ are equal to the ratio of the power of the heat flux into the coolant and the heat flux transferred by thermal conductivity to the power of the Joule heating, respectively. They take into account the influence of the convective and conductive heat transfer mechanisms on the electrodynamics processes in the superconductor and relate as

$$E_h / E_\lambda = H \qquad (61)$$

For this reason, the voltage-current characteristic of the superconductor depends not only on its critical properties but also on its thermal resistance H. Thus, the growth of the voltage-current characteristic of the high-T_c superconductor will depend on the character of the temperature variation, as it was shown above. According to (36), the temperature of superconductor rewritten in the form of

$$T(x) = T_0 + \left(T_{cB} - T_0\right)\left[1 - \frac{1}{1+\frac{E}{E_h}\left(\frac{E}{E_c}\right)^{1/n}\frac{\tanh\sqrt{\left(E/E_\lambda\right)\left(E/E_c\right)^{1/n}}}{\sqrt{\left(E/E_\lambda\right)\left(E/E_c\right)^{1/n}}}}\frac{\cosh\left(\sqrt{\frac{E}{E_\lambda}\left(\frac{E}{E_c}\right)^{1/n}}\frac{x}{a}\right)}{\cosh\left(\sqrt{\frac{E}{E_\lambda}\left(\frac{E}{E_c}\right)^{1/n}}\right)}\right]$$

increases with the reduction in values E_h and E_λ. As a result, this formula allows to formulate the thermal nature of the characteristic quantities E_h and E_λ considering the limiting cases at $E_\lambda \to \infty$, i.e. at $\lambda_0 \to \infty$, and at $E_h \to \infty$, i.e. at $h \to \infty$.

Indeed, it is easy to find that the temperature distribution in the superconductor becomes uniform, when $E_\lambda \to \infty$. In this case, the temperature and current can be expressed as

$$T = T_0 + \left(T_{cB} - T_0\right)\left[1 - \frac{1}{1+\left(E/E_h\right)\left(E/E_c\right)^{1/n}}\right] \qquad (62)$$

$$I = I_{c0}\frac{\left(E/E_c\right)^{1/n}}{1+\left(E/E_h\right)\left(E/E_c\right)^{1/n}} \qquad (63)$$

Therefore, the quantity E_λ is the characteristic electric field exceeding which there will be a non-uniform temperature distribution in the slab. It exists because of the power of the conductivity heat flux is finite due to the finite value of λ_0. Further, as follows from (62) and (63), the non-isothermal $E(J)$ characteristic of superconductor in the uniform current distribution becomes isothermal ($T = T_0$) at $E_h \to \infty$. Consequently, the quantity E_h is the characteristic electric field allowing one to estimate the influence of the external cooling intensity on the character of growth of its voltage-current characteristics taking into account the thermal deviation of electrodynamics states from those that exist under the ideal cooling conditions ($h \to \infty$, $\lambda_0 \to \infty$).

Obviously, ideal conditions $E_h \to \infty$ and $E_\lambda \to \infty$ should be kept in the experiments in which the critical current of superconductor is measured. However, the quantities E_h and E_λ are finite. Therefore, the contribution of the convective and conductive mechanisms in the formation of the voltage-current characteristic of the high-T_c superconductor will depend on the difference between E_h and E_λ. First, if $E_h/E_\lambda << 1$, i.e. at $H << 1$, then according to (60) - (63) the formation of the voltage-current characteristic of the superconductor at $dI/dt \to 0$ will be practically uniform and isothermal in the range of electric fields $E << E_h$. Second, if $E_h << E << E_\lambda$, then the non-isothermal formation of the electrodynamics states will be spatially homogeneous. Third, the voltage-current characteristic of the superconductor will be influenced by the non-uniform distribution of temperature in its cross-sectional area when $E >> E_\lambda$. Therefore, if $E >> E_h >> E_\lambda$, i.e. at $H>>1$, then non-isothermal electrodynamics processes occurring in the superconductor during current charging depends on the spatial heterogeneity of the temperature. In this case, the difference between non-isothermal voltage-current characteristics of the superconductor described by the equalities (60) and (63), which are defined in the framework of one-dimensional and zero-dimensional approximations, respectively, increases with increasing its thermal resistance (Fig. **8**). Therefore, the analysis of the formation of the thermo-electrodynamics states of superconductor should be done allowing for the non-uniform distribution of temperature at $E_h >> E_\lambda$, i.e. taking into account the thermal size effect.

The comparison between the critical current of the superconductor with the corresponding values of the current of instability calculated in accordance with the zero-dimensional and one-dimensional approximations is shown in Fig. (**14a**). These quantities are normalized to the width of the slab ($I^*_c=0.5I_c/b$, $I^*_{q,0}=0.5I_{q,0}/b$, $I^*_q=0.5I_q/b$). Fig. (**14b**) presents the electric field preceding the onset of instability as a function of the critical current density. This curve was defined by the solution of equation (40). The calculations were made for the examined $Bi_2Sr_2CaCu_2O_8$ superconductor at $\lambda_0=0.92\times10^{-3}$ W/(cm×K).

The results presented clearly demonstrate the existence of the character features of the variation in the current stability boundary of high-T_c superconductors. First, the zero-dimensional approximation leads to the overestimated boundary values of the current (also of the electric field) comparing with the corresponding values determined in the framework of the one-dimensional model. As discussed above, this feature is determined by the relevant difference between values T_v and $T_{v,0}$. Second, it is apparent that the values of E_q and I_q may be both lower and higher than the critical quantities E_c and I_c. Therefore, the stability areas may be both sub-critical and over-critical. In particular, the current instabilities may happen at $E_q < E_c$ and $I_q < I_c$ (sub-critical stability area). As follows from Fig. (**14a**), the higher the value J_{c0}, the more the difference between the values of I_q and I_c and also the difference between the values of I^*_q and $I^*_{q,0}$ calculated in terms of the one-dimensional and zero-dimensional models. However, the given peculiarities are modified with the decrease in the quantity J_{c0}, as follows from Fig. (**14b**). First, one can observe the over-critical regimes of stability when the allowed values of electric field and current exceed simultaneously the relevant values of E_c and I_c. Second, Fig. (**14**) also shows possible existence of the intermediate stability area when the sub-critical stable currents ($I_q < I_c$) exist at the over-critical values of the electric field ($E_q > E_c$).

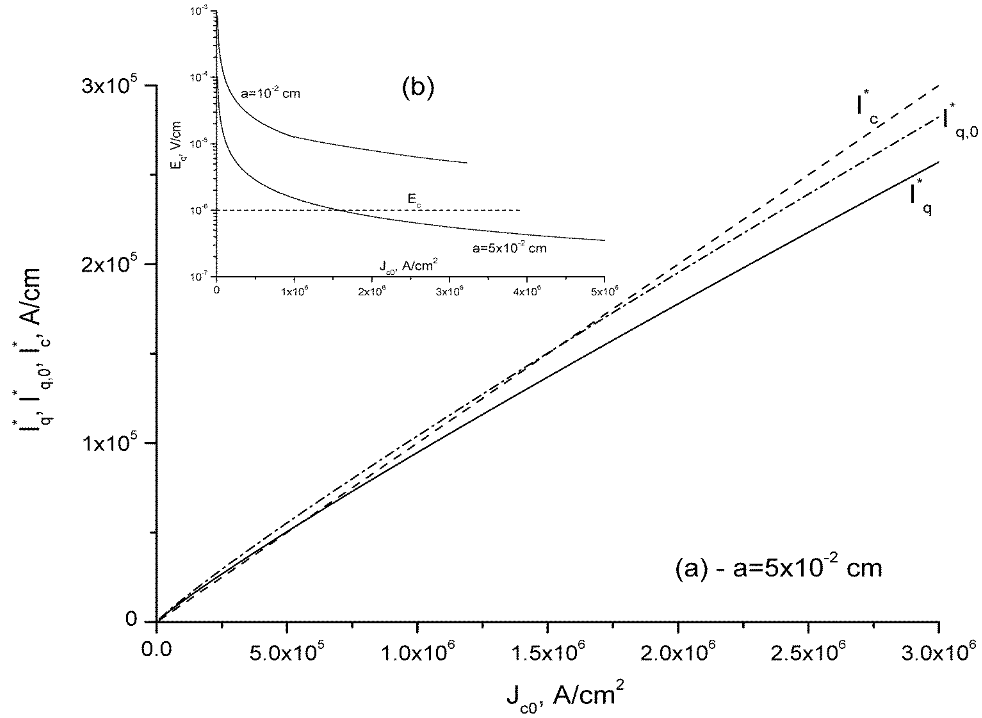

Fig. (14). Influence of the critical current density of the superconductor on the allowable values of charged current (**a**) and electric field (**b**) before current instability onset.

In order to find the boundary of the stability areas discussed, let us use the relations (42) and (43). Then, the boundary between the sub-critical and over-critical values of the electric field follows from the equality $E_q = E_c$. In this case, the current of the instability is obviously lower than the critical current of the superconductor. Accordingly, the condition of existence of the sub-critical stability area ($E_q < E_c$, $I_q < I_c$) is described by the inequality

$$E_h / E_c < n + H(n-1)/3$$

In the thermally uniform states ($H << 1$), this condition rewritten as

$$E_h / E_c < n$$

If these conditions are violated, the electric field induced in the superconductor before the onset of instability will exceed the value of E_c. In this case, as noted above, the current of the instability may be lower than I_c. Therefore, the equality $I_q = I_c$ allows to find the over-critical stable states ($E_q > E_c$, $I_q > I_c$). The latter will occur at

$$\frac{E_h}{E_c} > 1 + \sqrt{H\left[n + H(n-1)/3\right]} / \tanh\sqrt{\frac{H}{n + H(n-1)/3}}$$

in the framework of the non-uniform approximation or if inequality

$$E_h / E_c > (n+1)\left[(n+1)/n\right]^n$$

takes place in the zero-dimensional approximation. Then the intermediate current stability area, i.e. over-critical electric field and sub-critical currents ($E_q > E_c$, $I_q < I_c$), will take place at

$$1 + \sqrt{H\left[n + H(n-1)/3\right]} / \tanh\sqrt{\frac{H}{n + H(n-1)/3}} > \frac{E_h}{E_c} > n + H(n-1)/3$$

or

$$(n+1)\left[(n+1)/n\right]^n > E_h / E_c > n$$

when $H << 1$.

As follows from Fig. (**14**), the degradation of the current-carrying capacity of the superconductor takes place. Namely, the boundaries values of applied currents may be not only lower than a priori defined value of I_c, but also do not increase proportionally to the increase in this value. This effect is because the penetration of the charged current depends on the corresponding change in the superconductor's temperature, which inevitably increases with increasing current. Fig. (**15**) depicts the dependence of degradation parameters $\Delta = (I_c - I_q)/I_q$ and $\Delta_0 = (I_c - I_{q,0})/I_{q,0}$ on the dimensionless values of thermal resistance H and the quantity $\varepsilon_2 = (E_\lambda / E_c)^{n/(n+1)}$, which were calculated at n=10 according to the one-dimensional and zero-dimensional approximations, respectively. These results make it possible to determine the value of H and ε_2 at which the thermal degradation is most noticeable.

As it is expected, a significant reduction in the instability current relative to the critical current of the superconductor will be observed at $H << 1$, i.e. under the non-intensive cooling conditions. In this case (more exactly at $H < 0.1$), the

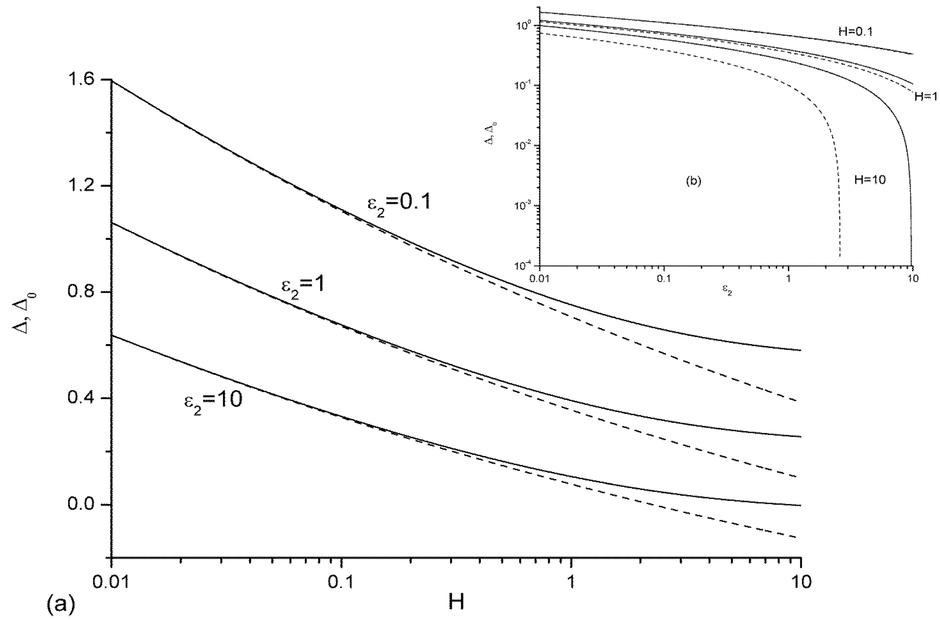

Fig. (15). Dependence of the degradation parameters determined in the zero- and one-dimensional approximations on the thermal resistance of superconductor (**a**) and value ε_2 (**b**): (——) - Δ, (- - - -) - Δ_0.

values of Δ and Δ_0 do not practically differ from each other, and the size effect is insignificant. Therefore, the degradation parameter can be estimated in the spatially uniform approximation. Accordingly, it is easy to find

$$\Delta \approx \Delta_0 = \left(nE_h / E_c\right)^{1/(n+1)}\left(n+1\right)/\,n-1$$

This expression shows that the degradation of current-carrying capacity of the high-T_c superconductors may be essential during non-intensive cooling conditions due to the finite value of the exponent n of their voltage-current characteristics. However, at $H > 10$, for example, in the intensive cooling, this estimation is too rough, because, as disused above, in this case the calculations of the stability conditions must take into account the thermal heterogeneity of the electrodynamics states induced in high-T_c superconductors by current charging. In these cases, the thermal degradation depends on the efficiency of the conductive transfer of the Joule heating and the critical properties of the superconductor. The influence of the latter on the thermal degradation of current-carrying capacity can be determined analyzing the influence of value ε_2. As follows from Fig. (**15b**), the reduction in the stability currents in comparison with the critical current is the most noticeable at $\varepsilon_2 << 1$. In this case, the degradation of the current-carrying capacity will exceed 20% even at intensive cooling (H = 10). At the same time, the effect of degradation might be not so significant at $\varepsilon_2 >> 1$ because in this case it would depend, fist of all, on the heat removal to the coolant. Thereby, the characteristic value ε_2 = 1 defines the boundary between the areas of high or low thermal degradation of current-carrying capacity of the superconductor under the intensive cooling conditions.

Thus, there exist two typical values of the electric field, which affect the thermal intensity of electrodynamics phenomena in superconductors. They depend on the cooling condition, the thickness of the superconductor, its thermal conductivity and the critical properties, defining the role of the convective and conductive thermal mechanisms in the current penetration into superconductors. In accordance with the thermal peculiarities of the stable electrodynamics state formation of the superconductor, the conditions of current stability change. The written criteria of current instability onset show that the allowable values of electric field and current can be sub-critical or over-critical relative to a priori chosen critical parameters of the superconductor E_c and J_c. As a result, the thermal degradation effect of current-carrying capacity of the superconductor exists. In this case, allowable values of applied current do not increase with the proportional increase in cross-sectional area of the superconductor and its critical current density. The effect of the thermal degradation should be taken into account when the permissible applied currents are determined for superconductors with high critical currents.

10. CONCLUSION

The performed analysis shows that the macroscopic electrodynamics phenomena in high-T_c superconductors may depend on their thermal states. To describe them correctly, the following features of the non-isothermal electrodynamics state evolution must take into consideration.

Thermo-electrodynamics phenomena in the superconductors have fission-chain-reaction character. Herewith, even their stable development depend on the combined temperature variation of $C(T)$, $\lambda(T)$, $J_c(T)$ and dJ_c/dT.

There may exist three stable operating modes of applied current penetration: partially penetrated mode with negligible overheating fully penetrated mode with inconsiderable but finite overheating and fully penetrated mode with noticeable overheating.

Applied current instability of modern high-T_c superconductors occurs during fully penetrated states. Therefore, instability onset of current penetration does not depend on the current charging rate but depicted by finite temperature overheating of superconductor. Such resistive states extend allowable operating modes of high-T_c superconductors permitting to utilize the maximum current-carrying capacity of superconductors.

The stable temperature rise of a high-T_c superconductor may essentially differ from the coolant temperature in the over-critical modes. It depends on the cross section of superconductor, cooling conditions, critical current of superconductor. Therefore, unlike the low-T_c superconductors, the $E(J)$ characteristics of high-T_c superconductors have only a positive slope during continuous current charging. As a result, they do not allow one to find the boundary between stable and unstable operating states. This peculiarity has to be considered during experiments at which the critical current of high-T_c superconductors is defined.

It is proved that the calculated current of the instability is reduced if the thermal heterogeneity of the electrodynamics states is taken into consideration in the theoretical analysis of the stability conditions. This is due to the increase in average temperature of superconductor, which is inevitable before the instability onset. The analysis made shows that the non-uniform temperature distribution in superconductor depends not only on its thermal resistance but also on its critical parameters. As a result, the limiting stable values of the electric field and current depend nonlinearly on the thickness of the superconductor, its critical properties as well as on the cooling conditions. Therefore, the current of instability will not increase proportionally to the increase in the thickness of superconductor or its critical current density. This leads to the thermal degradation of the current-carrying capacity of superconductor. This effect must be considered in investigations of current-carrying capacity of superconductors having a high critical current density.

The allowable stable values of electric field and current can be below or above those determined by a priori chosen critical parameters of the superconductor. The criteria allowing one to estimate the boundary of sub-critical and over-critical stability areas are written taking into account the size effect. As a result, the critical values E_c and I_c do not define the limiting values of the electric field and current stably existing in high-T_c superconductors.

The violation features of the stable current distribution in high-T_c superconductors cooled by liquid helium, hydrogen and nitrogen are studied. It is shown that the mechanism of the current instability depends on the type of coolant. Firstly, the destruction of stable current states may occur after trivial transition of the cooling conditions on the surface of superconductor from the nucleate to film boiling regimes. This instability mechanism is more probable for superconductors cooled by liquid helium. Secondly, the current instability may occur due to the disruption of the stable formation of the electrodynamics states of a superconductor even in the nucleate boiling regime of cooling. Such stability violation of the charged current is most probable when liquid nitrogen cools the superconductor. The necessary criteria allowing one to determine the influence of the properties of superconductor and coolant on the current instability mechanism are written.

Thus, the correct investigation of the macroscopic electrodynamics phenomena in high-T_c superconductors must be done under the discussed non-trivial thermal mechanism considering that stable temperature variation of superconductors may occur during macroscopic penetration of applied current.

ACKNOWLEDGEMENT

None declared.

CONFLICT OF INTEREST

None declared.

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Received: October 22, 2011 Revised: December 22, 2011 Accepted: January 03, 2012