Lightning Channel-Base Current Identification as Solution of a Volterra-like Integral Equation

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Abstract: In this paper, a novel procedure to reconstruct the lightning channel-base current starting from the measurement of the induction field generated by it is presented. The procedure is based on a suitable mathematical manipulation of the equation expressing the induction field in the time domain, in order to transform it into a Volterra-like integral equation. Such kind of equations can be easily numerically solved without resorting to any sort of regularization techniques as they are not affected by the typical ill-conditioning of the inverse problems. The developed algorithm has been validated by means of several numerical simulations, which have shown its effectiveness also in presence of measurement noise on the induction field values.

Keywords: Lightning, inverse problems, integral equations.

1. INTRODUCTION

A detailed model for the return stroke current is important in the context of the lightning general research, since it allows to evaluate properly the related electromagnetic fields and, in turn, the voltages and currents induced on overhead power lines.

The term return stroke current refers to the current wave that propagates from the ground to the cloud, after the stepped leader has reached the ground. When assessing lightning interactions e.g. with power lines and related risks of potential damage or fault, attention is typically focused on the return stroke phase of the lightning phenomenon, as it is during this phase that lightning effects are usually more intense and thus more dangerous. A model for the return stroke current is a function that relates the current i in the discharge channel to the time t and the channel axial coordinate l' [1-8].

Rakov and Uman [3] divided the return stroke models into four categories, according to the kind of equations that have to be solved in order to obtain the expression for the return stroke current. Such categories are:

1. *Gas discharge or physical models*: the current can be found as the solution of a system involving also hydrodynamic equations (in terms of temperature, pressure and mass density).
2. *Electromagnetic models*: the lightning channel is considered as a vertical antenna and the channel current is obtained solving the Maxwell equations with a numerical technique.
3. *Distributed circuit models*: the current is determined as the solution of a suitable distributed parameter circuit, which approximates the behavior of the system.
4. *Engineering models*: they directly provide an expression for the current in the lightning channel, whose parameters have numerical values that can be determined comparing the simulated results with the measured data. In these models, the channel current is related to the channel-base one by means of a suitable attenuation function P of the discharge channel axial coordinate l'.

In this paper, the attention is focused on the engineering models and a new procedure to obtain information on the lightning current starting from induction field measurements is presented and discussed.

Typically, the validation of the engineering models consists of a direct procedure, in which, starting from a given expression for the current, the electromagnetic fields are calculated and then compared with the measured ones. The numerical values of the model parameters are then modified in order to make the difference between the two field waveforms minimum.

In recent works [9, 10], an inverse algorithm, which does not make any assumption on the height dependent attenuation function P, but calculates it solving the integral equation expressing the link between the electromagnetic fields and the return stroke current was proposed. The primary aim of such algorithm was to derive the attenuation function P, given an expression for the channel-base current. It is important to highlight that practical application of the algorithm was hampered by its frequency domain nature, requiring the evaluation of a reliable Fourier spectrum of the measured fields. Unfortunately, experimental data are typically available in a time interval much less than that necessary for the field to vanish, as it would be required to compute an accurate Fourier transform.

Furthermore, the height dependent attenuation function P does not exhibit great influence on field values for measurements relatively near to the lightning impact point. In such cases therefore it would be much more significant to recon-

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struct the channel-base current, assuming a given expression for *P*.

In light of these considerations, a complete reformulation of the methodology derived in [9, 10] is presented in this paper, in order to develop a simple and efficient inverse procedure able to identify the lightning channel-base current directly in the time domain. The key point of the algorithm is the mathematical manipulation of the equation expressing the induction field in the time domain, in order to transform it into a Volterra-like integral equation [11], which can be numerically solved without resorting to any sort of regularization techniques, as it is not affected by the typical ill-conditioning of the inverse problems.

Estimates of lightning peak currents from measured lightning electromagnetic fields can be obtained by way of empirical formulas [12, 13], statistical equations [14] and model-based theoretical equations [15] relating the electromagnetic field and the lightning current.

The present paper aims at bringing a contribution to the last category (model-based theoretical equations). Indeed, available theoretical equations are all based on expressions relating far electromagnetic fields and associated return stroke currents at the channel base, which have been derived in the literature for various lightning return stroke models [14]. The proposed approach makes it possible to derive the channel base current for a given engineering model relaxing the far-field assumption.

The paper is organized as follows: in section 2, the Volterra-like integral equation expressing the link between the induction field and the lightning channel-base current is derived starting from the time domain expression of the induction field at ground level. Next, in section 3, the numerical treatment of such integral equation is presented and discussed. Then, in section 4, several numerical simulations are performed in order to test and validate the proposed algorithm. Finally, in section 5, some concluding remarks are drawn and the perspectives of future work outlined.

2. DERIVATION OF THE VOLTERRA-LIKE INTEGRAL EQUATION

In the following, the expression of the induction field at ground level as a function of the channel base current $i_0(t)$ will be manipulated in order to obtain a relation similar to that of a Volterra integral equation.

The mathematical manipulations are similar to that carried out in [16] and, as in this latter paper, the TL model is used (i.e., the current propagates along the channel without attenuation). With reference to Fig. (**1**), at the observation point *P*, placed (without loss of generality) along the *x*-axis at distance ρ from the origin, the induction field is along the *y*-axis and reads [16, 17]:

$$B_y\left(t\right) = \frac{\mu_0 \rho \cos(\theta)}{2\pi}\left[\int_0^{L(t)} \frac{1}{R^3(\ell)} i_0\left(t - \frac{R(\ell)}{c} - \frac{\ell}{v}\right) d\ell + \right.$$
$$\left. + \int_0^{L(t)} \frac{1}{cR^2(\ell)} i_0'\left(t - \frac{R(\ell)}{c} - \frac{\ell}{v}\right) d\ell \right] \quad (1)$$

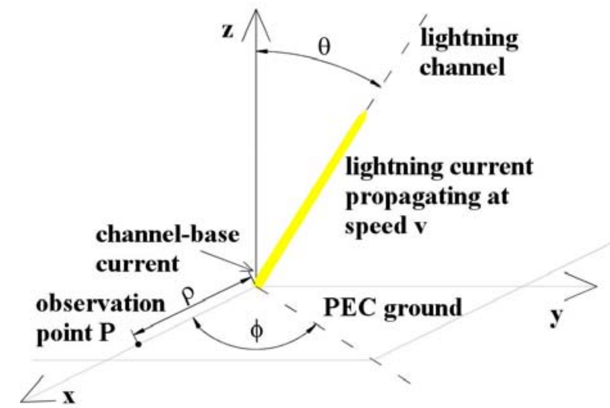

Fig. (1). Problem geometry.

where:

$$R(\ell) = \sqrt{\rho^2 + \ell^2 - \rho\ell\cos(\phi)\sin(\theta)} \quad (2)$$

is the distance between the point along the channel at abscissa l and the observation point *P*.

In the previous relations, $i_0'(t)$ is the time domain derivative of the channel base current, *c* is the speed of light, *v* is the current wavefront speed, l is the abscissa along the lightning channel, μ_0 is the vacuum permeability; finally, ϕ is the azimuth, θ is the elevation and *L(t)* is the upper integration limit depending on geometry, speed and time. In equation (1), it is assumed that the current starts propagating upward at t=0 and that $i_0(t) = 0$ for *t<0*, so the induction field in *P* "starts" at t=ρ/c; therefore it is convenient to perform the following change of variable and function:

$$\begin{cases} \tilde{t} = t - \dfrac{\rho}{c} \\ \tilde{B}\left(\tilde{t}\right) = B_y\left(\tilde{t} + \dfrac{\rho}{c}\right) \end{cases} \quad (3)$$

Substituting (3) in (1) we can rewrite it as:

$$\tilde{B}\left(\tilde{t}\right) = \frac{\mu_0 \rho \cos(\theta)}{2\pi}\left[\int_0^{\tilde{L}(\tilde{t})} \frac{1}{R^3(\ell)} i_0\left(\tilde{t} - \frac{R(\ell)-\rho}{c} - \frac{\ell}{v}\right) d\ell + \right.$$
$$\left. + \int_0^{\tilde{L}(\tilde{t})} \frac{1}{cR^2(\ell)} i_0'\left(\tilde{t} - \frac{R(\ell)-\rho}{c} - \frac{\ell}{v}\right) d\ell \right] \quad (4)$$

where $\tilde{L}(\tilde{t})$ is now the upper integration limit as a function of $\tilde{t}$.

This way, the time reference is set to the instant when a relevant measure of the induction field starts being available.

With reference to equation (4), the time dependent channel length can be defined as:

$$\tilde{L}(\tilde{t}) \text{ s.t. } \tilde{t} - \frac{R(\ell)-\rho}{c} - \frac{\ell}{v} \geq 0 \ \forall \ \ell \in \left[0, \tilde{L}(\tilde{t})\right] \quad (5)$$

The value $\tilde{L}(\tilde{t})$ can be found as one of the radices of the equation:

$$\tilde{t}-\frac{R(\tilde{L}(\tilde{t}))-\rho}{c}-\frac{\tilde{L}(\tilde{t})}{v}=0 \quad (6)$$

when solved for $\tilde{L}(\tilde{t})$; this implies that, if we define:

$$T(\ell)=\frac{R(\ell)-\rho}{c}+\frac{\ell}{v} \quad (7)$$

the following relation holds:

$$\tilde{t}-T(\tilde{L}(\tilde{t}))=0 \Rightarrow T\left(\tilde{L}(\tilde{t})\right)=\tilde{t}. \quad (8)$$

Solving equation (6) and defining:

$$\begin{cases} b(\tilde{t})=c(\rho+c\tilde{t})\text{-}\rho\sin(\theta)\cos(\phi)v \\ r(\tilde{t})=\sqrt{b^2(\tilde{t})+(v^2\text{-}c^2)(c\tilde{t}+2\rho)c\tilde{t}} \end{cases} \quad (9)$$

the radix expressing $\tilde{L}(\tilde{t})$ reads:

$$\tilde{L}(\tilde{t})=\frac{b(\tilde{t})-r(\tilde{t})}{c^2\text{-}v^2} \quad (10)$$

(this is the only radix that satisfies $\tilde{L}(0)=0$).

In order to simplify the notation, we define two auxiliary functions:

$$\begin{cases} f_1(\ell)=\dfrac{\mu_0\rho\cos(\theta)}{2\pi R^3(\ell)} \\ f_2(\ell)=\dfrac{\mu_0\rho\cos(\theta)}{2\pi c R^2(\ell)} \end{cases} \quad (11)$$

and we split the induction field into two terms:

$$\begin{cases} \tilde{B}_1(\tilde{t})=\displaystyle\int_0^{\tilde{L}(\tilde{t})} f_1(\ell)i_0(\tilde{t}-T(\ell))d\ell \\ \tilde{B}_2(\tilde{t})=\displaystyle\int_0^{\tilde{L}(\tilde{t})} f_2(\ell)i_0'(\tilde{t}-T(\ell))d\ell \end{cases} \quad (12)$$

so that:

$$\tilde{B}(\tilde{t})=\tilde{B}_1(\tilde{t})+\tilde{B}_2(\tilde{t}) \quad (13)$$

Now, with:

$$a=1\text{-}\cos^2(\phi)\sin^2(\theta) \quad (14)$$

we introduce:

$$h(\ell)=\frac{\mu_0\cos(\theta)}{2\pi\rho a}\left(\frac{v(\ell-\cos(\phi)\sin(\theta)\rho)}{(cT(\ell)v+\ell c-v\rho)}-1\right) \quad (15)$$

This function is a primitive of $f_1(\ell)$:

$$h'(\ell)=f_1(\ell) \quad (16)$$

and furthermore satisfies the condition $\lim_{\ell\to+\infty} h(\ell)=0$, although this is inessential in this context. Inserting it in (12), the expression for $\tilde{B}_1(\tilde{t})$ can be modified as follows:

$$\tilde{B}_1(\tilde{t})=\int_0^{\tilde{L}(\tilde{t})} h'(\ell)i_0(\tilde{t}-T(\ell))d\ell=\left[h(\ell)i_0(\tilde{t}-T(\ell))\right]_{\ell=0}^{\ell=\tilde{L}(\tilde{t})}+$$

$$-\int_0^{\tilde{L}(\tilde{t})} h(\ell)\frac{d\left[i_0(\tilde{t}-T(\ell))\right]}{d\ell}d\ell \quad (17)$$

Taking into account that:

$$\left[h(\ell)i_0(\tilde{t}-T(\ell))\right]_{\ell=0}^{\ell=\tilde{L}(\tilde{t})}=h(\tilde{L}(\tilde{t}))i_0(0)-h(0)i_0(\tilde{t}) \quad (18)$$

$$\frac{d\left[i_0(\tilde{t}-T(\ell))\right]}{d\ell}=-i_0'(\tilde{t}-T(\ell))T'(\ell) \quad (19)$$

recalling that $i_0(0)=0$ and defining $h_0\triangleq h(0)$, equation (17) becomes:

$$\tilde{B}_1(\tilde{t})=-h_0i_0(\tilde{t})+\int_0^{\tilde{L}(\tilde{t})} h(\ell)i'_0(\tilde{t}-T(\ell))T'(\ell)d\ell \quad (20)$$

Now, it is possible to make the following change of variables:

$$\ell\to\tilde{\tau}=T(\ell)$$

obtaining:

$$\tilde{B}_1(\tilde{t})=-h_0i_0(\tilde{t})+\int_0^{\tilde{t}} h(L(\tilde{\tau}))i'_0(\tilde{t}-\tilde{\tau})d\tilde{\tau} \quad (21)$$

The same variable change can be applied to $\tilde{B}_2(\tilde{t})$, obtaining:

$$\tilde{B}_2(\tilde{t})=\int_0^{\tilde{t}} f_2(L(\tilde{\tau}))i'_0(\tilde{t}-\tilde{\tau})L'(\tilde{\tau})d\tilde{\tau} \quad (22)$$

Summing up, if we define:

$$g(\tilde{\tau})=h(L(\tilde{\tau}))+f_2(L(\tilde{\tau}))L'(\tilde{\tau}) \quad (23)$$

the expression for the induction field reads:

$$\tilde{B}(\tilde{t})=-h_0i_0(\tilde{t})+\int_0^{\tilde{t}} g(\tilde{\tau})i'_0(\tilde{t}-\tilde{\tau})d\tilde{\tau} \quad (24)$$

The actual expression for $g(\tilde{\tau})$ can be written as follows; with the dummy functions:

$$\begin{cases} A(\tilde{\tau})=\text{-}cv^2\tilde{\tau}+c\rho v\sin(\theta)\cos(\phi)+r(\tilde{\tau})c\text{-}v^2\rho \\ B(\tilde{\tau})=\text{-}c^2\tilde{\tau}v\text{-}c\rho v+r(\tilde{\tau})v+c^2\rho\sin(\theta)\cos(\phi) \end{cases} \quad (25)$$

$g(\tilde{\tau})$ reads:

$$g(\tilde{\tau})=\frac{\mu_0\cos(\theta)}{2\pi A(\tilde{\tau})}\left(\frac{\rho v(c^2\text{-}v^2)}{r(\tilde{\tau})}-\frac{B(\tilde{\tau})+A(\tilde{\tau})}{a\rho}\right) \quad (26)$$

Finally, with another variable change $\tilde{\tau}\to\tau=t-\tilde{\tau}$, equation (24) becomes:

$$\tilde{B}(\tilde{t})=-h_0i_0(\tilde{t})+\int_0^{\tilde{t}} g(\tilde{t}-\tau)i'_0(\tau)d\tau \quad (27)$$

Further manipulations can be applied to (27), in order to obtain an actual Volterra equation (i.e. with $i_0(\tau)$ under the integral sign, instead of $i'_0(\tau)$), but the solution procedure outlined in the next section can be conveniently applied to equation (27) in this form.

3. NUMERICAL SOLUTION

The algorithm used to solve equation (27) is carried out with reference to a practical situation, in which we have *N* samples of the induction field at instances $\tilde{t}_n = n\Delta t, \quad n = 1..N$, and we want to evaluate the channel base current at the same instances. Sampling equation (27) in the time instances $\tilde{t}_n$, we have:

$$\tilde{B}(n\Delta t) = -h_0 i_0(n\Delta t) + \int_0^{n\Delta t} g(n\Delta t - \tau) i'_0(\tau) d\tau \qquad (28)$$

This relation can be rewritten as follows:

$$\tilde{B}(n\Delta t) = -h_0 i_0(n\Delta t) + \\ + \sum_{m=1}^{n} \int_{(m-1)\Delta t}^{m\Delta t} g(n\Delta t - \tau) i'_0(\tau) d\tau \qquad (29)$$

If we use a piecewise approximation of the equation kernel:

$$g(n\Delta t - \tau) \simeq g\left((n-m)\Delta t + \frac{\Delta t}{2}\right), \\ \tau \in \left[\left(m-1\right)\Delta t, m\Delta t\right] \qquad (30)$$

defining:

$$g_j \triangleq g\left(j\Delta t + \frac{\Delta t}{2}\right), \quad j = 0..N-1 \qquad (31)$$

and inserting this approximation in (29), the expression for the sampled induction field measures becomes:

$$\tilde{B}(n\Delta t) = -h_0 i_0(n\Delta t) + \\ + \sum_{m=1}^{n} g_{n-m} \int_{(m-1)\Delta t}^{m\Delta t} i'_0(\tau) d\tau \qquad (32)$$

allowing to find the needed discrete approximation of (27):

$$\tilde{B}(n\Delta t) = -h_0 i_0(n\Delta t) + \\ + \sum_{m=1}^{n} g_{n-m} \left[i_0\left(m\Delta t\right) - i_0\left((m-1)\Delta t\right) \right] \qquad (33)$$

Equation (33) can be rewritten in a more compact form with the following definitions:

$$\begin{cases} B_n \triangleq \tilde{B}(n\Delta t), \quad n = 1..N \\ i_n \triangleq i_0(n\Delta t), \quad n = 1..N \\ W_j = g_j - g_{j-1}, \quad j = 1..N-1 \\ W_0 = -h_0 + g_0 \end{cases} \qquad (34)$$

this way the final system of equations reads:

$$B_n = \sum_{m=1}^{n} W_{n-m} i_m \qquad (35)$$

The solution of this system can be straightforwardly computed as:

$$\begin{cases} i_1 = \dfrac{B_1}{W_0}, \quad n = 1 \\ i_n = \dfrac{1}{W_0}\left(B_n - \sum_{m=1}^{n-1} W_{n-m} i_m \right), \quad n = 2..N \end{cases} \qquad (36)$$

In the expression of each current sample, the induction field sample at the same instant and the current samples evaluated in the previous equations appear: i_1 can be calculated knowing B_1; i_2 can be evaluated from B_2 and i_1; i_3 depends on B_3, i_1 and i_2, and so on.

4. SIMULATIONS

The outlined procedure has been numerically tested simulating an induction field due to a lightning and trying to reconstruct the channel base current used to compute it.

A classic Heidler expression has been used for $i_0(t)$:

$$\begin{cases} i_0(t) = \dfrac{I_0}{\eta} \dfrac{(t/\tau_1)^n}{(t/\tau_1)^n + 1} e^{-t/\tau_2} \\ \eta = e^{(-\tau_1/\tau_2)\sqrt[n]{(n\tau_2/\tau_1)}} \end{cases} \qquad (37)$$

with the following parameters: I_0 =10 kA, τ_1 =0.25 μs, τ_2 =2.5 μs, n =2.

The observation point *P* is located at ρ=100 m from the impact point, while elevation and azimuth of the lightning channel are respectively θ=20° and ϕ=30°; the induction field in *P* is reported in Fig. (**2**) and it is called "original": 1000 samples have been used in the time window 0 ÷ 5 μs.

The results here presented aim at reaching three goals:

1. check method consistency;
2. show its behavior in presence of noisy data;
3. show the effect of uncertainties on model parameters.

As far as the first issue is concerned, Fig. (**3**) shows that the algorithm is able to exactly reconstruct the channel base current from the "original" induction field of Fig. (**2**); this is not a result that can be taken for granted when dealing with inverse problems [18], especially without applying any sort of regularization: the sole presence of rounding errors very often leads to non negligible discrepancies between "reconstructed" and "real" values, if not, in the worst cases, to instability. This result, on the contrary, highlights the "inherent" stability and well-posedness of the method.

To face with the second issue, random noise has been added to the computed induction field, obtaining the "noisy" waveform in Fig. (**2**), which has been used as a new input. Comparison between original and reconstructed currents is reported in Fig. (**4**); of course, the noise affecting the induction field leads to noise on the channel-base current, but the algorithm does not suffer from instability and does not amplify the error.

In order to show the effect of model uncertainties, the procedure has been applied (with the "exact" induction field as input) with errors on some of the relevant model data: wavefront speed v and distance ρ have been perturbed by a ±10% error, while on θ a ±5° error has been used.

Obtained results are reported in Figs. (**5-7**), where perturbed parameters are marked with a prime. The main consideration these results seem to highlight is that, at least in the condition here assumed, uncertainties on geometry (distance and also azimuth) have an importance comparable or greater than that on wavefront speed; this could be a concern, as data as channel elevation and azimuth can be (roughly) known for triggered lightning when video recording of the lightning event are available, but not for natural ones.

Another comment, related to this issue, concerns the possibility to infer wavefront speed, i.e. one of the key lightning parameters on which researchers focus their attention, from measured fields: one has to face with the fact that errors on the geometry could have an influence on measures greater than that of the speed itself, making it impossible to identify its value.

Finally, it should be noticed from Fig. (**5**) that variations in wavefront speed seem to mostly influence the behavior of the channel-base current at early times and its rise time especially.

This evidence suggests that in order to identify the wavefront speed the attention should be focused on such time window and on the induction field derivative.

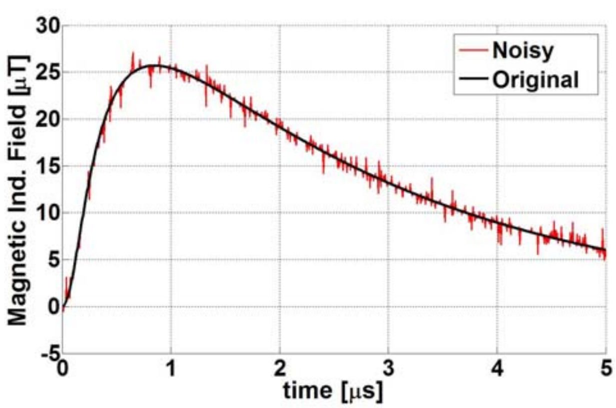

Fig. (2). Original and noisy induction field.

5. CONCLUSIONS AND PERSPECTIVES OF FUTURE WORK

This paper has addressed the problem of the lightning current identification, proposing an inverse algorithm that enables to reconstruct the channel-base current directly from the measurement of the induction field. Such algorithm is based on a suitable mathematical manipulation of the equation expressing the induction field in the time domain, in order to transform it into a Volterra-like integral equation, which can be numerically treated without resorting to any sort of regularization techniques, since it is not affected by the typical ill-conditioning of the inverse problems.

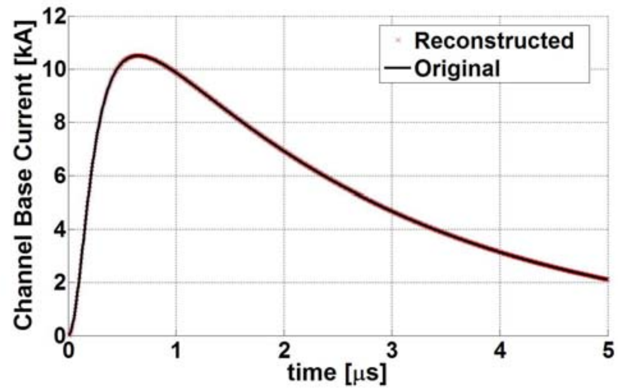

Fig. (3). Original *vs* reconstructed channel-base current (no noise on induction field).

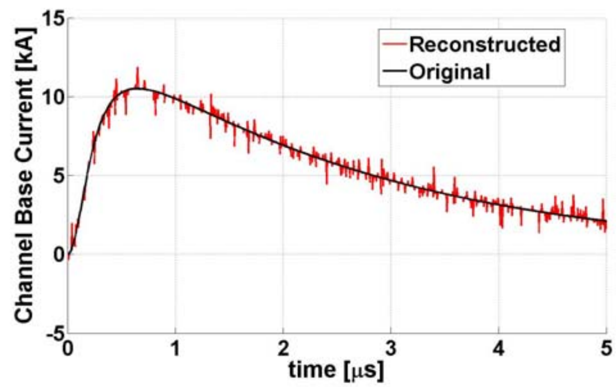

Fig. (4). Original *vs* reconstructed channel-base current (noise on induction field).

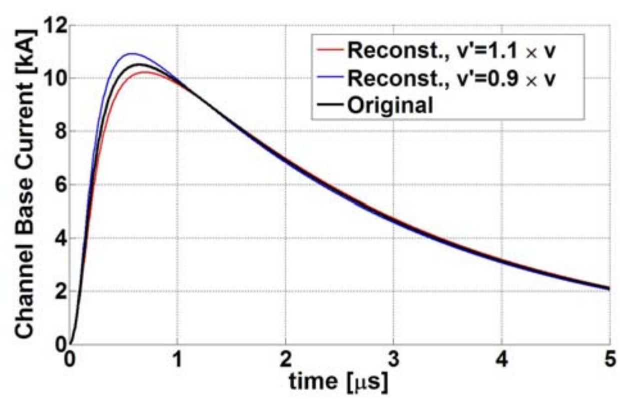

Fig. (5). Original *vs* reconstructed channel-base current (error on wavefront speed).

Several numerical simulations have been performed in order to validate the developed procedure and to test its effectiveness also in presence of noisy input data or of uncertain model parameters. The obtained results have shown the robustness of the algorithm towards experimental noise in the induction field and have pointed out that uncertainties on geometry have an importance comparable or greater than that on the wavefront speed.

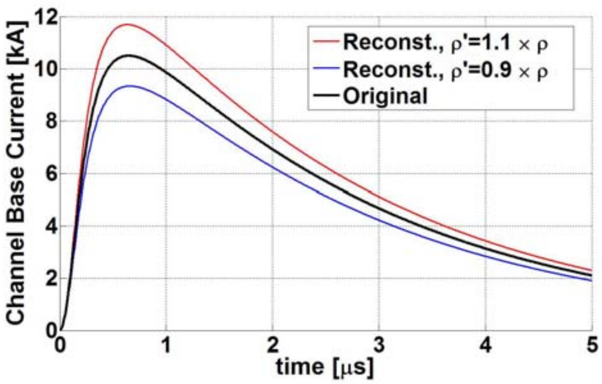

Fig. (6). Original *vs* reconstructed channel-base current (error on distance ρ).

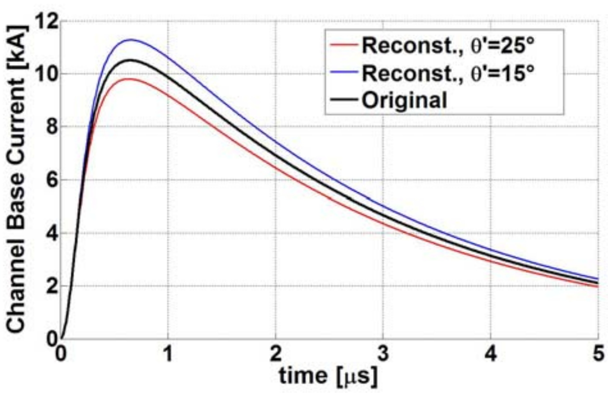

Fig. (7). Original *vs* reconstructed channel-base current (error on θ).

Further activity is in progress in order to carry out an experimental validation of the proposed algorithm, by using real (measured) induction field waveforms and other components of the lightning electromagnetic field (electric field, field derivatives). Finally, future activity will concern also the enhancement of the developed procedure in order to exploit measurements in different observation points to perform also the identification of other discharge channel parameters, as for instance the current wavefront speed.

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