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Fault Estimation for Uncertain Nonlinear Networked Control Systems

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Abstract: This paper proposes the design of a robust fault estimator for a class of nonlinear uncertain NCSs that ensures the fault estimation error is less than prescribed $\mathcal{H}_\infty$ performance level, irrespective of the uncertainties and network-induced effects. T-S fuzzy models are firstly employed to describe the nonlinear plant. Markov processes are used to model these random network-induced effects. Sufficient conditions for the existence of such a fault estimator for this class of NCSs are derived in terms of the solvability of bilinear matrix inequalities. An iterative algorithm is proposed to change this non-convex problem into quasi-convex optimization problems, which can be solved effectively by available mathematical tools. The effectiveness of the proposed design methodology is verified by a numerical example.

1. INTRODUCTION

In order to avoid production deteriorations or damages, system faults have to be identified and decisions that stop the propagation of their effects have to be made. This gives the rise to the research on fault detection and isolation (FDI) and in recent years, the problem has attracted lots of attention of researchers. Among them, the model-based approach is the common approach, see survey papers [1-4]. The prime importance [5, 6] in designing model-based fault-detection system is the increasing robustness of residual to unknown inputs and modelling errors and enhancing the sensitivity to faults. Two approaches are mainly applied in FDI to address these two issues. One is to use the $\mathcal{H}_\infty$ norm of transfer function matrix from fault to residual signal as a measure to estimate the sensitivity to the faults [7, 8]. Another method is to adopt the $\mathcal{H}_\infty$-filtering formulation to make the error between residual and fault as small as possible [9, 10]. Furthermore, the existence of time delays is commonly encountered in dynamic systems and has to be dealt with in the realm of FDI. Some results have been obtained to address this issue, see [11-14]. However, these results are mostly obtained for systems with state delays.

On the other hand, due to the expansion of system physical setups and functionality, networked control systems (NCSs) have been introduced into the design of control systems. NCSs are a type of distributed control systems where sensors, actuators, and controllers are interconnected by communication networks. It can improve the efficiency, flexibility and reliability of integrated applications, and reduce installation, reconfiguration and maintenance time and costs. Due to its low cost, flexibility, and less wiring, the use of NCSs is rapidly increasing in industrial applications, including telecommunications, remote process control, altitude control of airplanes, and so on, and therefore considerable attention has been devoted to the problem of networked control systems [15-23].

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Network-induced delays and data packet dropouts are two main issues raised in the research of NCSs, see [15, 21-23]. In the NCS, data is sent through the network in packets. Due to this network characteristic, therefore, any continuous-time signal from the plant are first sampled to be carried over the communication network. Chances are that those packets can be lost during transmission because of uncertainty and noise in communication channels. It may also occur at the destination when out of order delivery takes place. Furthermore, the network-induced delays are also a challenging problem in control of NCSs that occurs while exchanging data among devices connected by the communication network. Depending on network characteristics, such as their topologies, routing schemes, etc., these delays can be constant, time varying, or even random. They can degrade the performance of control systems can even destabilize the system. The severity of the network-induced delays is aggravated when data packet dropouts occur during a network transmission.

On the other hand, the study of Markovian jump linear systems has attracted a great deal of attention; see [24-31]. This class of systems is normally used to model stochastic systems which change from one mode to another randomly or according to some probabilities. Some of these results [28-31] are applied to Markovian jump linear systems with mode-dependent time delays. In [31], stabilization of networked control systems with the sensor-to-controller and controller-to-actuator delays is considered in the discrete-time domain. According to the characteristics of NCSs, the Markov process is an ideal model of the random time delays happen in the communication network.

According to the characteristics of NCSs, the Markov process is an ideal model of the random time delays happen in the communication network. Due to the characteristics of communication network, furthermore, network-induced time delays are input delays. It should be noted that in FDI with time-varying input delays, it is difficult to analyze $\mathcal{H}_\infty$ performance or disturbance attenuation based on the gain characterization, because of the state variation depends not only on the current but also the history of exterior disturbance input. To the best of authors' knowledge, fault estimation problem has not been well studied for systems with input-delays.

Motivated by the aforementioned issues, this paper firstly introduces a new disturbance attenuation notation for systems with input delays. We approximate the nonlinear plant by a Takagi-Sugeno model [32]. This fuzzy modelling is simple and natural. The system dynamics are captured by a set of fuzzy implications which characterize local relations in the state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models. In light of such formulation, this paper proposes a robust fault estimator that ensures the fault estimation error is less than prescribed $\mathcal{H}_\infty$ performance level, irrespective of the uncertainties and network-induced effects, i.e., network-induced delays and packet dropouts in communication channels, which are to be modeled by the Markov processes. Based on the Lyapunov-Razumikhin method, the existence of a delay-dependent fault estimator for the nonlinear plant is given in terms of the solvability of bilinear matrix inequalities (BMIs). An iterative algorithm is proposed to change this non-convex problem into quasi-convex optimization problems, which can be solved effectively by available mathematical tools.

This paper is organized as follows. Problem formulation and preliminaries are given in Sections 2. Section 3 gives the main results of this paper. An illustrating example is presented in Section 4. Finally, concluding remarks are drawn in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we describe the nonlinear plant as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\upsilon(t))[(A_i + \Delta A_i)x(t) + B_i\omega(t) + G_i f(t)] \\ y(t) = \sum_{i=1}^{r} \mu_i(\upsilon(t))[(C_i + \Delta C_i)x(t) + D_i\omega(t) + J_i f(t)] \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\omega(t) \in \mathbb{R}^p$ and $f(t) \in \mathbb{R}^q$ are, respectively, exogenous disturbances and faults which belong to $\mathcal{L}_2[0; \infty)$, $y(t) \in \mathbb{R}^l$ denotes the measurement output.

Furthermore, $i \in \mathcal{I}_R = \{1,\ldots,r\}$, r is the number of fuzzy rules; $v_k(t)$ are premise variables, $M_{\iota\kappa}$ are fuzzy sets, $k = 1,\ldots, p$, p is the number of premise variables

$$v(t) = [v_1(t), v_2(t),\ldots, v_p(t)]T,$$

$$\omega_i(\upsilon(t)) = \prod_{k=1}^{p} M_{lk}(\upsilon_k(t)), \omega_i(\upsilon(t)) \ge 0, \sum_{i=1}^{r} \omega_i(v(t)) > 0,$$

$$\mu_i(\upsilon(t)) = \frac{\omega_i(\upsilon(t))}{\sum_{i=1}^{r} \omega_i(\upsilon(t))}, \mu_i(\upsilon(t)) \ge 0, \sum_{i=1}^{r} \mu_i(v(t)) = 1.$$

Here, $M_{\iota\kappa}(v_k(t))$ denote the grade of membership of $v_k(t)$ in $M_{\iota\kappa}$.

In addition, matrices ΔA_i and ΔC_i characterize the uncertainties in the system and satisfy the following assumption:

Assumption 2.1.

$$\begin{bmatrix} \Delta A_i \\ \Delta C_i \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} F(t) E_i,$$

where H_{1i}, H_{2i}, and E_i are know read constant matrices of appropriate dimensions, and $F(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F(t)^T F(t) \le I$, in which I is the identity matrix of appropriate dimension.

In this paper, we consider a nonlinear networked control system of which the plant is described by the T-S model (1). The setup of the overall configuration is depicted in Fig. (**1**), where $\tau(t) \ge 0$ is the random time delay from sensor to controller. These delays are assumed to be upper bounded.

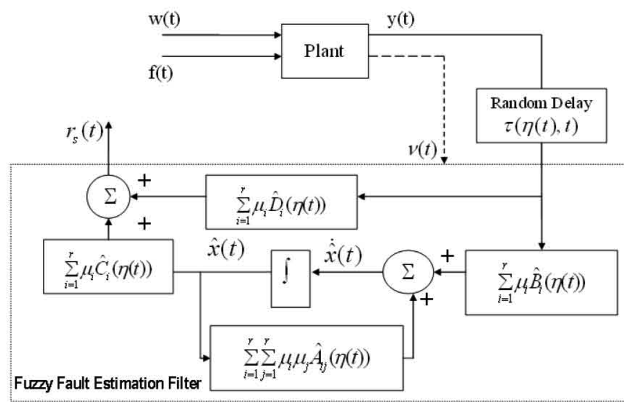

Fig. (1). Block diagram of a fault estimator for a nonlinear networked control system.

The plant outputs are sampled with periodic sampling interval h^s and sent through the network at times kh^s, $k \in \mathbb{N}$. In the absence of data dropouts, it can be noted that the measurement signals $\{y(kh^s), k \in \mathbb{N}\}$ are received by the controller side at times $kh^s + \tau_k^s$ where τ_k^s is the delay that measurement sent at kh^s experiences. A fault estimation filter is therefore constructed $\forall t \in [kh^s + \tau_k^s, (k + 1)h^s + \tau_{k+1}^s]$ as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\upsilon(t))\mu_j(\upsilon(t))\left[\hat{A}_{ij}\hat{x}(t) + \hat{B}_i y(kh^S)\right]$$
$$\hat{x}(0) = 0, \quad (2)$$
$$r_S(t) = \sum_{i=1}^{r} \mu_i(\upsilon(t))\left[\hat{C}_i\hat{x}(t) + \hat{D}_i y(kh^S)\right]$$

where $y(kh^s)$ is equal to the last successfully received measurement signal, $\hat{x}(t)$ is the filter's state vector, $r_s(t)$ is the residual signal, and matrices $\hat{A}_{ij}, \hat{B}_i, \hat{C}_i$ and $\hat{D}_i$ the filter's parameters.

It should be noted that in this system setup, the premise vector $v(t)$ is connected to the fault estimator via point-to-point architecture, which is immune to network-induced delays.

Defining

$$\tau(t) := t - kh^s, \forall t \in \left[kh^s + \tau_k^s, (k+1)h^s + \tau_{k+1}^s\right], \quad (3)$$

(2) can be rewritten as:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\upsilon(t))\mu_j(\upsilon(t))\left[\hat{A}_{ij}\hat{x}(t)+\hat{B}_i y(t-\tau(t))\right],\\ r_s(t) &= \sum_{i=1}^{r}\mu_i(\upsilon(t))\left[\hat{C}_i\hat{x}(t)+\hat{D}_i y(t-\tau(t))\right],\end{aligned} \tag{4}$$

where

$$\tau(t)\in[\min_k\{\tau_k^s\}, h^s+\max_k\{\tau_{k+1}^s\}], \forall k\in\mathbb{N}\ \dot{\tau}(t)=1, \tag{5}$$

Fig. (**2**) shows $\tau(t)$ with respect to time where for all $k, \tau_k^s=\tau^s$, and constant sampling interval h^s with $T = kh^s + \tau^s$. The derivative of $\tau(t)$ is almost always one, except at the sampling times, where $\tau(t)$ drops to τ^s.

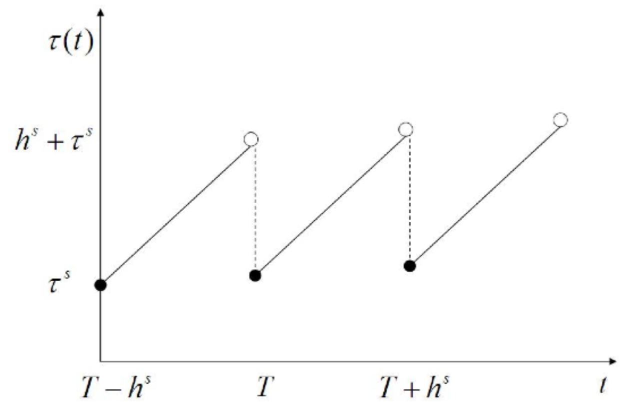

Fig. (2). Evolution of $\tau(t)$ with respect to time without packet dropout.

Furthermore, data packet dropout can be viewed as a delay grows beyond the defined boundary in (5). Let us define n^s as the number of consecutive dropouts in the network channel. Then we can get:

$$\tau(t)\in[\min_k\{\tau_k^s\}, (n^s+1)h^s+\max_k\{\tau_{k+1}^s\}], \forall k\in\mathbb{N}\ \dot{\tau}(t)=1, \tag{6}$$

If the measurement packet sent at kh_s is lost, for instance, then $\tau(t)$ increases up to $2h^s + \tau^s$. We can see this scenario from Fig. (**3**).

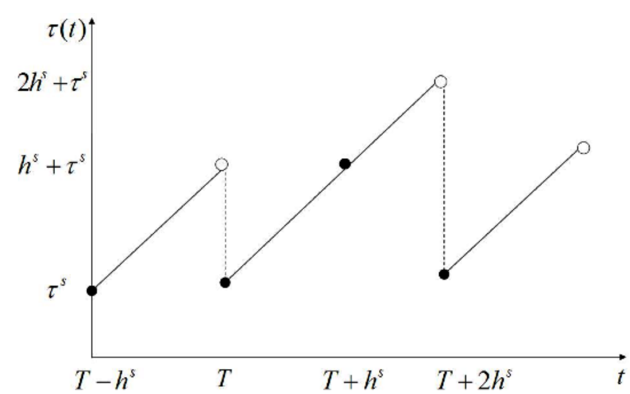

Fig. (3). Evolution of $\tau(t)$ with respect to time with packet dropout at kh^s.

In [33], a Markov chain is utilised to model network delays. Modes of the Markov chain are defined as different network load conditions. For each mode of in the Markov chain, a corresponding delay is assumed to be time-varying but upper bounded by a known constant.

Following the same line as [33], we use a Markov process $\{\eta(t)\}$ to model $\tau_k^s\cdot$ $\{\eta(t)\}$ is a continuous-time discrete-state Markov process taking values in a finite set $S = \{1, 2, \cdots, s\}$ with transition probability matrix given by:

$$Pr\{\eta(t+\Delta)=\jmath \mid \eta(t)=\imath\} = \begin{cases}\lambda_{\imath\jmath}\Delta+o(\Delta), & \imath\neq\jmath\\ 1+\lambda_{\imath\imath}\Delta+o(\Delta), & \imath=\jmath,\end{cases} \tag{7}$$

where $\Delta > 0$, and $\lim_{\Delta\to 0}\dfrac{o(\Delta)}{\Delta}=0$ Here $\lambda_{\imath\jmath}\geq 0$ is the transition rate from mode $\imath$ to mode $\jmath\,(\imath = \jmath)$, and $\lambda_{\imath\imath}=-\sum_{\jmath=1,\jmath\neq\imath}^{S}\lambda_{\imath\jmath}$.

Together with each mode in the Markov process, the corresponding delay is assumed to be time-varying but upper bounded by a known constant. Furthermore, we assume that the mode of the Markov process or state of the network load condition is accessible by the controller and the sensor. The sensor sends the mode of the network load condition and the measurement to the controller. These assumptions are reasonable and they are employed in [33].

From (6), $\{\eta(t)\}$ can be regarded without loss of generality as the model of $\tau(t)$.

Therefore, following the modelling procedure presented in paper 2, for the nonlinear plant represented by (1), the fuzzy fault estimator at time t is inferred as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\upsilon(t))\mu_j(\upsilon(t))\Big[\hat{A}_{ij}(\eta(t))\hat{x}(t)\\ &\qquad +\hat{B}_i(\eta(t))y(t-\tau(\eta(t),t))\Big]\\ r_S(t) &= \sum_{i=1}^{r}\mu_i(\upsilon(t))\Big[\hat{C}_i(\eta(t))\hat{x}(t)\\ &\qquad +\hat{D}_i(\eta(t))y(t-\tau(\eta(t),t))\Big]\end{aligned} \tag{8}$$

where $\hat{A}_{ij}(\eta(t)), \hat{B}_i(\eta(t)), \hat{C}_i(\eta(t))$ and $\hat{D}_i(\eta(t))$ in each plant rule are parameters of the fault estimator which are to be designed.

Substituting (8) into (1) yields

$$\begin{aligned}\dot{\tilde{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\upsilon(t))\mu_j(\upsilon(t))\Big[\mathcal{A}_{ij}(\eta(t))\tilde{x}(t)\\ &\quad +\mathcal{B}_i(\eta(t))\tilde{x}(t-\tau(\eta(t),t))+\mathcal{C}_{ij}(\eta(t))\omega(t)\Big]\\ e(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\upsilon(t))\mu_j(\upsilon(t))\Big[\mathcal{D}_{1i}(\eta(t))\tilde{x}(t)\\ &\quad +\mathcal{D}_{2ij}(\eta(t))\tilde{x}(t-\tau(\eta(t),t))+\mathcal{D}_{3ij}(\eta(t))\omega(t)\Big]\end{aligned} \tag{9}$$

where $e(t) = r_s(t) - f(t)$ is the fault estimation error, $\omega(t) = [\omega^T(t)\ f^T(t)\ \omega^T(t-\tau(\eta(t),t))\ f^T(t-\tau(\eta(t),t))]^T$, $\tilde{x}(t) = [x_T(t)\ \tilde{x}(t)]^T$, and

$$\mathcal{A}_{ij}(\eta(t)) = \begin{bmatrix}A_i+\Delta A_i & 0\\ 0 & \hat{A}_{ij}(\eta(t))\end{bmatrix},$$

$$\mathcal{B}_{ij}(\eta(t)) = \begin{bmatrix}0 & 0\\ \hat{B}_{ij}(\eta(t))(C_j+\Delta C_j) & 0\end{bmatrix},$$

$$\mathcal{C}_{ij}(\eta(t)) = \begin{bmatrix}B_i & G_i & 0 & 0\\ 0 & 0 & \hat{B}_i(\eta(t))D_j & \hat{B}_i(\eta(t))J_j\end{bmatrix},$$

$$\mathcal{D}_{1i}(\eta(t)) = [0\ \hat{C}_i(\eta(t))],$$
$$\mathcal{D}_{2ij}(\eta(t)) = [\hat{D}_i(\eta(t))(C_j + \Delta C_j)\ 0],$$
$$\mathcal{D}_{3ij}(\eta(t)) = [0 - I\ \hat{D}_i(\eta(t))D_j\ \hat{D}_i(\eta(t))J_j].$$

The aim of this paper is to design a fault estimator of the form (8) such that the following inequality holds:

For (9) with its zero state response $(x(\phi) = 0, \omega(\phi) = 0, -\chi \le \phi \le 0$,

$$\mathbf{E}\left[\int_0^{T_f} e^T(t)e(t)\,dt\right] \le \gamma^2 \mathbf{E}\left[\int_0^{T_f} \sup_{-x\le\phi\le 0} \omega^T(t+\phi)\omega(t+\phi)dt\right] \tag{10}$$

for any nonzero $\omega(t) \in \mathcal{L}_2[0, T_f]$ and $T_f \ge 0$, provided $x = \{x(\xi) : t - 2\chi \le \xi \le t\} \in L^2_{\mathcal{F}_t}([-2\chi, 0]; \mathbb{R}^n)$ satisfying:

$$\mathbf{E}\left[\min_{\eta(t)\in\mathcal{S}} V(x(\xi), \eta(\xi), \xi\right] < \delta\mathbf{E}\left[\max_{\eta(t)\in\mathcal{S}} V(x(t), \eta(t), t\right] \tag{11}$$

for all $t - 2\chi \le \xi \le t$, then a fault estimator is designed satisfying a disturbance attenuation level γ.

In this paper, we assume $u(t) = 0$ before the first control signal reaches the plant. From here, $\mu_i(v(t))$ and $\mu_j(v(t))$ are denoted as μ_i and μ_j respectively for the convenience of notations. In the symmetric block matrices, we use (*) as an ellipsis for terms that are induced by symmetry. $\hat{A}ij(\eta(t))$ is denoted as $\hat{A}\,ij(\iota)$ if $\eta(t) = \iota$.

3. MAIN RESULT

The following theorem provides sufficient conditions for the existence of a mode-dependent fault estimator for the system (9) that guarantees disturbance attenuation level γ.

Theorem 3.1. *Consider the system (9) satisfying Assumption 2.1. For given positive delay-free attenuation constant γ_{d_f}, positive constants $\tau^*(\iota)$, $\varepsilon_{1_{ij\iota}}$, $\varepsilon_{2_{ij\iota}}$, ε_{3ij_ι}, and $\varepsilon_{4_{ij\iota}}$, if there exist symmetric positive matrices $X(\iota)$, $Y(\iota)$, R_{1_ι}, R_{2_ι}, R_{3_ι}, and R_{4_ι}, and matrices $F_i(\iota)$, $L_i(\iota)$, and $\hat{D}_i(\iota)$, and positive scalars β_{1_ι}, β_{2_ι}, such that the following inequalities hold where $\iota \in \mathcal{S}$:*

$$\begin{bmatrix} Y(\iota) & I \\ I & X(\iota) \end{bmatrix} > 0, \tag{12}$$

$$\Upsilon_{ii}(\iota) < 0, \ for\ i \in \mathcal{I}_R \tag{13}$$

$$\Upsilon_{ij}(\iota) + \Upsilon_{ji}(\iota) < 0, \ for\ i < j < r \tag{14}$$

$$\Phi_{ij}(\iota) < 0, \ for\ \{i, j\} \in \mathcal{I}_R \times \mathcal{I}_R \tag{15}$$

$$\begin{bmatrix} R_{4_\iota} & (*)^T \\ \Lambda_\iota T & \mathcal{Q}(\iota) \end{bmatrix} > 0, \tag{16}$$

$$\begin{bmatrix} -R_{1_\iota} & (*)^T & (*)^T & (*)^T \\ 0 & -I & (*)^T & (*)^T \\ 0 & -Y(\iota) & -R_{2_\iota} & (*)^T \\ 0 & 0 & 0 & -R_{3_\iota} \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} -\beta_{2_\iota} Y(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ -\beta_{2_\iota} I & -\beta_{2_\iota} X(\iota) & (*)^T & (*)^T & (*)^T & (*)^T \\ 0 & Y(\iota)C_i^T F_i^T(\iota) & -Y(\iota) & (*)^T & (*)^T & (*)^T \\ 0 & C_i^T F_i^T(\iota) & -I & -X(\iota) & (*)^T & (*)^T \\ 0 & \varepsilon_{4ij_\iota} H_{2j}^T F_i^T(\iota) & 0 & 0 & -\varepsilon_{4ij_\iota} I & (*)^T \\ 0 & 0 & E_j Y(\iota) & E_j & 0 & -\varepsilon_{4ij_\iota} I \end{bmatrix} < 0, \ ) \tag{18}$$

for $\{i, j\} \in \mathcal{I}_R \times \mathcal{I}_R$

$$\begin{bmatrix} -Y(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ -I & -X(\iota) & (*)^T & (*)^T & (*)^T & (*)^T \\ B_i^T & B_i^T X(\iota) & -I & (*)^T & (*)^T & (*)^T \\ G_i^T & G_i^T X(\iota) & 0 & -I & (*)^T & (*)^T \\ 0 & D_i^T F_i^T(\iota) & 0 & 0 & -I & (*)^T \\ 0 & J_i^T F_i^T(\iota) & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{19}$$

for $\{i, j\} \in \mathcal{I}_R \times \mathcal{I}_R$

where $\Phi_{ij}(\iota)$ and $\Upsilon_{ij}(\iota)$ are represented at the next page and

$$Z(\iota) = [\sqrt{\lambda_{\iota 1}} Y(\iota) \cdots \sqrt{\lambda_{\iota(\iota-1)}} Y(\iota)$$
$$\sqrt{\lambda_{\iota(\iota-1)}} Y(\iota) \cdots \sqrt{\lambda_{\iota 1}} Y(\iota)],$$
$$\Lambda_\iota = [\sqrt{\lambda_{\iota 1}} I \cdots \sqrt{\lambda_{\iota(\iota-1)}} I \sqrt{\lambda_{\iota(\iota+1)}} I \cdots \sqrt{\lambda_{\iota\mathcal{S}}} I],$$
$$Q(\iota) = diag\{Y(1), \cdots Y(\iota-1), Y(\iota+1), \cdots Y(s)\},$$
$$\Xi_{1i}(\iota) = A_i Y(\iota) + Y(\iota) A_i^T + (\beta_{1_\iota} + 4\beta_{2_\iota})\tau^*(\iota) Y(\iota) + \lambda_{\iota\iota} Y(\iota),$$
$$\Xi_{2ij}(\iota) = X(\iota) A_i + A_i^T X(\iota) + F_i(\iota) C_j + C_j^T F_i^T(\iota) + (\beta_{1_\iota} + 4\beta_{2_\iota})\tau^*(\iota) X(\iota) + \sum_{\jmath=1}^{s} \lambda_{\iota\jmath} X(\jmath),$$

then (9) *holds for delay $\tau(\iota, t)$ satisfying $\tau(\iota, t) \le \tau^*(\iota)$* with $\gamma^2 = \gamma_{d_f} + \max(\tau^*(\iota))$ *for $\iota \in \mathcal{S}$. Furthermore, the mode dependant fault estimator is obtained of the form* (8) *with*

$$\hat{A}_{ij}(\iota) = [Y^{-1}(\iota) - X(\iota)]^{-1}[-A_i^T - X(\iota) A_i Y(\iota) - F_i(\iota) C_j Y(\iota) - \sum_{\jmath=1}^{\mathcal{S}} \lambda_{\iota\jmath} Y^{-1}(\jmath) Y(\iota)] Y^{-1}(\iota), \tag{20}$$

$$\hat{B}_i(\iota) = [Y^{-1}(\iota) - X(\iota)]^{-1} F_i(\iota), \tag{21}$$

$$\hat{C}_i(\iota) = L_i(\iota) Y^{-1}(\iota). \tag{22}$$

Proof: Note that for each $\eta(t) = \iota \in S$ for the system (9) at time t, it follows from Leibniz-Newton Formula

$$\Upsilon_{ij}(\iota)=\begin{bmatrix}
\Xi_{1i}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
(\beta_{1_\iota}+4\beta_{2_\iota})\tau^*(\iota)I & \Xi_{2ij}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
0 & 0 & -I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
0 & 0 & 0 & -I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
B_i^T & B_i^T X(\iota) & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
G_i^T & G_i^T X(\iota) & 0 & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
0 & D_j^T F_i(\iota) & 0 & 0 & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
0 & J_j^T F_i(\iota) & 0 & 0 & 0 & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
L(\iota) & 0 & \hat{D}_i(\iota)C_j & 0 & 0 & -I & \hat{D}_i(\iota)D_j & \hat{D}_i(\iota)J_j & -I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\
Z^T(\iota) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{Q}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T \\
\varepsilon_{1ij_\iota}H_{1i}^T & \varepsilon_{1ij_\iota}H_{1i}^T X^T(\iota)+H_{2j}^T F_i^T(\iota)) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij_\iota}I & (*)^T & (*)^T & (*)^T \\
E_iY(\iota) & E_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij_\iota}I & (*)^T & (*)^T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_{2ij_\iota}H_{2j}^{T_o}\hat{D}_i^T(\iota) & 0 & 0 & 0 & -\varepsilon_{2ij_\iota}I & (*)^T \\
0 & 0 & E_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2ij_\iota}I
\end{bmatrix}$$

$$\begin{aligned}
&\hat{x}(t-\tau(\iota,t)) \\
&=\hat{x}(t)-\int_{-\tau(\iota,t)}^{0}\dot{\hat{x}}(t+\theta)\,d\theta \\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big\{\tilde{x}(t)-\int_{-\tau(\iota,t)}^{0}[\mathcal{A}_{ij}(\iota)\tilde{x}(t+\theta) \\
&+\mathcal{B}_{ij}(\iota)\tilde{x}(t-\tau(\iota,t)+\theta)+\mathcal{C}_{ij}(\iota)\omega(t+\theta)]d\theta\Big\}
\end{aligned}$$

Then the closed-loop system (9) can be rewritten as:

$$\begin{aligned}
\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r}\mu_i\mu_j\mu_k\mu_l\Big\{\varepsilon_{ij}\tilde{x}(t)=\mathcal{B}_{ij}\int_{-\tau(\iota,t)}^{0}[\mathcal{A}_{kl}\tilde{x}(t+\theta) \\
+\mathcal{B}_{kl}\tilde{x}(t-\tau(\iota,t)+\theta)+\mathcal{C}_{kl}\omega(t+\theta)]d\theta+\mathcal{C}_{ij}\omega(t)\Big\}
\end{aligned} \tag{23}$$

where $\tau(\iota,\ t)$ is constant and $\mathcal{E}_{ij}$ = $\mathcal{A}_{ij}$ + $\mathcal{B}_{ij}$ for the sake of simplification of notation.

Select a stochastic Lyapunov function candidate as

$$V(\tilde{x}(t),\eta(t),t)=\tilde{x}^T(t)P(\eta(t))\tilde{x}(t) \tag{24}$$

where $P(\eta(t))$ is the positive constant symmetric matrix for each $\eta(t)=\iota\in\mathcal{S}$. It follows

$$\alpha_1\left\|\tilde{x}(t)\right\|^2\le V(\tilde{x}(t),\eta(t),t)\le\alpha_2\left\|\tilde{x}(t)\right\|^2 \tag{25}$$

where $\alpha_1=\lambda_{min}(P(\eta(t)))$ and $\alpha_2=\lambda_{max}(P(\eta(t)))$.

The weak infinitesimal operator $\tilde{A}$ can be considered as the derivative of the function of $V(\tilde{x}(t),\eta(t),t)$ along the trajectory of the joint Markov process $\{\tilde{x}(t),\eta(t),t\ge 0\}$ at the point $\{\tilde{x}(t),\eta(t)=\iota\}$ at time t; see [24] and [34].

$$\begin{aligned}&\tilde{A}V(\tilde{x}(t),\eta(t),t)\\&=\frac{\partial V(\cdot)}{\partial t}+\dot{\tilde{x}}^T(t)\frac{\partial V(\cdot)}{\partial\tilde{x}}\Big|_{\eta=\iota}+\sum_{j=1}^{\mathcal{S}}\lambda_{\iota j}V(\tilde{x}(t),j,t)\end{aligned} \tag{26}$$

Then we can get (27) at the next page following (26).

$$\mathcal{A}_{kl}P^{-1}(\iota)\mathcal{A}_{kl}^T<\beta_{1_\iota}P^{-1}(\iota) \tag{28}$$

$$\mathcal{B}_{kl}P^{-1}(\iota)\mathcal{B}_{kl}^T<\beta_{2_\iota}P^{-1}(\iota) \tag{29}$$

$$\mathcal{C}_{kl}\mathcal{C}_{kl}^T<P^{-1}(\iota) \tag{30}$$

Then (27) becomes:

$$\begin{aligned}&\tilde{A}V(\tilde{x}(t),\eta(t),t)\\
&\le\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big\{\tilde{x}^T(t)[\varepsilon_{ij}^TP(\iota)+P(\iota)\varepsilon_{ij}]\tilde{x}(t)\\
&+\tilde{x}^T(t)P(\iota)\mathcal{C}_{ij}\omega(t)+\omega^T(t)\mathcal{C}_{ij}^TP(\iota)\tilde{x}(t)\\
&\sum_{j=1}^{\mathcal{S}}\lambda_{\iota j}\tilde{x}^T(t)P(j)\tilde{x}(t)+\tau(\iota,t)[3\beta_{2_\iota}\tilde{x}^T(t)P(\iota)\tilde{x}(t)\\
&+\beta_{1_\iota}\tilde{x}^T(t+\theta)P(\iota)\tilde{x}(t+\theta)\\
&+\beta_{2_\iota}\tilde{x}^T(t-\tau(\iota,t)\\
&+\theta)P(\iota)\tilde{x}(t-\tau(\iota,t)+\omega^T(t+\theta)\omega(t+\theta)]\Big\}\\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big\{\tilde{x}^T(t)[\varepsilon_{ij}^TP(\iota)+P(\iota)\varepsilon_{ij}+3\tau(\iota,t)\beta_{2_\iota}P(\iota)\\
&+\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})P(\iota)+\sum_{j=1}^{\mathcal{S}}\lambda_{\iota j}P(j)]\tilde{x}(t)\\
&+\tilde{x}^T(t)P(\iota)\mathcal{C}_{ij}\omega(t)+\omega^T(t)\mathcal{C}_{ij}^TP(\iota)\tilde{x}(t)\\
&+\tau(\iota,t)[\beta_{1_\iota}\tilde{x}^T(t+\theta)P(\iota)\tilde{x}(t+\theta)\\
&+\beta_{2_\iota}\tilde{x}^T(t-\tau(\iota,t)+\theta)P(\iota)\tilde{x}(t-\tau(\iota,t)+\theta)\\
&+\omega^T(t+\theta)\omega(t+\theta)]-\tilde{x}^T(t)\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})P(\iota)\tilde{x}(t)\Big\}\end{aligned} \tag{31}$$

Furthermore, by adding and subtracting $-e^T(t)e(t)+\gamma_{d_f}\omega^T(t)\omega(t)$ to and from (31), we can get:

$$\begin{aligned}&\tilde{A}V(\tilde{x}(t),\eta(t),t)\\
&\le\tilde{x}_e^T(t)\mathcal{M}_\iota(\tau(\iota,t),\delta)\tilde{x}_e(t)+\tilde{x}^T(t-\tau(\iota,t))\tilde{x}(t-\tau(\iota,t))\\
&-e^T(t)e(t)+\gamma_{d_f}\omega^T(t)\omega(t)\\
&+\tau(\iota,t)[\beta_{1_\iota}\tilde{x}^T(t+\theta)P(\iota)\tilde{x}(t+\theta)\\
&+\beta_{2_\iota}\tilde{x}^T(t-\tau(\iota,t)+\theta)P(\iota)\tilde{x}(t-\tau(\iota,t)+\theta)\\
&+\omega^T(t+\theta)\omega(t+\theta)]-\tilde{x}^T(t)\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})P(\iota)\tilde{x}(t)\end{aligned}$$

where $\tilde{x}_e(t)=[\tilde{x}^T(t)\quad\tilde{x}^T(t-\tau(\iota,t))\quad\omega^T(t)]^T$, is given by:

$$\begin{aligned}&\mathcal{M}_\iota(\tau(\iota,t),\delta)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Bigg\{\\
&\begin{bmatrix}\begin{pmatrix}\varepsilon_{ij}^TP(\iota)+P(\iota)\varepsilon_{ij}^T\\+3\tau(\iota,t)\beta_{2_\iota}P(\iota)\\+\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})P(\iota)\\+\sum_{j=1}^{\mathcal{S}}\lambda_{\iota j}P(j)\end{pmatrix}&(*)^T&(*)^T\\0&-I&(*)^T\\\mathcal{C}_{ij}^TP(\iota)&0&-\gamma_{d_f}I\end{bmatrix}\\
&+\begin{bmatrix}\mathcal{D}_{1_i}^T\\\mathcal{D}_{2_{ij}}^T\\\mathcal{D}_{3_{ij}}^T\end{bmatrix}\begin{bmatrix}\mathcal{D}_{1_i}&\mathcal{D}_{2_{ij}}&\mathcal{D}_{3_{ij}}\end{bmatrix}\Bigg\}\end{aligned}$$

We denote

$$\begin{aligned}&\nabla_\iota(i,j)=\begin{bmatrix}\begin{pmatrix}\varepsilon_{ij}^TP(\iota)+P(\iota)\varepsilon_{ij}^T\\+3\tau(\iota,t)\beta_{2_\iota}P(\iota)\\+\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})P(\iota)\\+\sum_{j=1}^{\mathcal{S}}\lambda_{\iota j}P(j)\end{pmatrix}&(*)^T&(*)^T\\0&-I&(*)^T\\\mathcal{C}_{ij}^TP(\iota)&0&-\gamma_{d_f}I\end{bmatrix},\\
&\bar{\mathcal{D}}^T(i,j)=\begin{bmatrix}\mathcal{D}_{1_i}^T\\\mathcal{D}_{2_{ij}}^T\\\mathcal{D}_{3_{ij}}^T\end{bmatrix}\end{aligned} \tag{32}$$

Then

$$\begin{aligned}&\mathcal{M}_\iota(\tau(\iota,t),\delta)\\
&=\sum_{i=1}^{r}\mu_i^2\left[\nabla_\iota(i,i)+\bar{\mathcal{D}}^T(i,i)\bar{\mathcal{D}}(i,i)\right]\\
&+2\sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\mu_j\frac{1}{2}\left[\nabla_\iota(i,j)+\bar{\mathcal{D}}^T(i,j)\bar{\mathcal{D}}(i,j)\right]\end{aligned} \tag{33}$$

In this paper the time delays are assumed to be bounded, hence $\tau(\iota,t)$ can also be assumed to be bounded, that is, $\tau(\iota,t)\le\tau^*(\iota)$, where $\tau^*(\iota)$ is the constant given in the theorem. Using this fact, we learn that:

$$\mathcal{M}_\iota(\tau(\iota,t),\delta)\le\mathcal{M}_\iota(\tau^*(\iota),\delta)$$

Hence, if (13) and (14) hold, it can be shown later that $\mathcal{M}_\iota(\tau^*(\iota),\delta)<0$ for $\delta=1$. Then we get

$$\begin{aligned}&\tilde{A}V(\tilde{x}(t),\eta(t),t)\\
&<-\sigma\tilde{x}_e^T(t)\tilde{x}_e(t)+\tilde{x}^T(t-\tau(\iota,t))\tilde{x}(t-\tau(\iota,t))-e^T(t)e(t)\\
&+\gamma_{d_f}\omega^T(t)\omega(t)+\tau(\iota,t)[\beta_{1_\iota}\tilde{x}^T(t+\theta)P(\iota)\tilde{x}(t+\theta)\\
&+\beta_{2_\iota}\tilde{x}^T(t-\tau(\iota,t)+\theta)P(\iota)\tilde{x}(t-\tau(\iota,t)+\theta)\\
&+\omega^T(t+\theta)\omega(t+\theta)]-\tilde{x}^T(t)\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})P(\iota)\tilde{x}(t)\end{aligned} \tag{34}$$

$$\begin{aligned}
&\tilde{A}V(\tilde{x}(t),\eta(t),t)\\
&\dot{\tilde{x}}(t)P(\iota)\tilde{x}(t)+\tilde{x}^T(t)P(\iota)\dot{\tilde{x}}(t)+\sum_{\jmath=1}^{\mathcal{S}}\lambda_{\iota\jmath}\tilde{x}^T(t)P(\jmath)\tilde{x}(t)\\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r}\mu_i\mu_j\mu_k\mu_l\Big\{\tilde{x}^T(t)[\varepsilon_{ij}^T P(\iota)+P(\iota)\varepsilon_{ij}]\tilde{x}(t)\tilde{x}^T(t)P(\iota)\mathcal{C}_{ij}\omega(t)+\omega^T(t)\mathcal{C}_{ij}^T P(\iota)\tilde{x}(t)\\
&\quad+\sum_{\jmath=1}^{\mathcal{S}}\lambda_{\iota\jmath}\tilde{x}^T(t)P(\jmath)\tilde{x}(t)-2\int_{-\tau(\iota,t)}^{0}\tilde{x}^T(t)P(\iota)\mathcal{B}_{ij}[\mathcal{A}_{kl}\tilde{x}(t+\theta)+\mathcal{B}_{kl}\tilde{x}(t-\tau(\iota,t)+\theta)+\mathcal{C}_{kl}\omega(t+\theta)]d\theta\Big\}\\
&\le\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r}\mu_i\mu_j\mu_k\mu_l\Big\{\tilde{x}^T(t)[\varepsilon_{ij}^T P(\iota)+P(\iota)\varepsilon_{ij}]\tilde{x}(t)\tilde{x}^T(t)P(\iota)\mathcal{C}_{ij}\omega(t)+\omega^T(t)\mathcal{C}_{ij}^T P(\iota)\tilde{x}(t)\\
&\quad+\sum_{\jmath=1}^{\mathcal{S}}\lambda_{\iota\jmath}\tilde{x}^T(t)P(\jmath)\tilde{x}(t)+\tau(\iota,t)[\frac{1}{\beta_{1_\iota}}\tilde{x}^T(t)P(\iota)\mathcal{B}_{ij}\mathcal{A}_{kl}P^{-1}(\iota)\mathcal{A}_{kl}^T\mathcal{B}_{ij}^T P(\iota)\tilde{x}(t)+\beta_{1_\iota}\tilde{x}^T(t+\theta)P(\iota)\tilde{x}(t+\theta)\\
&\quad+\frac{1}{\beta_{2_\iota}}\tilde{x}^T(t)P(\iota)\mathcal{B}_{ij}\mathcal{B}_{kl}P^{-1}(\,)\mathcal{B}_{kl}^T\mathcal{B}_{ij}^T P(\iota)\tilde{x}(t)+\beta_{2_\iota}\tilde{x}^T(t-\tau(\iota,t)+\theta)P(\iota)\tilde{x}(t-\tau(\iota,t)+\theta)\\
&+\tilde{x}^T(t)P(\iota)\mathcal{B}_{ij}\mathcal{C}_{kl}\mathcal{C}_{kl}^T\mathcal{B}_{ij}^T P(\iota)\tilde{x}(t)+\omega^T(t+\theta)\omega(t+\theta)]
\end{aligned}\tag{27}$$

where

$$\sigma=\min\{\lambda_{min}(-\mathcal{M}_\iota(\tau^*(\iota),1))\}.$$

It is easy to see that $\sigma > 0$.

In this paper, we assume that for all $\psi \in [-\tau(\iota, t), 0]$, a scalar $\varepsilon > 0$ exists such that

$$\|\tilde{x}(t+\psi)\|\le\epsilon\|\tilde{x}(t)\|. \tag{35}$$

It can be noted from [35] that (35) is not restrictive since we allow ε to be any value, greater or smaller than 1. In the sequel, therefore, we assume there exists $\varepsilon < \alpha$. Hence, by Dynkin's formula [27], (34) becomes:

$$\begin{aligned}
&\mathbf{E}\{V(x(t),\eta(t),t)\}-\mathbf{E}\{V(x(0),\eta(0),0)\}\\
&\mathbf{E}\{\int_0^{T_f}e^T(t)e(t)\,dt\}\\
&+\gamma\mathbf{E}\{\int_0^{T_1}\sup_{-x\le\phi\le0}\omega^T(t+\phi)\omega(t+\phi)dt\}\\
&+\tau(\iota,t)\Big[\beta_{1_\iota}\mathbf{E}\{\int_0^{T_f}\tilde{x}^T(t+\theta)P(\iota)\tilde{x}(t+\theta)dt\}\\
&\beta_{2_\iota}\mathbf{E}\{\int_0^{T_f}\tilde{x}^T(t-\tau(\iota,t)+\theta)P(\iota)\times\tilde{x}(t-\tau(\iota,t)+\theta)dt\}\Big]\\
&-\tau(\iota,t)\delta(\beta_{1_\iota}+\beta_{2_\iota})\mathbf{E}\{\int_0^{T_f}\tilde{x}^T(t)P(\iota)\tilde{x}(t)dt\}
\end{aligned}$$

with $\gamma = \max(\tau^*(\iota)) + \gamma_{d_f}$ and $\chi = \max(\tau^*(\iota))$.

Applying the Razumikhin-type theorem for stochastic systems [36], we assume that for any $\delta > 1$, the following inequality holds:

$$\mathbf{E}\Big[\min_{\eta(t)\in\mathcal{S}}V(x(\xi),\eta(\xi),\xi)\Big]<\delta\mathbf{E}\Big[\{\max_{\eta(t)\in\mathcal{S}}V(x(t),\eta(t),t)\Big] \tag{36}$$

Using the fact that $\tilde{x}(0) = 0$and $V(\tilde{x}(T_f)) \ge 0$ for all $T_f \ne 0$ and bearing in mind the assumption that $\tau(\iota, t) \le \tau^*(\iota)$, we have:

$$\begin{aligned}
&\mathbf{E}\{\int_0^{T_f}e^T(t)e(t)\,dt\}\\
&\le\gamma\mathbf{E}\{\int_0^{T_1}\sup_{-x\le\phi\le0}\omega^T(t+\phi)\omega(t+\phi)dt\}
\end{aligned}$$

This satisfies the conditions set in Definition 2.1 and we can say the system (9) has a disturbance attenuation level γ.

Hereinafter, we will show that (13) and (14) guarantees $\mathcal{M}_\iota(\tau^*(\iota),1)<0.$

Applying Schur complement to $\mathcal{M}_\iota(\tau^*(\iota),1)<0,$ we can have:

$$\begin{bmatrix}
\begin{pmatrix}\varepsilon_{ij}^T P(\iota)+P(\iota)\varepsilon_{ij}\\ +\tau^*(\iota)\beta_{1_\iota}+4\beta_{2_\iota}P(\iota)\\ +\sum_{\jmath=1}^{\mathcal{S}}\lambda_{\iota\jmath}P(\jmath)\end{pmatrix} & (*)^T & (*)^T & (*)^T\\
0 & -I & (*)^T & (*)^T\\
\mathcal{C}_{ij}^T P(\iota) & 0 & -\gamma_{d_f}I & (*)^T\\
\mathcal{D}_{1_i} & \mathcal{D}_{2_{ij}} & \mathcal{D}_{3_{ij}} & -I
\end{bmatrix}<0 \tag{37}$$

Using the partition $P(\iota)=\begin{bmatrix}X(\iota) & Y^{-1}(\iota)-X(\iota)\\ Y^{-1}(\iota)-X(\iota) & X(\iota)-Y^{-1}(\iota)\end{bmatrix},$

multiplying (37) to the left by $\vee_\iota$ and to the right by $\vee_\iota^T$

$$\begin{bmatrix} \Xi_{1i}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ (\beta_{1_\iota}+4\beta_{2_\iota})\tau^*(\iota)I & \Xi_{2i}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ 0 & 0 & -I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ 0 & 0 & 0 & -I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ B_i^T & B_i^T X(\iota) & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ G_i^T & G_i^T X(\iota) & 0 & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T & (*)^T \\ 0 & D_j^T F_i(\iota) & 0 & 0 & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T & (*)^T \\ 0 & J_j^T F_i(\iota) & 0 & 0 & 0 & 0 & 0 & -\gamma d_f I & (*)^T & (*)^T \\ L(\iota) & 0 & \hat{D}_i(\iota)C_j & 0 & 0 & -I & \hat{D}_i(\iota)D_j & \hat{D}_i(\iota)J_j & -I & (*)^T \\ Z^T(\iota) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q(\iota) \end{bmatrix}$$

$$+\begin{bmatrix} H_{1i} \\ X(\iota)H_{1i}+F_i(\iota)H_{2j} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F(t)\left[E_iY(\iota)\ E_i\ 0\,0\,0\,0\,0\,0\,0\,0\right]+\left[E_iY(\iota)\ E_i\ 0\,0\,0\,0\,0\,0\,0\,0\right]^T F^T(t)\begin{bmatrix} H_{1i} \\ X(\iota)H_{1i}+F_i(\iota)H_{2j} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \tag{38}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hat{D}_i(\iota)H_{2j} \\ 0 \end{bmatrix} F(t)\left[0\,0\ E_i\ 0\,0\,0\,0\,0\,0\,0\right]+\left[0\,0\ E_i\ 0\,0\,0\,0\,0\,0\,0\,0\right]^T F^T(t)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hat{D}_i(\iota)H_{2j} \\ 0 \end{bmatrix}^T$$

$$< 0.$$

where $\vee_\iota = \begin{bmatrix} J_\iota^T & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$ with $J_\iota = \begin{bmatrix} Y(\iota) & I \\ Y(\iota) & 0 \end{bmatrix}$, using Assumption 2.1 and Schur complement, and applying the controllers defined as in (20)-(22) yields (38) at the same page.

Using Lemma 2.1, it is easy to see that (13) guarantees the existence of (38), which infers $\mathcal{M}_\iota(\tau^*(\iota),1) < 0$. Using the continuity property of the eigenvalues of $\mathcal{M}_\iota(\cdot,\cdot)$ with respect to δ, there exists a sufficiently small $\varepsilon > 0$ such that $\mathcal{M}_{\iota}(\tau^*(\iota), 1+\epsilon) < 0$. Hence, there exists a $\delta > 1$ such that $\mathcal{M}_{\iota}(\tau^*(\iota), \delta) < 0$ still holds.

Next, it will be shown that (15)-(19) are derived from (28)-(30).

Firstly, the inequality (28) can be rewritten as follows by applying Schur complement:

$$\begin{bmatrix} -\beta_{1_\iota} P^{-1}(\iota) & \mathcal{A}_{ij} \\ \mathcal{A}_{ij}^T & -P(\iota) \end{bmatrix} < 0 \tag{39}$$

Using Assumption 2.1, multiplying (39) to the left by $\begin{bmatrix} J_\iota^T P(\iota) & 0 \\ 0 & J_\iota^T \end{bmatrix}$, and to the right by $\begin{bmatrix} P(\iota) J_\iota & 0 \\ 0 & J_\iota \end{bmatrix}$, and using the controllers defined as (20)-(22) yields:

$$\begin{bmatrix} -\beta_{1_\iota} Y(\iota) & (*)^T & (*)^T & (*)^T \\ -\beta_{1_\iota} I & -\beta_{1_\iota} X(\iota) & (*)^T & (*)^T \\ Y(\iota) A_i^T & \begin{pmatrix} -A_i - Y(\iota) C_j^T F_i^T(\iota) \\ -\sum_{j=1}^{\mathcal{S}} \lambda_{\iota j} Y(\iota) Y^{-1}(j) \end{pmatrix} & -Y(\iota) & (*)^T \\ A_i^T & A_i^T X(\iota) & -I & -X(\iota) \end{bmatrix} + \begin{bmatrix} H_{1i} \\ X(\iota) H_{1i} \\ 0 \\ 0 \end{bmatrix} F(\iota) \begin{bmatrix} 0 & 0 & E_i Y(\iota) & E_i \end{bmatrix} + \begin{bmatrix} 0 & 0 & E_i Y(\iota) & E_i \end{bmatrix}^T F^T(\iota) \begin{bmatrix} H_{1i} \\ X(\iota) H_{1i} \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{40}$$

To address the term containing $-\sum_{j=1}^{\mathcal{S}} \lambda_{\iota j} Y(\iota) Y^{-1}(j)$ we first rewrite (40) into the following equivalent form:

$$\begin{bmatrix} -R_{1_\iota} & (*)^T & (*)^T & (*)^T \\ 0 & \begin{pmatrix} -(\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j)) \\ \times(\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j)) \end{pmatrix} & (*)^T & (*)^T \\ 0 & -\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j) & -R_{2_\iota} & (*)^T \\ 0 & 0 & 0 & -R_{3_\iota} \end{bmatrix} + \begin{bmatrix} H_{1i} \\ X(\iota) H_{1i} \\ 0 \\ 0 \end{bmatrix} F(\iota) \begin{bmatrix} 0 & 0 & E_i Y(\iota) & E_i \end{bmatrix} + \begin{bmatrix} 0 & 0 & E_i Y(\iota) & E_i \end{bmatrix}^T F^T(\iota) \begin{bmatrix} H_{1i} \\ X(\iota) H_{1i} \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{41}$$

On the left hand side of (41), if the second term is less than zero, we get:

$$\begin{bmatrix} -\beta_{1_\iota} Y(\iota) + R_{1_\iota} & (*)^T & (*)^T & (*)^T \\ -\beta_{1_\iota} I & \begin{pmatrix} -\beta_{1_\iota} X(\iota) \\ +(\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j)) \\ \times(\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j)) \end{pmatrix} & (*)^T & (*)^T \\ Y(\iota) A_i^T & -A_i - Y(\iota) C_j^T F_i^T(\iota) - \lambda_{\iota\iota} I & -Y(\iota) + R_{2_\iota} & (*)^T \\ A_i^T & A_i^T X(\iota) & -I & -X(\iota) + R_{3_\iota} \end{bmatrix} + \begin{bmatrix} H_{1i} \\ X(\iota) H_{1i} \\ 0 \\ 0 \end{bmatrix} F(\iota) \begin{bmatrix} 0 & 0 & E_i Y(\iota) & E_i \end{bmatrix} + \begin{bmatrix} 0 & 0 & E_i Y(\iota) & E_i \end{bmatrix}^T F^T(\iota) \begin{bmatrix} H_{1i} \\ X(\iota) H_{1i} \\ 0 \\ 0 \end{bmatrix}^T < 0 \tag{42}$$

By defining new variables R_{4_ι} and using (16), we get

$$R_{4_\iota} R_{4_\iota} > \left[\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j) \right] \left[\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j) \right]$$

$R_{4_\iota} > \lambda_{\iota j} Y^{-1}(j)$, which also implies that

Therefore, by applying Lemma 2.1 and Schur complement, it is not hard to see that if (15) holds, (42) is guaranteed and (28) is thereby satisfied.

Furthermore, we address the negativeness of the second term on the left hand side of (41). Firstly, we want the second term is less than zero, that is:

$$\begin{bmatrix} -R_{1_\iota} & (*)^T & (*)^T & (*)^T \\ 0 & \begin{pmatrix} -(\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j)) \\ \times(\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j)) \end{pmatrix} & (*)^T & (*)^T \\ 0 & -\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j) & -R_{2_\iota} & (*)^T \\ 0 & 0 & 0 & -R_{3_\iota} \end{bmatrix} < 0 \tag{43}$$

By multiplying (43) both sides by

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & (\sum_{j=1, j\neq\iota}^{\mathcal{S}} \lambda_{\iota j} Y^{-1}(j))^{-1} & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

we can see that if there exists (17) , (43) holds. It is straightforward to obtain that the third term is negative as well if (17) holds.

(18)-(19) can be derived from (29)-(30) using the same procedure.

Besides, $P(\iota) > 0$ is equivalent to

$$J_{\iota}^T P(\iota) J_{\iota} = \begin{bmatrix} Y(\iota) & I \\ I & X(\iota) \end{bmatrix} > 0 \quad (44)$$

We therefore have the inequality condition (12).

This completes the proof.

It should be noted that terms $Y(\iota)C_j^T F_i^T(\iota), \beta_{1_\iota} X(\iota)$ and $\beta_{1_\iota} Y(\iota)$ in (13)-(19) are not convex constraints, which are difficult to solve. We therefore propose the following algorithm to change this non-convex feasibility problem into quasi-convex optimization problems [37].

Iterative linear matrix inequality (*ILMI*) *algorithm*

Step 1. Find $X(\iota), Y(\iota), \hat{D}_i(\iota), F_i(\iota)$ and $L_i(\iota)$ subject to (12) and (13) with $\tau^*(\iota) = 0$.

Let $n = 1$ and $X_n(\iota) = X(\iota)$ and $Y_n(\iota) = Y(\iota)$.

Step 2. Solve the following optimization problem for $\alpha_n \hat{D}_i(\iota), F_i(\iota)$ and $L_i(\iota)$ with the given $\tau^*(\iota)$ and $X_n(\iota)$ and $Y_n(\iota)$ obtained in the previous step:

OP1: Minimize α_n subject to the following LMI constraints:

$$\text{Left hand-side of (13)} \; -\alpha_n \left[\begin{array}{cc|c} Y_n(\iota) & I & 0 \\ I & X_n(\iota) & \\ \hline 0 & & 0 \end{array} \right] < 0 \quad (45)$$

and (12), (15)-(19).

Step 3. If $\alpha_n < 0$, $X_n(\iota)$, $Y_n(\iota)$ and $\hat{D}_i(\iota)$, $F_i(\iota)$, and $L_i(\iota)$ are a feasible solution to the BMIs and stop.

Step 4. Set $n = n + 1$. Solve the following optimization problem for α_n, $X_n(\iota)$ and $Y_n(\iota)$ with $\hat{D}_i(\iota)$, $F_i(\iota)$, and $L_i(\iota)$ obtained in the previous step:

OP2: Minimize α_n subject to LMI constraints (45), (12), and (15)-(19).

Step 5. If $\alpha_n < 0$, $X_n(\iota)$, $Y_n(\iota)$ and $\hat{D}_i(\iota)$, $F_i(\iota)$, and $L_i(\iota)$ are a feasible solution to the BMIs and stop.

Step 6. Set $n = n + 1$. Solve the following optimization problem for $X_n(\iota)$ and $Y_n(\iota)$ with α_n, $\hat{D}_i(\iota)$, $F_i(\iota)$, and $L_i(\iota)$ obtained in the previous step:

OP3: Minimize trace ($\begin{bmatrix} Y_n(\iota) & I \\ I & X_n(\iota) \end{bmatrix}$) subject to $IX_n(\iota)$ LMI constraints (45), (12), and (15)-(19).

Step 7. Let $T_n = T_n = \begin{bmatrix} Y_n(\iota) & I \\ I & X_n(\iota) \end{bmatrix}$. If $\|T_n - T_{n-1}\| / \| T_n \| < \zeta$, ζ is a prescribed tolerance, go to Step 8. Else, set $n = n + 1$, $X_n(\iota) = X_{n-1}(\iota)$ and $Y_n(\iota) = Y_{n-1}(\iota)$, then go to Step 2.

Step 8. A fault estimator for the system may not be found, stop.

Remark 3.1.

(1) *In Step 1, the initial data is obtained by assuming that the system has no time delay.*

(2) *A term* $-\alpha_n \left[\begin{array}{cc|c} Y_n(\iota) & I & 0 \\ I & X_n(\iota) & \\ \hline 0 & & 0 \end{array} \right]$ *is introduced in* (13) *to relax the LMI constraints. It is referred as* α/2-*stabilizable problem in* [38]. *If an* $\alpha_n < 0$ *can be found, the robust fault estimator can be obtained. The rationale behind this concept can also be found in* [39].

(3) *The optimization problem in Step 2 and Step 4 is a generalized eigenvalue minimization problem. These two steps guarantee the progressive reduction of* α_n. *Step 6 guarantees the convergence of the algorithm.*

4. NUMERICAL EXAMPLE

To illustrate the validation of the results obtained previously, we consider the following problem of balancing an inverted pendulum on a cart. The equations of motion of the pendulum are described as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2 \sin(2x_1)/2 - \alpha\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + \omega \quad (46)$$

where x_1 denotes the angle of the pendulum from the vertical position, and x_2 is the angular velocity. $g = 9.8m/s_2$ is the gravity constant, m is the mass of the pendulum, $a = 1/(m + M)$, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart. In the simulation, the pendulum parameters are chosen as $m = 2kg$, $M = 8kg$, and $2l = 1.0m$.

We approximate the system (46) by the following T-S fuzzy model:

Rule 1: *If* $x_1(t)$ is M_1, *then*

$$\dot{x}(t) = (A_1 + \Delta A_1)x(t) + B_1\omega(t) + (B_{2_1} + \Delta B_{2_1})u(t)$$
$$(t) = C_1 x(t) + D_{12} u(t)$$
$$y(t) = C_2 x(t)$$

Rule 2: *If* $x_1(t)$ is M_2, *then*

$$\dot{x}(t) = (A_2 + \Delta A_2)x(t) + B_1\omega(t) + (B_{2_2} + \Delta B_{2_2})u(t)$$
$$(t) = C_1 x(t) + D_{12} u(t)$$
$$y(t) = C_2 x(t)$$

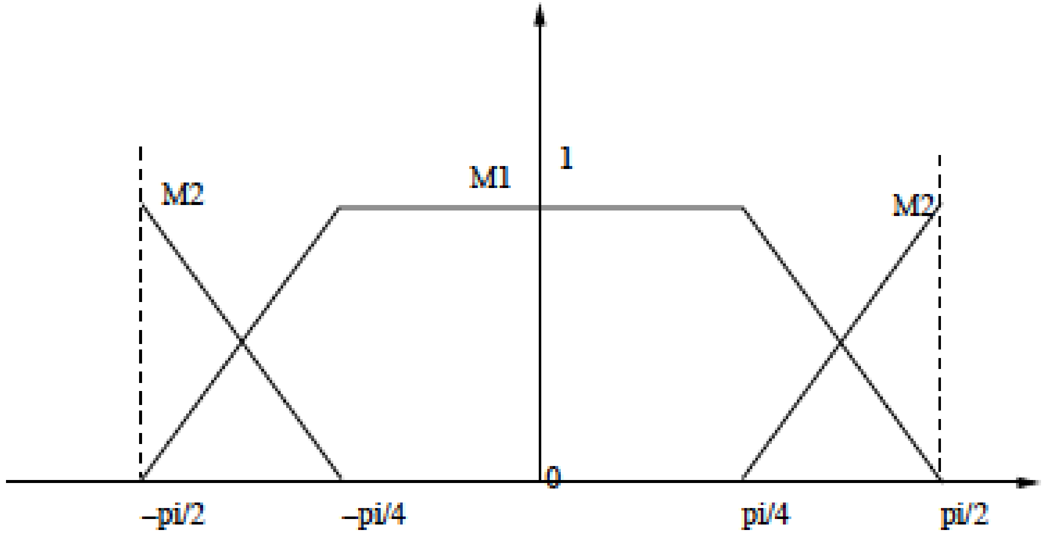

Fig. (4). Membership function.

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4l/3 - aml} & 0 \end{bmatrix}, \quad B_{2_1} = \begin{bmatrix} 0 \\ -\dfrac{\alpha}{4l/3 - aml} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}, \quad B_{2_2} = \begin{bmatrix} 0 \\ -\dfrac{\alpha\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0.3 \end{bmatrix}, D_{12} = 0.01, C_2 = \begin{bmatrix} 9 & 0.1 \end{bmatrix}$$

$$H_{1_1} = H_{1_2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, E_{1_1} = E_{1_2} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$E_{2_1} = E_{2_2} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

and $\beta = \cos(88^\circ)$. The disturbance attenuation level γ is set to be equal to 1 in this example and $\varepsilon_1 = \varepsilon_2 = 1$. The membership functions for Rule 1 and Rule 2 are shown in Fig. (**4**).

In our simulation, we assume $\tau^*(1) = 0.045$ and $\tau^*(2) = 0.025$. We assume the sampling period is 0.01, that is, $h^s = 0.01$, and $n^s = 0$ which means no data packet dropout happens in the communication channel.

The random time delays exist in $\mathcal{S} = \{1, 2\}$, and its transition rate matrices are given by:

$$\Lambda = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

In this example, the fault signal is simulated as follows:

$$f(t) = \begin{cases} 1 & t \in [5,10] \\ 0 & others \end{cases}$$

For the sake of simplicity, $\hat{D}_i(\iota)$ is assumed to be a zero matrix in this example. By applying Theorem 3.1 and the iterative algorithm, we get the following fault estimator for $\iota \in \mathcal{S} = \{1, 2\}$ of the form (20)-(22) where:

$$\hat{A}_{11}(1) = \begin{bmatrix} -5.2761 & -42.358 \\ 79.949 & -18.168 \end{bmatrix},$$

$$\hat{A}_{12}(1) = \begin{bmatrix} -6.3274 & -41.749 \\ 82.695 - 18.11 & \end{bmatrix},$$

$$\hat{A}_{21}(1) = \begin{bmatrix} -6.1284 & -44.1547 \\ 74.265 & -19.541 \end{bmatrix},$$

$$\hat{A}_{22}(1) = \begin{bmatrix} -6.1147 & -43.224 \\ 78.4474 & -19.3218 \end{bmatrix},$$

$$\hat{B}_1(1) = \begin{bmatrix} 1.0018 \\ 0.0029931 \end{bmatrix}, \hat{B}_2(1) = \begin{bmatrix} 0.91953 \\ 0.18201 \end{bmatrix},$$

$$\hat{C}_1(1) = \begin{bmatrix} 2.06 - 7.8782 \end{bmatrix}, \hat{C}_2(1) = \begin{bmatrix} 1.9243 - 7.6107 \end{bmatrix},$$

$$\hat{A}_{11}(2) = \begin{bmatrix} -10.386 & -41.1 \\ 96.295 & -17.874 \end{bmatrix},$$

$$\hat{A}_{12}(2) = \begin{bmatrix} -2.4862 & -42.937 \\ 70.708 & -18.469 \end{bmatrix},$$

$$\hat{A}_{21}(2) = \begin{bmatrix} -8.546 & -44.587 \\ 85.4447 & -17.214 \end{bmatrix},$$

$$\hat{A}_{22}(2) = \begin{bmatrix} -2.5548 & -45.254 \\ 88.214 & -17.228 \end{bmatrix},$$

$$\hat{B}_1(2) = \begin{bmatrix} 1.1029 \\ -0.27829 \end{bmatrix}, \hat{B}_2(2) = \begin{bmatrix} 1.0011 \\ -0.0094 \end{bmatrix},$$

$$\hat{C}_1(2) = \begin{bmatrix} 2.0784 - 8.0286 \end{bmatrix}, \hat{C}_2(2) = \begin{bmatrix} 1.9683 - 7.7673 \end{bmatrix},$$

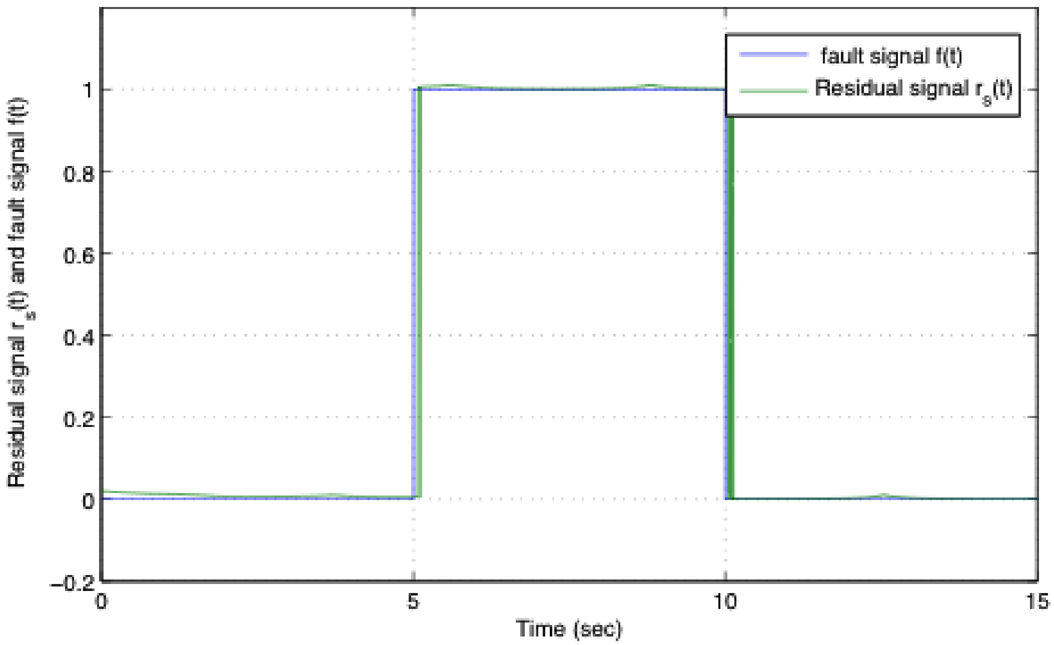

Fig. (5). Residual signals $r^s(t)$ and $f(t)$.

Histories of the residual signals $r^s(t)$ along with the fault signal $f(t)$ are shown in Fig. (**5**). The results demonstrate that the designed fault estimator meets the performance requirement.

5. CONCLUSION

In this paper, a technique of designing a delay-dependant fuzzy fault estimator for an nonlinear uncertain net-worked control system with random communication network-induced delays and data packet dropouts has been proposed. The Lyapunov-Razumikhin method has been employed to derive such a fault estimator for this class of systems. Sufficient conditions for the existence of such a fault estimator for this class of nonlinear NCSs are derived in a form of bilinear matrix inequalities. We finally use a numerical example to demonstrate the effectiveness of this methodology at the last section.

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