

# Planetary-Scale Vortical Structures in the Conducting Atmosphere

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**Abstract:** The problem of generation and existence of large-scale vortices in the ionosphere is analyzed in this paper. Some exact solutions of the magneto-hydrodynamic (MHD) equations have been found and new invariants have been constructed allowing to discover mechanisms of generation of large-scale vortices and planetary waves in the electroconducting atmosphere under the action of non-conservative Coriolis and Ampere forces.

## 1. INTRODUCTION

Recently the problem of study of dynamics of large-scale ( $10^3 - 10^4$  km) motions in the ionosphere against the background of which occur almost all physicochemical processes is in the focus of attention of the researches investigating the upper atmosphere. This is caused by the fact that at the considered altitudes (80 – 600 km) atmosphere is weakly ionized ionospheric plasma the charged component of which instantly reacts to any changes of the dynamic mode of the neutral component of ionosphere. The response to the dynamic impact has electromagnetic character. It propagates in the medium with velocity above  $1 \text{ km} \cdot \text{s}^{-1}$  in the form of natural (background) oscillations and contains valuable information about external sources and electrodynamic processes in the upper atmosphere. The response is registered by ionospheric and magnetic observatories during magnetic storms, substorms [1], earthquakes [2-4], spacecraft startups [5,6] etc. In the latter case, the response represents solitary, large-scale cyclonic and anticyclonic type vortex structure. Interpretation of the response of ionospheric plasma is a main goal of the researchers of the upper atmosphere and near-earth cosmic space [7-11].

The problem of generation and conditions of existence of large-scale vortices in the ionosphere is analyzed in this paper. Some exact solutions of the magneto-hydrodynamic (MHD) equations have been found and new invariants have been constructed allowing to reveal the mechanisms of generation of large-scale vortices and planetary waves in the electroconductive atmosphere under the action of non-conservative Coriolis and Ampere forces.

## 2. MODEL OF MEDIUM AND BASIC EQUATIONS

Weather-forming low frequency ( $10^{-4} - 10^{-6} \text{ s}^{-1}$ ) planetary-scale processes proceed very slowly in the

troposphere, with local wind velocities ( $5 - 20 \text{ m} \cdot \text{s}^{-1}$ ) [12-14]. As observations have shown [12,15,16], large-scale dynamic processes in the ionosphere have long time intervals (from ten seconds up to several hours for electromagnetic planetary waves, and from two days up to two weeks and more for Rossby-type waves) and velocities from  $10 - 100 \text{ m} \cdot \text{s}^{-1}$  up to several tens of  $\text{km} \cdot \text{s}^{-1}$ . Time of global impact of the above sources on the ionosphere corresponds to the frequencies of electromagnetic planetary waves [6,12]. This leads to the resonance amplification of the amplitudes of these wave oscillations and allows registering them by delay of perturbation by ionospheric and magnetic observatories located at the same latitude and thousands of kilometres distant from each other. Characteristics of the dynamic processes in the upper atmosphere is determined by the presence of electroconductive component and by effect of geomagnetic field on this component. Presence of anisotropic conductivity and inhomogeneous geomagnetic field gives additional electromagnetic elasticity to the upper atmosphere of the earth. As a result, dynamic processes in the ionosphere can be described by three-component liquid model: pressure of neutral molecules (neutral gas)  $P_m$ , pressure of electrons (electron gas)  $P_e$  and ions (ion gas)  $P_i$ , and also pressure of geomagnetic field  $P_H = H_0^2 / 8\pi = Q^2 / 8\pi r^6$ , where  $H_0$  is a strength of geomagnetic field,  $Q = 8,1 \cdot 10^{25} \text{ Gauss} \cdot \text{sm}^{-3}$  - magnetic dipole moment of the Earth,  $r$  - distance between the centre of the Earth and the selected point. Magnetic pressure  $P_H$  in the  $E$  and  $F$  regions of ionosphere (80 – 600 km) hardly changes with height and approximately equals  $4 \cdot 10^{-3} \text{ dyne} \cdot \text{sm}^2$ . On the contrary, the pressure of the molecules  $P_m$ , rapidly (exponentially) decreases with height and at the altitude of 130 km  $P_m \approx P_H$  [17]. Pressure of ionospheric plasma  $P_{pl} = P_e + P_i \approx 2NkT_e$  is always

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much less than  $P_m$  and  $P_H$ . For example, even for the maximum values of the ionospheric plasma concentration  $N \sim 10^7 \text{ sm}^{-3}$  and temperature of electrons  $T_e \approx 2000^0 \text{ K}$  the plasma pressure -  $P_{pl} = 10^{-5} \text{ dyne} \cdot \text{sm}^2$ . Therefore, excluding diffusion processes, the effect of plasma pressure on the ionosphere can be neglected at the interval of latitudes 80–600 km [18]. Consequently, dynamic processes in the ionosphere, depending on the altitude, will be determined either by pressure of the neutral gas  $P_m$  (region of altitudes 80–130 km) or by magnetic pressure (region of altitudes above 130 km). The intensity of the influence of some or other factor essentially depends on the ionization degree of the medium  $\eta = N / N_m$ , as well as on the values of the gyrofrequencies of the electrons  $\omega_e = eH_0 / mc$ , ion's  $\omega_i = eH_0 / Mc$ , frequencies of collision of charged particles with each other  $v_{ei}$  and with neutral molecules  $v_{em}, v_{im}$ . Here  $e$  - elementary charge,  $m$  and  $M$  - masses of electrons and ions respectively,  $c$  - light speed,  $N_m$  - concentration of neutral molecules. In the ionosphere, in the region of altitudes 80–600km:  $\omega_{\tilde{a}} \approx 10^7 \text{ s}^{-1}$ ,  $\omega_i \approx (1.5-3) \cdot 10^2 \text{ s}^{-1}$ . Maximum value of collision frequencies in the lower  $E$  region (80–130 km) equal respectively:  $v_{ei} \approx 10^4 \text{ s}^{-1}$ ,  $v_{em} \approx 10^5 \text{ s}^{-1}$ ,  $\omega_{im} \approx 10^3 - 10^4 \text{ s}^{-1}$ . Therefore, here are the following inequalities:

$$\omega_e \gg v_e, \omega_i \ll v_{im}, \quad (1.1)$$

where  $v_e = v_{ei} + v_{em}$ . Consequently, in this region of the upper atmosphere the electrons are magnetized (geomagnetic force lines are frozen-in the electronic component), however ions are not. Ions, as passive impurities, are completely entrained by neutral particles [12,17]. As frequencies of collisions very rapidly decrease with altitude, beginning from 120 km and above, the second inequality (1.1) is violated and takes the following form:

$$\omega_i > v_{im}. \quad (1.2)$$

As a result, plasma component in the upper  $E$ - and  $F$  regions will be completely magnetized. Taking into account the above inequalities, the general expressions are simplified for the coefficients of Hall's  $\sigma_H$  and Pedersen's  $\sigma_{\perp}$  conductivities (transversal conductivity) and in the lower  $E$  region (80–130 km), (Hall's region), and take the following form [12]:

$$\sigma_H = \frac{eNc}{H_0}, \sigma_{\perp} = \frac{e^2N}{Mv_{im}}, \frac{\sigma_H}{\sigma_{\perp}} = \frac{v_{im}}{\omega_i} \gg 1, \quad (1.3)$$

For low frequency, slow planetary waves ( $L \sim 10^3 - 10^4 \text{ km}$ ) in this region of atmosphere the

inequalities always are as follows  $\omega \ll \omega_i < v_{im}$ , i.e. frequency of collisions  $v_{im}$  is more than characteristic wave frequencies  $\omega$  and ions cyclotron frequency  $\omega_i$ . However, wave equation in this region of the upper atmosphere does not contain frequency of collisions due to the Hall's effect (see Eq. (1.3)) and the decisive role of collisions manifests in the form of wave equation allowing for the gyroscopic effect conditioned by geomagnetic field.

Correspondingly, for the upper  $E$  and  $F$  regions (130–600 km) we have:

$$\sigma_H = e^2N \left( \frac{1}{m\omega_e} - \frac{1}{M\omega_i} \right) = 0, \sigma_{\perp} = \frac{NMc^2v_{im}}{H_0^2}. \quad (1.4)$$

From equation (1.3) it follows that in the lower  $E$  region transversal conductivity  $\sigma_{\perp}$  can be neglected in comparison with Hall's conductivity  $\sigma_H$ . The latter, as follows from equation (1.3) does not depend on collision frequency of particles and does not contribute to dissipation of motion energy. Electromagnetic Ampere force  $\mathbf{F}_A = [\mathbf{j} \cdot \mathbf{H}_0] / \rho c$  per unit mass caused by Hall current has gyrotropic nature and acts on medium like Coriolis force  $\mathbf{F}_A = N[\mathbf{V}\omega_i] / N_m$ . In the upper  $E$  and  $F$  regions ampere force caused by Pedersen conductivity has dissipative nature and takes form of Rayleigh friction  $\mathbf{F}_{\perp} = -Nv_{im}\mathbf{V}_{\perp} / N_m = -\lambda\mathbf{V}_{\perp}$ , where  $\mathbf{V}_{\perp} = \mathbf{V} - (\mathbf{V}\mathbf{H}_0)\mathbf{H}_0 / H_0^2$ ,  $\mathbf{V}$  - velocity of neutrals. In the region of altitudes of 80–115 km turbulent mixing is also a substantial factor of motion dissipation [13,14]. The reason of turbulence in this region of atmosphere is destruction of the internal gravity and planetary waves. Turbulent motions transform into laminar motions in the region above 115 km.

Taking into account the above equation, the medium motion equation for lower  $E$  region can be presented in the following form [17]:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla P + \rho \mathbf{g} + \rho[\mathbf{V} \cdot 2\boldsymbol{\omega}_0] + \rho_i[\mathbf{V}\omega_i] + v \frac{\partial^2 \mathbf{V}}{\partial z^2}. \quad (1.5)$$

As equation (1.5) does not contain magnetic field  $\mathbf{h}$  induced by motion, equation (1.5) together with continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{V}) = 0 \quad (1.6)$$

and the heat influx:

$$\frac{dP}{dt} + \gamma P \nabla \cdot \mathbf{V} = \mathcal{E}. \quad (1.7)$$

with a given heat influx  $\mathcal{E}$  forms a closed system. Here  $P$  and  $\rho = MN_m$  - pressure and density of neutrals,  $\mathbf{g}$  - acceleration vector of gravity force,  $\boldsymbol{\omega}_0$  - angular velocity vector of the earth rotation (always directed from the south to the north),  $v$  - turbulent mixing coefficient,  $\rho_i = M_i N_i$  -

ions density,  $z$  - vertical coordinate,  $\gamma$  - polytropic index. The system (1.5)-(1.7) represents equations of atmospheric hydrodynamics, in which additional Coriolis-type mechanical force of magnetic nature is caused by existence of geomagnetic field  $\mathbf{H}_0$  and Hall's electroconductivity. In this approximation, in the ionosphere  $E$  region generation of large-scale waves of electromagnetic nature (due to the full entrainment  $V \approx V_i$ ) is impossible. Velocity of electrons in the ionosphere  $E$  region, taking into account  $V_e \gg V \approx V_i$  [17], is directly determined by current density  $\mathbf{j}$ :

$$\mathbf{V}_e \approx -\frac{1}{eN} \mathbf{j} = -\frac{c}{4\pi eN} \nabla \times \mathbf{h}. \quad (1.8)$$

In this case induced magnetic field  $\mathbf{h}$  is determined from the Maxwell's equation:

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times [\mathbf{V}_e \cdot \mathbf{H}_0] = -\frac{c}{4\pi eN} \nabla \times [\nabla \times \mathbf{h} \cdot \mathbf{H}_0], \quad (1.9)$$

here  $\mathbf{H}_0$  - geomagnetic field vector (always directed from the north to the south),  $\mathbf{h}$  - its perturbation (deviation from  $\mathbf{H}_0$ ). Equation (1.9) for the middle-scale processes ( $L \leq 10^3$  km) in the  $E$  region, as an exact solution, contains oscillating branch of helicons ("atmospheric whistles"); for large-scale processes ( $L \sim 10^3 - 10^4$  km) when inhomogeneous geomagnetic field effect can not be neglected, ( $\nabla \mathbf{H}_0 \neq 0$ ), as shown below, this equation describes electromagnetic planetary waves (new branch of electromagnetic oscillations of the ionospheric resonator).

In the  $F$  region, plasma component of atmosphere is completely magnetized and Ampere force takes form of elastic electromagnetic force:

$$\mathbf{F}_A = \frac{1}{4\pi} [\nabla \times \mathbf{h} \cdot \mathbf{H}_0]. \quad (1.10)$$

As a result, a closed system of equations of single-component magnetic hydrodynamics with given heat influx  $\varepsilon$ , taking into account equations (1.6) and (1.7), can be presented in the following form [12]:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla P + \rho \mathbf{g} + \rho [\mathbf{V} \cdot 2\omega_0] + \frac{1}{4\pi} [\nabla \times \mathbf{h} \cdot \mathbf{H}_0], \quad (1.11)$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times [\mathbf{V} \cdot \mathbf{H}_0] + \nabla \times \left[ \mathbf{H}_0 \frac{1}{\rho_i v_{im}} \frac{1}{4\pi} [\nabla \times \mathbf{h} \cdot \mathbf{H}_0] \right]. \quad (1.12)$$

For the lower  $E$  region, taking  $rot$  from both parts of equation (1.5) on condition of the absence of dissipative forces, we will find condition of conservation of the new invariant [19,20]:

$$helm \left( \nabla \times \mathbf{V} + 2\omega_0 + \frac{N}{N_m} \frac{e}{M c} \mathbf{H}_0 \right) = 0. \quad (1.13)$$

Here operator *helm* for any vector field  $\mathbf{a}$  has the following form [21]:

$$helm \mathbf{a} = \frac{\partial \mathbf{a}}{\partial t} - \nabla \times [\mathbf{V} \cdot \mathbf{a}] + \mathbf{V} \nabla \cdot \mathbf{a}. \quad (1.14)$$

Equality  $helm \mathbf{a} = 0$  means conservation ("freezing-in") of both, force lines of vector  $\mathbf{a}$  and intensity of vector tubes [21].

In the absence of magnetic field ( $\mathbf{H}_0 = 0$ ) from equation (1.13) we get a well-known condition of conservation ("freezing-in") of absolute vortex  $\nabla \times \mathbf{V} + 2\omega_0$  [22] which, as a particular case, contains slow weather-forming planetary Rossby waves caused by inhomogeneous angular velocity of the earth rotation  $\nabla \omega_0 \neq 0$ . In the minimums and maximums of planetary waves always are located tropospheric cyclones and anticyclones, which together with the wave propagate with the velocity of a medium zonal wind ( $\sim 10 \text{ m} \cdot \text{s}^{-1}$ ) and actually determine regional weather in the lower atmosphere of the Earth. From equation (1.13) it follows that in the lower part of the  $E$  region must exist slow planetary waves, caused by inhomogeneities of  $\nabla \omega_0$  and  $\nabla H_0$ . From (1.11) and (1.12) for the incompressible dissipativeless of ionosphere, we get:

$$helm(\nabla \times \mathbf{V} + 2\omega_0) = \nabla \times \frac{1}{4\pi\rho} [\nabla \times \mathbf{h} \cdot \mathbf{H}_0], \quad (1.15)$$

$$helm \mathbf{H} = 0, \quad (1.16)$$

where  $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ . Equation (1.15) shows partial freezing-in of the absolute vortex, and equation (1.16) - complete freezing-in of magnetic field  $\mathbf{H}$  in the  $F$  region. The equations (1.13) and (1.15) are generalized Fridman-Helmholtz equations for the ionospheric medium. When  $H_0 \rightarrow 0$  they change to Fridman's equation for the absolute vortex  $\nabla \times \mathbf{V} + 2\omega_0$ , and when  $H_0 \rightarrow 0$  and  $\nabla \omega_0 \rightarrow 0$  - to classical Helmholtz equation for vorticity  $rot \mathbf{V}$ . These equations have remarkable feature, in particular, for large-scale processes time derivative from vorticity  $d\nabla \times \mathbf{V} / dt$  is one of the main members (this property is absent in the Euler equation of motion (1.5) and (1.11), in which inertial member  $\rho d\mathbf{V} / dt$  is negligibly small in comparison with the other members). In the absence of certain information about the main forces (pressure gradient, gravity force), it allows composing prognostic equations and carry out their numerical integration. Another important feature of the Fridman-Helmholtz equation in comparison with the Euler equation is natural accounting of inhomogeneous effects of angular velocity of the earth rotation  $\omega_0$  and geomagnetic field  $\mathbf{H}_0$ . Finally, Fridman-Helmholtz equation is a basic condition of motion, in which  $\nabla \times \mathbf{V}$  is always nonzero.

Equations (1.13), (1.15) and (1.16) contain full information about evolution of vortices and planetary waves caused by nonconservative forces ( $rot \mathbf{F} \neq 0$ ) in the fluid. In the ionosphere, these are Coriolis forces  $\mathbf{F}_K = \rho[\mathbf{V} \cdot 2\omega_0]$  and the Ampere force  $\mathbf{F}_A = [\nabla \times \mathbf{h} \cdot \mathbf{H}_0] / 4\pi$ .

Thus, we can conclude that the  $E$  region (80–150 km) behaves as a neutral medium. Here ion component of plasma is present as passive admixture and moves together with the neutral component ( $V_i \approx V$ ) [12,17]. Dynamic processes in this region of the upper atmosphere are mainly caused by pressure  $P$  of the neutral gas. The electron component, which is completely magnetized here, considerably depends on the geomagnetic field ( $P_H / P_e \gg 1$ ). Regardless of the neutrals, it moves with the drift velocity  $\mathbf{V}_e = c[\mathbf{E} \cdot \mathbf{H}_0] / H_0^2 = \mathbf{V}_d$  caused by the vortical electric field, the value of which for large-scale processes substantially exceeds ionospheric dynamo field generated in the ionosphere by the wind mechanism  $\mathbf{E}_d = [\mathbf{V} \cdot \mathbf{H}_0] / c$  [23]. Generation and evolution of large-scale vortical and wave motions in this region of the upper atmosphere should be investigated on the bases of three-fluid model of hydrodynamics: applying equations (1.5)-(1.7) for the neutrals and ions ( $\mathbf{V} = \mathbf{V}_i$ ), and for the electrons - equations (1.8) - (1.9). Physical processes in the neutral component will have hydrodynamical character. The large-scale low-frequency processes proceed quite slowly – with the velocity of ionospheric winds ( $10\text{--}100 \text{ m} \cdot \text{s}^{-1}$ ). For the electrons large-scale processes are faster ( $800\text{--}900 \text{ m} \cdot \text{s}^{-1}$  –  $10 \text{ km} \cdot \text{s}^{-1}$ ) and wavy motions have electromagnetic nature.

In the  $F$  region (150 – 600km) electrons and ions are completely magnetized,  $\omega_e \gg v_e$ ,  $\omega_i \gg v_{im}$ . They are tightly connected with the geomagnetic lines of force and their motion is mainly determined by geomagnetic field pressure ( $(P_H / P) \gg 1$ ). Due to equality of the masses of molecules and ions, the neutrals will be effectively involved in the motion. As a result, perturbation in a neutral component will propagate with the characteristic velocity  $\mathbf{U}_A = \mathbf{H}_0 / \sqrt{4\pi\rho}$ , which in this sphere of the upper atmosphere varies from  $800\text{--}900 \text{ m} \cdot \text{s}^{-1}$  to  $1\text{--}5 \text{ km} \cdot \text{s}^{-1}$  [5,6]. Dynamic processes in the  $F$  region must be investigated on the bases of the single-fluid model of magnetic hydrodynamics of the ionosphere (equations (1.11), (1.12), (1.15), (1.16)). Here dynamic processes have magnetohydrodynamic character and proceed much faster than in the  $E$  region. Besides, from equation (1.12) follows that here the motion is kinematically possible only if velocities comply with Maxwell's induction equation (1.16). Firstly, this fact substantially limits kinematic arbitrariness of motion in the  $F$  region; secondly, it shows that dynamically possible motions can occur only with the velocities satisfying the equation (1.16). Induction equation

like Helmholtz–Fridman equation (1.15), naturally contains inhomogeneity of the geomagnetic field. Below, large-scale vortical structures of the type of moving cyclones (anticyclones) will be investigated on the bases of dynamic equations of the ionosphere.

### 3. EXCITATION, INTENSIFICATION AND PROPAGATION OF LARGE-SCALE VORTICES AND PLANETARY WAVES IN THE IONOSPHERE

As it was mentioned beginning from the altitude of 80 km and higher, the upper atmosphere of the Earth is strongly dissipative medium. The vertical coefficient of turbulent mixing in the lower ionosphere (70–125 km) according to all existent estimations is of the order of  $10^6 \text{ m}^2 \cdot \text{s}^{-1}$ . Often when modelling large-scale processes for this region of the upper atmosphere, effective coefficient of Rayleigh friction between ionospheric layers is introduced instead of turbulent mixing coefficient, which at the altitudes of about 100 km amounts to  $\lambda \approx 10^{-5} \text{ s}^{-1}$ . The role of the “ion” friction rapidly increases at the altitudes above 120 km and its analytical expression coincides with Rayleigh friction formula. Therefore, often during study of large-scale ( $10^3\text{--}10^4 \text{ km}$ ) vortex structures and low-frequency ( $10^{-4}\text{--}10^{-6} \text{ s}^{-1}$ ) planetary waves in the ionosphere we will apply the well-known Rayleigh formula  $\mathbf{F} = -\lambda_{Ra} \mathbf{V}$  assuming that for the altitudes 80–130km  $\lambda_{Ra} \approx 10^{-5} \text{ s}^{-1}$ , and for the altitudes above 130 km  $\lambda_{Ra} = Nv_{im} / N_m$ . Dissipative force  $F$  has an accumulative nature and its action becomes perceptible only after a certain time interval ( $t \sim 1 / \lambda_{Ra}$ ). Real mechanism of dissipation in the atmosphere against the background of baroclinic, nonlinear and dispersive effects generates in the ionosphere moving spatial structures representing the equilibrium stationary solutions of equations of magnetic hydrodynamics. In the ionosphere against the background of such stationary solutions (quasi-statics, quasi-geostrophicity, magneto-vortex rings, cyclone (anticyclone), solitons and so on) always appear weather-forming large-scale ( $10^3\text{--}10^4 \text{ km}$ ), small and average-scale ( $10^{-3}\text{--}10^3 \text{ km}$ ) nonstationary processes.

For example, simplified Helmholtz-Fridman equation:

$$\frac{\partial \nabla \times \mathbf{V}}{\partial t} = \mathbf{A} \quad (2.1)$$

describes generation of nonzero vorticity  $\nabla \times \mathbf{V}$  in the atmosphere under the action of barocline vector  $\mathbf{A}$  taking into account temperature contrasts in the form of advection of warm and cold. According to the observations [24] vector  $\mathbf{A}$  is a slowly-varying function of time and equation of vortex (2.1), with the initial conditions of Cauchy  $\nabla \times \mathbf{V}|_{t=0} = 0$  (at the initial moment in the atmosphere there were no vortices) has an unreal solution as under the action of barocline vector  $\mathbf{A}$  the generating vortex will be growing

unlimitedly in time  $\nabla \times \mathbf{V} = \mathbf{A} \cdot t$ . If a dissipation mechanism in the ionosphere is a Rayleigh friction, equation for the vorticity will take the following form:

$$\frac{\partial \nabla \times \mathbf{V}}{\partial t} = \mathbf{A} - \lambda_{Ra} \nabla \times \mathbf{V} \quad (2.2)$$

which with the same Cauchy conditions has the bounded solution:

$$\nabla \times \mathbf{V} = \frac{\mathbf{A}}{\lambda_{Ra}} \left( 1 - e^{-\lambda_{Ra} t} \right). \quad (2.3)$$

Indeed, from equation (3.3) follows that vorticity will increase linearly with time only at small time intervals ( $t \ll 1/\lambda_{Ra}$ ) under the action of baroclinicity. After a certain time, when the dissipation effect reaches a specific value, the vortex starts to decrease and for the large intervals of time ( $t \gg 1/\lambda_{Ra}$ ) tends to constant (equilibrium) value  $\mathbf{A} / \lambda_{Ra}$ . The value  $T = 1/\lambda_{Ra}$  can be called a time of relaxation of nonstationary vortex. For the E region T is of the order of twenty-four hours, and for the F region - of the order of one hour [12,15]. Stationary solution describes the equilibrium between baroclinicity and dissipation effects ( $\mathbf{A} = \lambda_{Ra} (\nabla \times \mathbf{V})_{st}$ ). As a result, dissipative structure of movement is formed in the ionosphere in the form of stationary cyclones and anticyclones. In the troposphere such immovable cyclones (anticyclones) are called as ‘‘centres of action of atmosphere’’ [24]. For example, to them are related Icelandic and Aleutian cyclones, Siberian and Canadian anticyclones, etc.

In the geostrophic approximation, in the presence of heat (cold) source  $\mathcal{E}$  and dissipative force  $\mathbf{F} = -\lambda_{Ra} \mathbf{V}$  the following formulae have been obtained [12] describing the structure of stationary cyclone (anticyclone):

$$\Omega_z = (\nabla \times \mathbf{V})_z = \frac{2\omega_{0z}}{\lambda_{Ra}^2 + 4\omega_{0z}^2} \frac{1}{\rho} \Delta P, \quad (2.4)$$

$$\nabla \cdot \mathbf{V} = -\frac{\lambda_{Ra}}{\lambda_{Ra}^2 + 4\omega_{0z}^2} (\nabla \times \mathbf{V})_z, \quad (2.5)$$

$$V_z = -\int_{z_0}^z \text{Div } \mathbf{V} dz, \quad (2.6)$$

$$V_z = \frac{\mathcal{E}}{\rho c_p (\gamma - \gamma_a)}, \quad (2.7)$$

here  $P$  and  $\rho$  - pressure and density of medium,  $\omega_{0z}$  - vertical component of the angular velocity of the earth rotation,  $\gamma$  and  $\gamma_a$  - vertical and adiabatic gradients of temperature,  $c_p$  - heat capacity at a constant pressure,

$$\text{Div } \mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} \quad \text{two-dimensional divergence,}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ - Laplace operator. From (2.4)-(2.7) follows,}$$

that when  $\varepsilon > 0$  in the medium appear vertical currents  $V_z > 0$ . Then from (2.6) we have  $\text{Div } \mathbf{V} < 0$  i.e. inflow to the center from the periphery (convergent region). For this case from (2.5) follows that vortex  $(\nabla \times \mathbf{V})_z$  will be positive and then, according to the right-hand screw rule, particles in this vortex will be rotating counter-clockwise. From (2.4) when  $(\nabla \times \mathbf{V})_z > 0$  follows that  $\Delta P > 0$ , i.e. minimum pressure will be in the vortex center. Thus, when  $\varepsilon > 0$  all structural elements of a stationary cyclone are defined sequentially. When  $\varepsilon > 0$  formulae (2.4)-(2.7) define structural elements of stationary anticyclone (see Fig. 1). As it will be shown below, stationary vortical formations generate also in the planetary Rossby wave when convective term of vorticity taking into account zonal dominant current  $\mathbf{u}(\partial(\nabla \times \mathbf{V})_z / \partial x)$  is exactly balanced by  $\beta$ -effect ( $-\beta V_y$ , where  $\beta = \partial 2\omega_{0z} / \partial y$  - Rossby parameter). In other words, when nonlinear term of vortex equation increases with time, it will be balanced by dispersive spread, etc. Generalizing all the above we can assert that stationary vorticity structures in the ionosphere is a consequence of invariants of Helmholtz-Fridman equations and Maxwell induction equation, taking into account Hall’s effect. These equations naturally include internal, self-consistent and opposite processes. In case of preservation of invariant, increase of one process is compensated by decrease of another and vice-versa. Let’s illustrate this from condition of invariance preservation (1.13). When  $\omega_0 = 0$ , equation (1.13) for a plane motion is reduced to the following formula:

$$\frac{d}{dt} \left( (\nabla \times \mathbf{V})_z + \frac{N}{N_n} \frac{e}{Mc} H_{0z} \right) = 0.$$

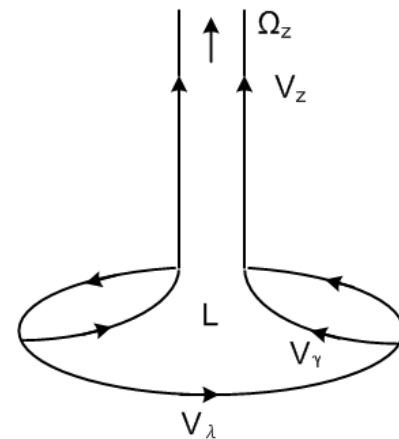


Fig. (1). Kinematic picture of a stationary cyclone motion.

Partial solution of this equation has the following form

$$(\nabla \times \mathbf{V})_z + \frac{N}{N_n} \frac{e}{Mc} H_{0z} = A = const. \quad (2.8)$$

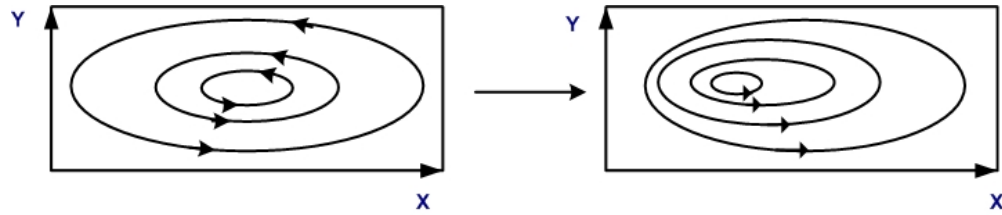


Fig. (2). Vortex intensification in the ionosphere. The dependence of inhomogeneous magnetic field  $H_{0z}$  versus  $y$ .

Geomagnetic field  $H_{0z}$  in the meridional direction has a maximum on a pole and decreases to zero on the equator. Now let's consider the case when at the initial point of time a movement is counter-clockwise, i.e.  $A > 0$  (Fig. 2). Here  $x$  is directed along latitude from the west to the east, and  $y$  – along meridian from the south to the north. In the plane  $xy$ , to the left of the instant centre of rotation, and in case of the motion of particles from the north to the south  $H_{0z}$  will decrease towards negative  $y$ . Consequently  $(\nabla \times \mathbf{V})_z$  must increase by equal value so that  $A$  remains constant. To the right of the instant centre of rotation, when the particles move towards positive  $y$ ,  $H_{0z}$  will increase and  $(\nabla \times \mathbf{V})_z$  decrease. Thus, presence of inhomogeneous magnetic field  $H_{0z}$  along meridian leads to the intensification of vortex to the left from the instant centre of rotation, i.e. particles on a left-hand side will be rotating faster, than to the right of the rotation centre. In case of clockwise rotation, constant  $A$  will be negative and taking this into consideration intensification of vortex will occur again on a left-hand side from the instant centre. Intensification mechanism of such vortex is similar to the amplification of currents at the western ocean coasts caused by the Coriolis force [22].

The problem of possible generation of vortical motions in the ionosphere is not less important, when at the initial point of time the motion is vortex-free and rectilinear along meridian (from the north to the south). At the initial moment from the condition of retaining the equation (2.8) we will have (Fig. 3)  $(eN H_{0z} / N_n M c) = A = const$ . When particles move towards negative  $y$ ,  $H_{0z}$  will decrease. Consequently positive vortex should generate  $(\nabla \times \mathbf{V})_z$  which will compensate the decrease of  $H_{0z}$ , so that  $A$  remains constant. Thus, appears rotational counter-clockwise motion further result of which will be intensification of the positive vortex to the left from the instant centre of rotation

and weakening to the right. If we have irregular vorticity along  $y$  axis,  $(\nabla \times \mathbf{V})_z = \alpha y + (\nabla \times \mathbf{V})_{z0}$ , where  $\alpha$  and  $(\nabla \times \mathbf{V})_{z0}$  are constants, similar to above considered case, geomagnetic field will intensify  $H_{0z} + h$  to the left from the rotation centre and weaken to the right from the centre of rotation  $H_{0z} - h$ . Thus, simple hydrodynamic mechanism of generation of geomagnetic perturbations is revealed in the ionosphere caused by non-uniform vortical structures along meridian.

When studying nonstationary processes in the ionosphere, consideration must be given to the fact that atmosphere density at ionospheric altitudes is nearly million times less than in the troposphere, and the wind velocity exceeds the wind velocity in the troposphere by an order of magnitude. Therefore, ionosphere (beginning from 120 km and higher) becomes nonstationary and nonlinear medium. As a rule, considering nonlinearity, steepness of the wave front increases leading to its breaking or formation of shock wave. However, as it is well known, shock waves do not arise spontaneously in the ionosphere. This indicates to the fact that in the real ionosphere for the planetary-scale motions when dissipative forces can be neglected, dispersive effects of the medium must be essential. As a result before breaking the wave must disintegrate either into separate nonlinear waves or into the vortex formations. If nonlinear increase of the steepness of wave front will be exactly compensated by dispersion spreading, then stationary waves may appear in the ionosphere, i.e. all values describing waves, which propagate without changing their shapes will depend on  $x - ct$ , where  $c$  - wave phase velocity. It has been shown for the first time [20] that planetary waves are partial exact solutions of nonlinear Helmholtz-Fridman vorticity equation (1.13) describing slow weather-forming processes in the atmosphere. By applying simplification of the theory of long waves and neglecting electromagnetic and viscous forces for barotropic, nondivergent atmosphere, after introduction of flow function  $\psi$  by formulae

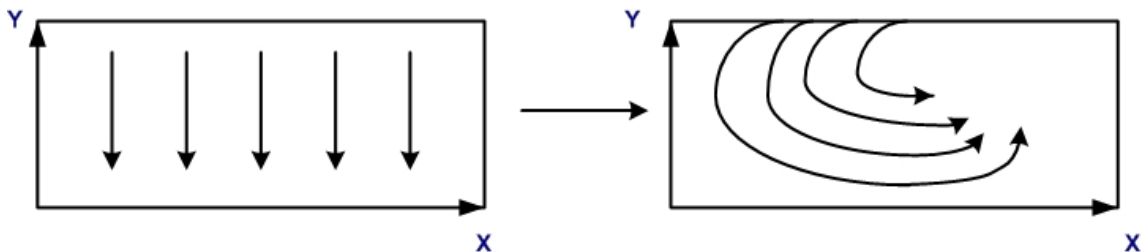


Fig. (3). Vortex generation in the ionosphere. The dependence of inhomogeneous magnetic field  $H_{0z}$  versus  $y$ .

$V_x = -\partial\psi / \partial y$ ,  $V_y = \partial\psi / \partial x$ , from equation (1.13) we get nonlinear equation for vorticity

$$\frac{\partial(\Delta\psi + f)}{\partial t} - \frac{\partial\psi}{\partial y} \frac{\partial}{\partial x}(\Delta\psi + f) + \frac{\partial\psi}{\partial x} \frac{\partial}{\partial y}(\Delta\psi + f) = 0, \quad (2.10)$$

where  $f = 2\omega_0 \sin \varphi$  - Coriolis parameter,  $\Omega_z = (\partial V_y / \partial x) - (\partial V_x / \partial y) = \Delta\psi$  - vortex. Below linear and nonlinear planetary waves on the bases of equation (2.10) are investigated.

Partial solutions of equations (2.10) containing nonlinearity and dispersion can be nonlinear waves either of solitary (solitons) or periodical (knoidal) type. To show this, let's suppose that  $\psi = \psi(\theta, y)$ ;  $\theta = x - ct$ . Then equation (2.10) will take the following form:

$$J(\psi + cy; \Delta\psi + f) = 0, \quad (2.11)$$

where  $J(a, b) = (\partial a / \partial \theta)(\partial b / \partial y) - (\partial a / \partial y)(\partial b / \partial \theta)$  Jacobian.

General solution of equation (2.11) has the following form

$$\Delta\psi + f = G(\psi + cy), \quad (2.12)$$

where  $G$  - arbitrary differentiable function of its own argument. It was shown [26,27] that in the presence of zonal flow with small inhomogeneity along meridian  $u = u_0 [1 + \alpha \sin(\alpha y_0)]$  and weak nonlinearity for the value  $\delta = y_0 - y$  from equation (2.12) the following nonlinear equation can be obtained

$$\Delta\delta + \frac{1}{\bar{u} - c} \frac{d\bar{u}}{dy_0} \left[ \left( \frac{\partial\delta}{\partial y} \right)^2 + \left( \frac{\partial\delta}{\partial x} \right)^2 + 2 \frac{\partial\delta}{\partial y} \right] + \frac{\beta}{\bar{u} - c} \delta = 0. \quad (2.13)$$

with homogeneous zonal flow ( $\bar{u} = const$ ) equation (2.13) becomes linear and coincides with the well known wave equations of Rossby [25]

$$\frac{\partial^2 \delta_0}{\partial \eta^2} + \frac{\beta h^2}{\bar{u} - c_0} \delta_0 = 0,$$

where  $\delta_0 = \delta / a$ ,  $\eta = y / h$ ,  $\xi = \theta / \lambda$  - nondimensional parameters;  $a$  - wave amplitude,  $h$  - thickness of a zonal flow,  $\lambda$  - wavelength. General solution of equation (2.13) with the boundary conditions  $\delta_0(\xi, 0) = \delta_0(\xi, 1) = 0$  has the following form

$$\delta_0(\xi, \eta) = F(\xi) \sin(n\pi\eta), \quad (2.14)$$

$$c_0 = u_0 - \frac{\beta}{m^2}, \quad (n = 1, 2, 3),$$

where  $m = n\pi / h$  - wavenumber,  $F(\xi)$  - arbitrary function. Solving equation (2.13) by applying theory of perturbation,

in the second approximation we will get nonlinear equation for the function  $F(\xi)$

$$\frac{d^2 F}{d\xi^2} + a_1 F^2 + a_2 F = 0. \quad (2.15)$$

Here the coefficients  $a_1$  and  $a_2$  are expressed with parameters  $\alpha$ ,  $u_0$ ,  $a$ ,  $\lambda$ ,  $\beta$ ,  $h$ ,  $n$  and we are not giving them here because of their bulkiness. Equation (2.15) investigated in detail in the above-mentioned papers is a stationary equation of Korteweg-de Vries, integrated once and describes structure of nonlinear waves. Depending on integration constants, the solution represents either solitary wave (soliton) or periodical (knoidal) wave. The solutions at infinity tending to zero are of interest in meteorology (since with  $\xi \rightarrow \infty$ ,  $y \rightarrow y_0$ ) and therefore the solution for equation (2.15) has a form of solitary-type wave

$$F(\xi) = A_1 \operatorname{sech}^2(A_2 \xi), \quad (2.16)$$

where  $A_1 = -3a_2 / 2a_1$ ,  $A_2^2 = -a_2 / 4$ . By the use of equation (2.16) current lines are constructed in a moving coordinate system  $(\eta, \xi)$  for the cases  $|\alpha| \ll 1$  and  $|\alpha| = \pi$  [23]. On examination of nonlinear wave process, depending on a thickness of zonal flow, large-scale closed vortices appear in the form of cyclons and anticyclons having diameters from 500 to 1000 km. Conditions of appearance of closed vortices in the disturbed flow have been derived. When  $|\alpha| \ll 1$  we have weak linear shift. Fig. (4) illustrates lines of flow for  $\alpha u_0 > 0$  and anticyclonic vortex formation in southern latitudes. When  $\alpha u_0 < 0$  i.e. in case of negative linear shift, cyclonic vortex appears in the northern latitudes. When  $|\alpha| = \pi$  we have symmetrical wind shift.

The Fig. (5), when  $\alpha u_0 > 0$ , shows lines of flow with cyclon and anticyclon formation, having common area of maximum velocities (jet flows). The Fig. (6), when  $\alpha u_0 < 0$ , where jet flows are absent, shows lines of flow of combined vortex pairs. Fig. (6) outwardly resembles the solution of two-dimensional solitary wave constructed in the work [28,29]. Also it is shown that with the real atmospheric parameters the obtained solutions make sense only for  $n$  ( $n = 1, 2$ ) corresponding to large-scale perturbations ( $\lambda \sim 2h / n$ ). Such system of solitary waves and large-scale vortices of cyclon and anticyclone-type often are observed in the atmosphere [24]. Thus, in the presence of inhomogeneity of zonal flow in meridional direction (as well as inhomogeneous geomagnetic field  $H_{0z}$  in equation. (2.8)) leads to formation of large-scale vortices, solitary waves and jet flows, playing an important role in weather-forming processes. Taking into consideration that zonal gradient of the wind velocity along meridian always exists in the ionosphere due to non-uniform heating of the atmosphere from the pole to the equator, caused by large-latitudinal heating source [12], the above considered mechanism of formation of nonlinear waves

should play an important role in studying the large-scale dynamic processes in the ionosphere. Below is shown that the solution of dipole-type vortices (see Fig. 6) should play an important role in dynamic processes of the ionosphere, since such vortical structures, as shown below, are long-living, having electromagnetic nature (this allows to register them through radio methods [16]) and can play essential role in the processes of heat and energy transfer and also in formation of strongly turbulent medium.

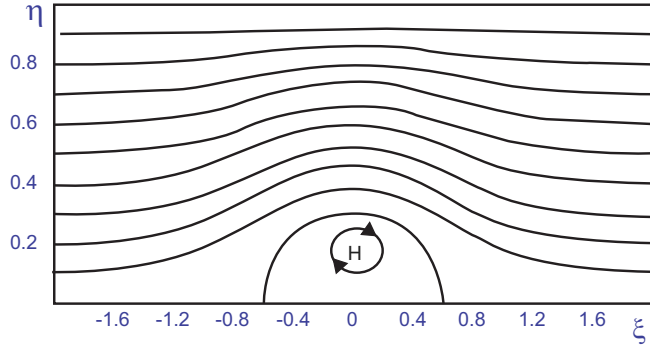


Fig. (4). Generation of anticyclone in the ionosphere at positive linear shear of a zonal wind.

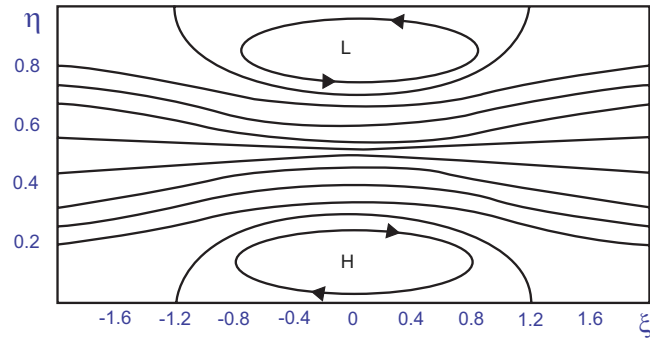


Fig. (5). Generation of cyclone-anticyclone couple and jet flows in the ionosphere at positive symmetrical shear of a zonal flow.

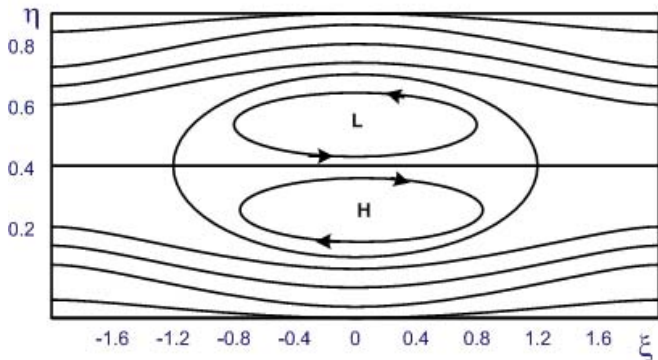


Fig. (6). Generation of cyclone-anticyclone couple (dipole vortex) in the ionosphere at negative symmetrical shear of a zonal flow.

Basic ionospheric equations of magnetic hydrodynamics for vorticity  $rot \mathbf{V}$  and magnetic field  $\mathbf{H}$  can be presented in the following form:

$$helm(\nabla \times \mathbf{V} + 2\omega_0) = \Gamma + \lambda \nabla \times \mathbf{V}, \quad helm\left(\frac{\mathbf{H}}{\alpha\rho}\right) = -\delta\Gamma, \quad (2.17)$$

where:  $\Gamma = \nabla \times \mathbf{F}_a$ ,  $\mathbf{F}_a = [\nabla \times \mathbf{H} \cdot \mathbf{H}] / 4\pi\rho$  - electromagnetic Ampere force per mass unit,  $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ ,  $\mathbf{H}_0$  - geomagnetic field,  $\mathbf{h}$  - its perturbation. When  $\delta = 1$  the the system equation. (2.17) describes dynamic processes in the  $E$  region of the ionosphere, and when  $\delta = 0$  - in the  $F$  region.

For the horizontal, incompressible, nondivergent flow in  $\beta$ -plane approximation the system (2.17) takes on the following form:

$$\frac{\partial \Delta \psi}{\partial t} + \beta \frac{\partial \psi}{\partial x} + c_H \frac{\partial h}{\partial x} + \lambda \Delta \psi = J(\psi, \Delta \psi),$$

$$\frac{\partial h}{\partial t} + \beta_H \frac{\partial \psi}{\partial x} + \delta c_H \frac{\partial h}{\partial x} = J(\psi, h). \quad (2.18)$$

Here are introduced the following designations:  
 $\beta = \partial 2\omega_{0z} / \partial y = -[\partial(2\omega_{0z}) / \partial \theta] / R = (2\omega_0 \sin \theta_0 / R) > 0$  - Rossby parameter,  
 $\beta_H = (eN / \rho c)(\partial H_{0z} / \partial y) = -(NeH_0 / N_n mc)(\sin \theta_0 / R) < 0$  - "magnetic Rossby parameter",

$c_H = (c / 4\pi eN)(\partial H_{0z} / \partial y) = -(cH_0 / 4\pi eN)(\sin \theta / R)$  - Phase velocity of fast electromagnetic planetary wave,  
 $h = (eN h_z / N_n M c)$  - perturbation,  $\theta_0 = 90^\circ - \varphi_0$  - certain average value of co-latitude near which motion in the medium is considered,  $\alpha = (c / eN)$  - Hall's parameter,  $H_p$  - magnetic field strength on the pole. From the system (2.18) follows law of evolution of energy  $E$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \left[ \frac{1}{2} \int dxdy \left( \Delta \psi^2 + \frac{h^2}{k_0^2} \right) \right] = \lambda \int dxdy (\Delta \psi)^2, \quad (2.19)$$

and potential enstrophy  $K$

$$\frac{\partial K}{\partial t} = \frac{\partial}{\partial t} \left[ \frac{1}{2} \int dxdy \left( \Delta \psi^2 + \frac{(\Delta h)^2}{k_0^2} \right) \right] = \lambda \int dxdy (\Delta \psi)^2, \quad (2.20)$$

of considered wavy structures. Here  $k_0^2 = N \omega_{pi}^2 / N_n c^2$  - square of characteristic wave number, wavelength of which substantially exceeds the earth radius  $R$ ,  $\omega_{pi}^2 = 4\pi e^2 N / M$  - ion plasma frequency. When  $\lambda_{Ra} = 0$ , the energy  $E$  and the enstrophy  $K$  of waves is retained.

For small perturbations, when  $\psi, h \sim \exp[i(\mathbf{k}\mathbf{r} - \omega t)]$  (where  $\mathbf{k}$  wave vector,  $\omega$  frequency of perturbation) and considering that  $\omega = \omega_0 + i\gamma$ ,  $|\gamma| \ll |\omega_0|$ ;  $k \gg k_0$ , from the system (2.18) for the  $E$  region of ionosphere ( $\delta = 1$ ), the following two branches of oscillations can be obtained [30,31]: for fast electromagnetic planetary wave we have

$$\omega_0^+ = k_x c_H, \quad c_\phi^+ = c_H, \quad \gamma^+ = -\lambda_{Ra} \frac{k_0^2}{k^2}, \quad (2.21)$$



and for slow planetary Rossby-type wave

$$\omega_0^- = -\beta' \frac{k_x}{k^2}, \quad c_\varphi^- = -\frac{\beta'}{k^2}, \quad \gamma^- = -\lambda_{Ra}. \quad (2.22)$$

Here  $k^2 = k_x^2 + k_y^2$ ,  $\beta' = \beta + \beta_H$ ;  $c_\varphi^\pm = \omega_0^\pm / k_x$  and  $\gamma^\pm$  - phase velocities and dumping decrements of fast and slow planetary waves respectively. From (2.21) follows that fast electromagnetic planetary wave will propagate from the east to the west ( $c_\varphi^+ < 0$ , since  $c_H < 0$ ) and practically does not attenuate,  $|\gamma^+| \ll \lambda_{Ra}$ . Phase velocity of the fast waves  $c_H$  is inversely proportional to the concentration of electrons  $N$  and their phase velocity  $c_\varphi^+$  on the night side exceeds phase velocity on the day side by one or two orders of magnitude,  $c_\varphi^+ = |c_H| = (1-7) \text{ km} \cdot \text{s}^{-1}$ . Period of fast waves  $T^+ = 2\pi / \omega_0^+$  varies from 2 to 10 minutes at night and from 10 minutes to 2 hours in the daytime, and the wavelength varies from 2000 to  $10^4 \text{ km}$ . The properties and parameters of these waves are in good agreement with mid-latitude long-period oscillations observed by ionospheric and magnetic observatories [2,3,15,16]. From equations (2.22) it follows that slow Rossby-type wave propagates westwardly, as well as eastwardly depending on wavelength and velocity of the average zonal wind. In dispersion of this wave together with the Rossby parameter  $\beta$ , magnetic Rossby parameter  $\beta_H$  ( $\beta_H \geq \beta$ ,  $\beta > 0$ ,  $\beta_H < 0$ ) plays an important role as well. Period of slow waves varies from 2 to 7 days, propagation velocity is of the order of  $100-300 \text{ m} \cdot \text{s}^{-1}$ . These waves strongly attenuate in the  $E$  region  $|\gamma^-| = \lambda_{Ra} \sim 10^{-5} \text{ s}^{-1}$ , however for very long waves ( $10^4 \text{ km}$  and longer) attenuation will be weak. In the  $F$  region, in the absence of rotation  $\omega_0 = 0$ , where Hall's effect is absent and  $\delta = 0$ , permanently acting factor - latitudinal gradient of geomagnetic field as in  $E$  region generates fast electromagnetic planetary waves having the following frequency

$$\omega_{on} = \pm \frac{1}{\sqrt{4\pi\rho}} \frac{\partial H_{0z}}{\partial y} = \mp \frac{H_p}{\sqrt{4\pi\rho}} \frac{\sin\theta}{R}, \quad \gamma^\pm = -\frac{\lambda_{Ra}}{2}. \quad (2.23)$$

Estimations show that phase velocity of the fast waves  $c_{on} = \omega_{on} / k_x$  at the altitudes (200-500km) varies from 5 to  $50 \text{ km} \cdot \text{s}^{-1}$ , the period varies ( $3-105 \text{ s}^{-1}$ ). Perturbation of geomagnetic field at these altitudes varies from 10 to 80 nT. The waves weakly attenuate  $|\gamma| = 0.5\lambda_{Ra} \approx 10^{-6} \text{ s}^{-1}$ . In case of magneto-ionospheric wavy perturbations in the  $F$  region values of parameters of  $c_{on}$  waves well coincide with the observed parameters in the  $F$  region. It is shown for the first time [30,31] that new branches of planetary waves  $\omega_0^+$ ,

$\omega_0^-$  and  $\omega_{0n}$  are self-oscillations of the ionospheric resonator. Physically, the branch  $\omega_0^+$  describes oscillations of magnetized electrons when the ions and neutrals are immovable,  $\omega_0^-$  - oscillations of ions and neutral particles when the electrons are immovable ( $\omega_0^+$  and  $\omega_0^-$  are defined from quasi-hydrodynamical equations using triple-fluid approximation),  $\omega_{0n}$  - oscillations of neutrals in single-fluid approximation. The properties of electromagnetic planetary waves are considered in more detail in [23, 32-35]. From the system (2.18) nonlinear solution  $\psi = \psi(\eta, y)$ ,  $h = h(\eta, y)$  in the form of stationary nonlinear waves is found in nondissipative approximation [29,32]. These waves propagate with the velocity  $u = \text{const}$  along latitudinal circles  $x$  without changing their shape  $\eta = x - ut$ . The solution of the system (2.18) along the polar coordinate system in the form of solitary waves has the following form:

$$\psi(r, \varphi, t) = \frac{c_H - u}{\beta_H} h(r, \varphi, t) = au F(r) \sin \varphi, \quad (2.24)$$

where  $r = (\eta^2 + y^2)^{1/2}$ ,  $\text{tg } \varphi = y / \eta$ ,  $a$  - circle radius, where the solution is being searched

$$F(r) = \begin{cases} \left(\frac{P}{k}\right)^2 \frac{J_1(kr)}{J_1(ka)} - (k^2 - p^2) \frac{r}{ak^2}, & \text{at } r < a \\ -\frac{K_1(pr)}{K_1(pa)}, & \text{at } r \geq a \end{cases}. \quad (2.25)$$

$J_1(x)$  - Bessel function of the first order,  $K_1(x)$  - McDonald function. Parameters  $P$  and  $k$  are connected by dispersion relation

$$p J_2(ka) K_1(pa) = -k J_1(kr) K_2(pa),$$

$$p^2 = \frac{\beta' c_H - \beta u}{u(c_H - u)} > 0. \quad (2.26)$$

Taking into consideration dispersion equation (2.26), in the solution (2.24) remain only two free parameters  $u$  and  $a$ . From equation (2.24) it follows that when  $r \rightarrow \infty$  the solution has asymptotic form  $\psi, h \sim r^{1/2} \exp(-pr)$ . Consequently the wave is localized along the surface of the Earth ( $\eta, y$ ). Lines of flow function have dipole character (resembles Fig. 6) and represent a pair of oppositely rotating vortices (cyclon-anticyclon) of equal intensity moving along latitudinal circles against the background of the zonal wind  $u$ . Motion of particles of medium in nonlinear structures (2.24) has nonzero vorticity  $\nabla \times \mathbf{V} = -\Delta \psi \mathbf{e}_z \neq 0$ , i.e. particles inside vortex rotate with velocity  $u_c \approx u$ . At the same time vortex entrains group of particles and rotating together they move with the vortical structure. This is also facilitated by "freezing-in" of the geomagnetic field in these

structures. Characteristic size for the slow vortices is  $d^- \geq 10^3$  km, and for the fast ones  $d^+ \geq 10^4$  km.

According to the equations of transfer (2.19) and (2.20), energy and enstrophy of large-scale vortices in the dissipative ionosphere considerably exceed dissipative term, and accordingly relaxation of such vortices occurs very slowly. As a result, in the ionosphere electromagnetic fast large-scale vortical structures are long-living (see equation (2.3)). Therefore, as it was mentioned above, they can play an important role in the processes of substance, heat and energy transfer and in formation of macro-turbulent horizontal transfer, where the above vortical structures can be considered as “turbulent particles”. Indeed, the above-discussed vortical structures, playing role of “turbulent particles”, can be considered as the elements of horizontal turbulent macroexchange in the global circulation processes in the ionosphere. Besides, horizontal turbulent exchange coefficient can be estimated by Obukhov formula  $k_1 = 10^{-2} d^{4/3} \text{ m}^2 \cdot \text{s}^{-1}$ . For vortices of sizes  $d \sim 10^3$  km (average size of cyclon and anticyclon) we get  $k_1 \approx 3 \cdot 10^6 \text{ m}^2 \cdot \text{s}^{-1}$ . This value exceeds vertical turbulent exchange coefficient hundreds of thousands of times and shows, that during global interaction of high and low latitudes (in the ionosphere pole is warmer than equator) meridional transfer (from the north to the south) must have macro-turbulent character. This extraordinary and interesting problem requires special examination and we will not discuss it here. Taking into consideration both components of geomagnetic field  $H_{0z}, H_{0y}$  obtained in the above-

mentioned formulae (3.21)-(3.23), a factor  $\sqrt{1 + 3\sin^2 \theta}$  appears for the natural frequencies, which shows that the considered large-scale waves have planetarywide character and they can be registered at all latitudes from the pole to the equator (their values double on the equator) [23,31]. Good agreement of theoretical results of the discussed electromagnetic planetary waves with experimentally observed data of large-scale mid-latitude long-period oscillations in the  $E$  region and magneto-ionospheric wavy perturbations in the  $F$  region indicate the existence of the sources of planetary wavy structures having electromagnetic nature at ionospheric altitudes. These sources are permanently acting factors in the upper atmosphere, latitudinal inhomogeneity of geomagnetic field and angular velocity of the earth rotation. The discussed wavy structures represent their own degrees of freedom of the ionospheric resonator [31], and the observed mid-latitude long-period oscillations and magneto-ionospheric wavy perturbations in the ionosphere can be considered as manifestation of these self-oscillation in the upper atmosphere. When ionosphere is affected from above or from below (magnetic storm, earthquake, artificial explosions, etc) in the first place wavy structures will intensify at these modes [15,38]. With a specific power of external sources, solitary vortices will generate [26,27,34,36,37] which is confirmed by observations [1,15,16,39,40].

#### 4. CONCLUSION

Summarizing all the above it should be noted that the main system of magnetic hydrodynamic equations of the ionosphere admits exact solutions constructed in the works [23,31,35] both, in linear as well as in nonlinear cases. It was shown that generation of linear planetary-scale vortical wavy structures having hydrodynamical and electromagnetic nature in the ionosphere is a result of constantly existing factors: latitudinal gradients of both angular velocity of the earth rotation and geomagnetic field. In nonlinear case, zonal shear flow factor is added to these fundamental factors, which is caused by inhomogeneous heating of the polar and equatorial regions. Thus, inhomogeneity of the earth rotation along meridian, geomagnetic field and zonal predominant flow can be considered among the real sources, generating planetary waves of hydrodynamic and electromagnetic nature in the ionosphere.

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