

Study on the Construction of Smart Transportation System Model under the Background of Smart City

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Abstract: “Smart city” construction has become the major trend of urban development in the world; it is the expression that city information has reached a higher stage. Smart transportation system has been considered as an important part of smart city, its stability control has become an urgent problem. The problem of robust control for a class of uncertain systems with time-varying delay is considered in this paper. Based on the Lyapunov stability theorem, with the linear matrix inequality approach, a sufficient condition is given to design the robust non-fragile control to make the systems stable.

Keywords: Smart city, smart transportation systems, linear matrix inequality.

1. INTRODUCTION

After the outbreak of the global financial crisis in 2008, IBM had proposed the idea of “smart planet” at the end of 2008. In 2009, IBM proposed again the slogan of “smart city” construction, if the government would construct the “smart planet”. Since then, the beautiful vision of “smart city” has been widely accepted. The “smart city” construction has become the major trend of urban development in the world, and has also become the new orientation of modern urban governance concept. Currently, more than 1200 “smart city” projects have being implemented in the world, the global number of the “smart city” is numerous, and each of “smart city” has distinct feature. “Smart city construction will cover all areas of the city, all classes, all regions, and would lead to fundamental change in the form of the modern city running, management mode, production and life style.” The impact of smart city construction on the city, the country and the world would be enormous and long-term, smart city would be the commanding heights of the future urban development, and even the national development [1-3].

Smart city is the product of city information development, it develops intelligent application by strong driving force of new generation information technology ,such as big data, cloud computing, IOT(Internet of things), geographic information, mobile Internet and others, and establishes a kind of new, sustainable urban development pattern. Its connotation focuses on the three points. The first, Smart city construction must regard information technology application as the main line. The second, Smart city is a complex and interacting system. In this system, information technology and other resources would be optimization and function together, make the city operation smarter. The third, Smart city is a new mode of city development. The essence of Smart

city lies in the fusion of information and urbanization; it is the expression that city information has reached a higher stage [4].

Smart Transportation is the construction of “digital transportation” project [5]. It would improve the public security, urban management, road monitoring system and the information network system through the technologies of the monitoring, surveillance, traffic flow distribution optimization, and establish the integrated intelligent urban traffic management and service system emphasized on a traffic guidance, emergency command , smart travel, taxi and bus management system; and achieve full sharing of traffic information , real-time monitoring road traffic conditions and dynamic management; Comprehensively enhance the monitor and intelligent management level, ensure transportation safety and smooth. The study of smart transportation has received much attention, and various methods have been given over the past years [6-8]. But, on the modeling and control of smart transportation, a few have been reported. In this paper, we consider the problem of robust control for a class of smart transportation systems. Based on a new model of smart transportation systems, with Lyapunov stability theorem, a sufficient condition will be obtained to make the smart transportation systems stable.

2. PROBLEM FORMULATION

Consider the following uncertain smart transportation systems with time-varying delay

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)) + Bu(t) \\ x(t) &= \phi(t)t \in [-d, 0] \end{aligned} \quad (1)$$

Where $x(t) \in R^n$ is systems state vector, $u(t) \in R^m$ is control input vector, $A, A_d \in R^{n \times n}$ are systems matrices, $B \in R^{n \times m}$ is control input matrix, $\phi(t) \in R^n$ is given initial

states on $[-d,0]$, $d(t)$ is time-varying delay and satisfying: $0 < d(t) < d$, $\Delta A, \Delta A_d \in R^{n \times n}$ are the systems uncertainties satisfying

$$\Delta A = D_1 F(t) E_1 \quad \Delta A_d = D_2 F(t) E_2$$

where D_1, D_2, E_1, E_2 are constant matrices with appropriate dimensions, and the time-varying matrix $F(t)$ satisfying

$$F^T(t)F(t) \leq I, \forall t \geq 0$$

For the systems (1), the following state feedback controller will be designed

$$u(t) = (k + \Delta k)x(t) \tag{2}$$

where $K \in R^{m \times n}$ is a constant matrix, ΔK is the control uncertainty satisfying

$$\Delta K = D_3 F(t) E_3$$

where D_3, E_3 are constant matrices with appropriate dimensions.

Substituting the controller (2) into (1) yields the closed-loop systems

$$\dot{x}(t) = \bar{A}x(t) + \bar{A}_d x(t-d(t)) \tag{3}$$

where $\bar{A} = A + \Delta A + BK + B\Delta K$, $\bar{A}_d = A_d + \Delta A_d$

3. MAIN RESULTS

Lemma 1: [6] For the given $n \times n$ symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, \text{ where } S_{11} \text{ is } r \times r, \text{ then the following}$$

three conditions are equivalent

- (1) $S < 0$;
- (2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;
- (3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2: [8] For the known constant matrices Y, D, E with appropriate dimensions, and the symmetric matrix Y , the following matrix inequality

$$Y + DF(t)E + E^T F^T(t)D^T < 0$$

is equivalent to the following matrix inequality

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0$$

Where the time-varying matrix $F(t)$ satisfying $F^T(t)F(t) \leq I$, and $\varepsilon > 0$ is a constant.

Theorem 1: For the uncertain delay systems (1), if there exist positive-definite P and S , such as the following matrix inequality holds

$$\begin{bmatrix} \Pi & P(A_d + D_2 F(t)E_2) \\ (A_d + D_2 F(t)E_2)^T P & -S \end{bmatrix} < 0 \tag{4}$$

$$\begin{aligned} \Pi = & (A + D_1 F(t)E_1 + BK + BD_1 F(t)E_1)^T P \\ & + P(A + D_1 F(t)E_1 + BK + BD_1 F(t)E_1) + S \end{aligned}$$

then the systems(1) is quadratic stable.

Proof: Consider the following Lyapunov functional

$$V(t) = x^T P x + \int_{t-d}^t x^T(\tau) S x(\tau) d\tau$$

Where P, S are positive-definite matrices.

Along the solution of the closed-loop systems (3), the difference of $V(t)$ is obtained

$$\begin{aligned} \dot{V}(t) = & \dot{x}(t)P x(t) + x^T(t)P \dot{x}(t) + x^T(t)P x(t) \\ & - x^T(t-d(t))S x(t-d(t)) \\ = & 2x^T(t)P[(A + \Delta A_d)x(t) \\ & + (A_d + \Delta A_d)x(t-d(t)) \\ & + B(K + \Delta K)x(t)] + x^T(t)S x(t) \\ & - x^T(t-d(t))S x(t-d(t)) \\ = & x^T(t)(2PA + 2P\Delta A + 2PBK \\ & + 2PB\Delta K + S)x(t) \\ & + 2x^T(t)(PA_d + P\Delta A_d)x(t-d) \\ & - x^T(t-d)S x(t-d) \end{aligned}$$

$$= \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}^T \Theta \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}$$

$$\text{where } \Theta = \begin{bmatrix} \Xi & P(A_d + D_2 F(t)E_2) \\ (A_d + D_2 F(t)E_2)^T P & -S \end{bmatrix}$$

$$\begin{aligned} \Xi = & (A + D_1 F(t)E_1 + BK + BD_3 F(t)E_3)^T P \\ & + P(A + D_1 F(t)E_1 + BK + BD_3 F(t)E_3) + S \end{aligned}$$

Theorem 2: The uncertain delay systems is quadratic stable, if there exist positive-definite matrices X, \bar{S} , and matrix \bar{K} with appropriate dimensions, and constants $\varepsilon_i > 0 (i=1,2,3)$, such as the following matrix inequality holds.

$$\begin{bmatrix} AX + B\bar{K} + (B\bar{K} + AX)^T \\ +\bar{S} + \varepsilon_1 D_1 D_1^T & A_d X & X E_1^T & 0 & X E_3^T \\ +\varepsilon_2 D_2 D_2^T + \varepsilon_3 D_3 D_3^T & * & * & * & * \\ * & -\bar{S} & 0 & X E_2^T & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (5)$$

Proof: The matrix inequality (4) can be rewritten as

$$\begin{aligned} Y + & \begin{bmatrix} PD_1 \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E_1 & 0 \end{bmatrix} + \begin{bmatrix} E_1 & 0 \end{bmatrix}^T F(t)^T \begin{bmatrix} PD_1 \\ 0 \end{bmatrix}^T \\ + & \begin{bmatrix} PD_2 \\ 0 \end{bmatrix} F(t) \begin{bmatrix} 0 & E_2 \end{bmatrix} + \begin{bmatrix} 0 & E_2 \end{bmatrix}^T F(t)^T \begin{bmatrix} PD_2 \\ 0 \end{bmatrix}^T \\ + & \begin{bmatrix} PBD_3 \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E_3 & 0 \end{bmatrix} + \begin{bmatrix} E_3 & 0 \end{bmatrix}^T F(t)^T \begin{bmatrix} PBD_3 \\ 0 \end{bmatrix}^T < 0 \end{aligned}$$

Where

$$Y = \begin{bmatrix} (A + BK)^T P + P(A + BK) + S & P A_d \\ A_d^T P & -S \end{bmatrix}$$

With the lemma 2, the above inequality is equivalent to the following matrix inequality

$$\begin{aligned} Y + \varepsilon_1 & \begin{bmatrix} PD_1 \\ 0 \end{bmatrix} \begin{bmatrix} PD_1 \\ 0 \end{bmatrix}^T + \varepsilon_1^{-1} \begin{bmatrix} E_1 & 0 \end{bmatrix}^T \begin{bmatrix} E_1 & 0 \end{bmatrix} \\ + \varepsilon_2 & \begin{bmatrix} PD_2 \\ 0 \end{bmatrix} \begin{bmatrix} PD_2 \\ 0 \end{bmatrix}^T + \varepsilon_2^{-1} \begin{bmatrix} 0 & E_2 \end{bmatrix}^T \begin{bmatrix} 0 & E_2 \end{bmatrix} \\ \varepsilon_3 & \begin{bmatrix} PBD_3 \\ 0 \end{bmatrix} \begin{bmatrix} PBD_3 \\ 0 \end{bmatrix}^T + \varepsilon_3^{-1} \begin{bmatrix} E_3 & 0 \end{bmatrix}^T \begin{bmatrix} E_3 & 0 \end{bmatrix} < 0 \end{aligned}$$

With the lemma1,the above inequality is equivalent to

$$\begin{bmatrix} \Lambda & P A_d & E_1^T & 0 & E_3^T \\ * & -S & 0 & E_2^T & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (6)$$

$$\begin{aligned} \Lambda = & (A + BK)^T P + P(A + BK) + S \\ & + \varepsilon_1 P D_1 D_1^T P + \varepsilon_2 P D_2 D_2^T P + \varepsilon_3 P D_3 D_3^T P \end{aligned}$$

Pre-and post-multiplying the inequality (6) by $\text{diag}\{P^{-1} \ P^{-1} \ I \ I \ I\}$, we obtain

$$\begin{bmatrix} \Omega & A_d P^{-1} & P^{-1} E_1^T & 0 & P^{-1} E_1^T \\ * & -P^{-1} S P^{-1} & 0 & P^{-1} E_2^T & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (7)$$

$$\begin{aligned} \Omega = & (A + BK) P^{-1} + P^{-1} (A + BK)^T \\ & + P^{-1} S P^{-1} + \varepsilon_1 D_1 D_1^T + \varepsilon_2 D_2 D_2^T + \varepsilon_3 D_3 D_3^T \end{aligned}$$

By given some transformations

$$P^{-1} S P^{-1} = \bar{S}, \quad K P^{-1} = \bar{K}, \quad P^{-1} = X$$

the inequality (7) is equivalent to the inequality(5).

CONCLUSION

This paper considers the problem of robust stable for a class of uncertain systems with time-varying delay. Based on the Lyapunov stability theorem, a sufficient condition and the controller design approach are given in term of linear matrix inequality.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interests regarding the publication of this paper.

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