The Study of 3D Digital Watermarking Algorithm Which is Based on a Set of Complete System of Legendre Orthogonal Function

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Abstract: As for the Encryption protection of 3D digital model, 3D digital watermark technology is inevitably the preferred plan. However, due to the curved surface property and three-dimension property in 3D model itself, there are no better plans that can effectively defend all kinds of watermark attacks among the numerous 3D digital watermark algorithms so far. Thus in this paper a modified algorithm will be proposed, which uses a set of complete system of Legendre orthogonal function to conduct the transformative processing to the geometry information of the digital model and embed the watermark into it. The system is a normal complete orthogonal function constituted by Legendre polynomial of times which not only shares the same property with the smooth orthogonal function system but also has the same property of discontinuous orthogonal function system. Therefore, from the result of the test we can see that the algorithm has strong invisibility, stability and robustness, which greatly enhances the security and the reducibility of 3D encrypted mode.

Keywords: Complete system of orthogonal function, Digital watermark, The function system of Legendre, 3D model.

1. INTRODUCTION

The digital watermarking algorithm of 3D model was proposed by Ohbuchi for the first time in 1997. As the gradual development and widely application of 3D print technology, 3D watermarking algorithm gets more and more attention and development. We can mainly divide this algorithm into spatial domain algorithm and transformed domain algorithm according to the position where the watermarking is embedded. Using the former algorithm, most embed watermarking information by changing the geometric coordinates, normal direction, texture coordinates or topological structure of 3D model. For example, the center-neighbored relative location of the vertex can be changed by using the distance from the vertex to the reference line [1]. Besides, the texture and the redundancy of 3D model can also be used to embed watermarking information in it. According to the latter algorithm, the representatives are as Kanai's 3D digital watermarking algorithm based on wavelet [2], Praun's progressive gird technology and the spread spectrum modulation ideology [3], Lili and others' 3D model watermarking algorithm which is based on spherical parameterization [4], Murotani's embedding the watermarking information into the spectrum of vertex sequence which is achieved by processing singular value decomposition to the track matrix of vertex sequence of 3D model [5], and Jeon's watermarking embedding and testing of DCT domain by the vertex coordinates which are achieved by sorting 3D model [6].

As for a 3D mesh model, generally speaking, no matter which algorithm you choose between spatial domain algorithm and transformed domain algorithm, they have to undergo the affine transformation such as displacement, rotation and scale and the watermarking attacks such as mesh compression and mesh simplification. Thus 3D model digital watermark should generally satisfy detectability, validity, high fidelity, robustness and its imperceptibility. [7] Therefore, considering all advantages and disadvantages of the above algorithms, this paper introduces a new kind of method that a complete system of Legendre orthogonal function is used to embed the watermark. The system is a normal complete orthogonal system constituted by Legendre polynomial of times which has properties of both Smooth orthogonal function system and discontinuous orthogonal function system.

2. AN OVERVIEW OF THE COMPLETE SYSTEM OF ORTHOGONAL LEGENDRE FUNCTION

2.1. The Complete System of Orthogonal Function

The complete system of orthogonal function is a orthogonal function system constituted by a sequence of orthogonal bases. The complete system of orthogonal function that we are familiar with has many systems such as the system of trigonometric function, orthogonal polynomials, Walsh function and Haar function [8-10]. The complete system of orthogonal function was proposed for the first time around 1983 when the relatively thorough theoretical basis was given. It consists of both a sequence of sectioned polynomial of times whose system is hierarchical structured and the composition of smooth function as well as each level of discontinuous functions [11].

n	$P_n(x)$
0	1
1	x ⊥
2	$(3x^2-1)/2$ ⊥
3	$(5x^3-3x)/2$ ⊥
4	$(35x^4-30x^2+3)/8$ ⊥
5	$(63x^5-70x^3+15x)/8$ ⊥

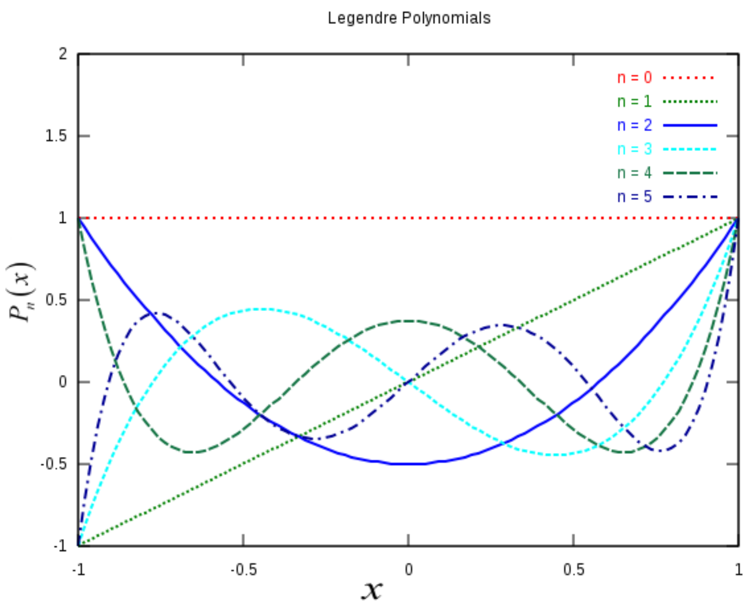

Fig. (1). 6 order curves of Legendre Polynomials.

Take Walsh's orthogonal function as an example, each of whose sequence is mutually orthogonal. Namely, if $m_0 \neq m_1$, then

$$\sum_{n=0}^{N-1} W[m_0,n]W[m_1,n]=0 \tag{1}$$

The following is an 8 point Walsh orthogonal function [12].

$$W_8=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{pmatrix} \tag{2}$$

2.2. Legendre Function

As for Legendre equation whose solution can be represented as a standard form of a power series, the bounded solutions are available (that is, convergence of solution set) if the equation satisfies $|x|<1$. And if is a nonnegative integer, namely, $n=0,1,2,L$, there are bounded solutions at the point: (as showed in Fig. (**1**). In this condition, a polynomial sequence constituted by a set of orthogonal polynomials is formed which is called Legendre Polynomials, namely, Legendre functions [13].

Mathematically, Legendre function is equal to the solutions of the following differential equation:

$$\left(1-x^2\right)\frac{d^2P(x)}{dx^2}-2x\frac{P(x)}{dx}+n(n+1)P(x)=0 \tag{3}$$

To be more convenient, we will turn the above equation into Sturm-Liouville form:

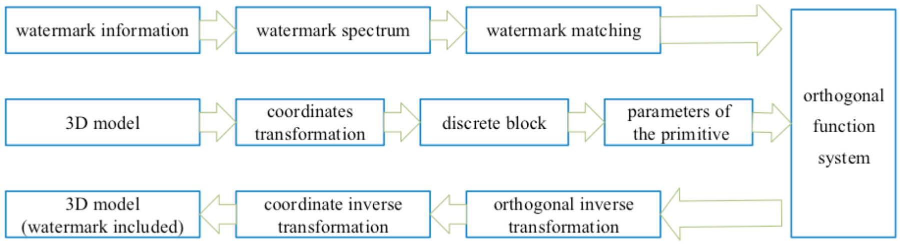

Fig. (2). The overall program for 3D digital watermarking algorithm which is based on a set of complete system of Legendre orthogonal function.

$$\frac{d}{dx}\left[\left(1-x^2\right)\frac{d}{dx}P(x)\right]+n(n+1)P(x)=0 \tag{4}$$

Translated into:

$$\frac{d}{dx}\left[\left(1-x^2\right)\frac{d}{dx}P(x)\right]=-n(n+1)P(x) \tag{5}$$

The solutions of Legendre equation can be written into standard formal power series. As the equation satisfies $|x|\le 1$, we can get the unbounded solutions (that is, convergence of solution set) [14-16]. And as is a non-negative integer, which means $n=0,1,2,\text{L}$, we can also get unbounded solutions at the point: $x=\pm 1$. In this condition, the solutions will vary as the variance of the n 's value, which constitutes a set of polynomial sequence that is consist of orthogonal polynomials [17]. This set of polynomials is called Legendre polynomials.

In this polynomial, is the order polynomial which can be expressed by Rodrigues equation:

$$P_n(x)=\frac{1}{2^n\times n!}\times\frac{d^n}{dx^n}\left[\left(x^2-x\right)^n\right] \tag{6}$$

The most essential property of Legendre polynomials is that the inner product concerned with in satisfies orthogonality [18]. That is:

$$\int_{-1}^{1}P_m(x)P_n(x)dx=\frac{2}{2n+1}\delta_{mn} \tag{7}$$

Inside, $\delta_{mn}=\begin{cases}1, & m=n\\ 0, & m\neq n\end{cases}$ 。

3. THE CONSTRUCTION OF THE COMPLETE SYSTEM OF ORTHOGONAL LEGENDRE FUNCTION

The process of constructing, piecewise times complete orthogonal function, is as follows:

(1) Take first Legendre polynomials from as first functions of times system, which can be called basis function of times system.

(2) The function generator of times system constructed by basis function has functions which are mutually orthogonal. This generator is orthogonal to the basis function in, which is called the generated function of times system.

(3) Recurrence generates subsequent sequence. As the three neighboring Legendre polynomials satisfy three recursions, whose recurrence relation formula is as follows:

This text takes the following functions as the original functions and lists the expression of 6 order Legendre polynomials as [19]. Then the original normal orthogonal bases are constructed.

4. THE BASIC PRINCIPLE OF THE ALGORITHM

The 3D digital watermarking algorithm mentioned in this text aims at the research on 3D mesh model. Meanwhile, the vertex coordinates of it is chosen to be an embedded carrier of the watermark [20]. To guarantee its robust, invisibility, safety and versatility, its global geometric characteristics, which is the distance from the vertex of the coordinates of 3D mesh model to the center of the model, is regarded as the embedded object.

In this text, the sequence of distance value will be embedded into watermark information through a set of complete system of orthogonal function and then by modifying the function coefficients [21]. The watermark can be added to the modified coefficient to make the influence of watermark spread to anywhere of the distance sequence. Thus the watermark information can not be removed easily. The flow chart of the watermarking algorithm is as follows:

5. THE PROCESS OF EMBEDDING WATERMARK

5.1. The Generation of Spread Spectrum Watermarking

Suppose the original watermarking information as m_j Use a larger spread spectrum factor to process the spread spectrum according to vertex's basic digits of the original 3D mesh model and get the spread spectrum sequence t_i .

$$t_i=m_j, j\cdot cr\le i\le (j+1)\cdot cr \tag{8}$$

Then use a set of one-dimensional binary pseudo-random sequence to modulate the spread spectrum t_i .

$$W=\{w_i\}=\{r_i\cdot t_i\}\in\{0,1\}, i=0,1,2,\text{L} ,m-1 \tag{9}$$

5.2. Transformation of Coordinates

Suppose 3D mesh model contains a set of vertexes U^c which are represented as$u_i^c = (x_i,\ y_i,\ z_i)$. In this quotation, x_i、y_i、$z_i$are the Cartesian coordinates values of its vertexes whose total number of is represented as (U^c). The barycentric coordinate of 3D model is represented as$O^c = (O_x,\ O_y,\ O_z)$, whose density is supposed to be uniform [22]. And the coordinates of its center $O^c$are the arithmetic mean values of all the vertexes in all their directions. Namely, O_x、O_y、$O_z$are separately

$$O_x = \frac{1}{N(U^c)}\sum_{i=1}^{N(U^c)} x_i$$

$$O_y = \frac{1}{N(U^c)}\sum_{i=1}^{N(U^c)} y_i \ ; \tag{10}$$

$$O_z = \frac{1}{N(U^c)}\sum_{i=1}^{N(U^c)} z_i$$

Then the barycentric coordinates of 3D model should be translated to the origin. And we use x_i'、y_i'、z_i' to represent the vertex coordinates of the translated 3D model as the followings:

$$\begin{cases} x_i' = x_i - O_x \\ y_i' = y_i - O_y \\ z_i' = z_i - O_z \end{cases} \tag{11}$$

In this way can we get the new vertex coordinate system which uniformly uses the center of 3D model as the origin. Allowing for the variability and the characteristic quality of 3D model, the coordinate vertex of 3D model can be translated to spatial coordinate by Cartesian coordinates to get it analyzed [23]. Represents $u_i^s = (\gamma_i,\ \theta_i,\ \varphi_i)$ the spatial coordinate value of 3D model and represents the spatial coordinate set of all its vertexes, among which:

$$\gamma_i = \gamma(u_i^s) = \sqrt{(x_i'')^2 + (y_i'')^2 + (z_i'')^2} \tag{12}$$

(4) In this way, we can get the set of required parameter primitives: $R = \{r_i\}$.

5.3. The Generation of Discrete Systems

At this point, another additional set of 8×8 discrete complete orthogonal system matrix is generated. The interval is divided into eight subintervals. Then discrete first eight primitives of piecewise linear complete orthogonal system and orthogonalize its unit to get the 8×8 transformation matrix T, whose transposed matrix is T'.

5.4. The Determination of Embedding the Watermark into Primitives

Select 64× S elements orderly from the set and divide it into groups, in each of which there are 64 elements. And use the 64 elements of each group to comprise the 8×8 matrix R_{ij}. Make this matrix pass the complete orthogonal system which is just generated, namely $R_{ij}' = TR_{ij}T'$. Select k low-frequency coefficients from R_{ij}', each of which is added one bit of watermark information. The total capacity of the added watermark happens to be that of the spread-spectrum watermark [24]. As for the defined step size α, we can get the coefficient matrix after the quantification of the selected coefficient.

As Legendre polynomials satisfy parity. Namely, as the order number is an even number, $P_k(x)$ is an even function. And as the order number is an odd, is an odd function. The quotation of it is: $P_k(-x) = (-1)^k P_k(x)$. In the process of embedding watermark, the embedded binary watermark can be embedded in corresponding point according to its parity. That is to say, the watermark embedding will be provided with fitter safety and stability. The detailed procedures are: as $w_i = 0$, modify the coefficient to make it equal to even times the value of which is closest to its coefficient [25]. And as $w_i = 1$, modify the coefficient to make it equal to odd times the value of which is closest to its coefficient.

5.5. The Recovery of 3D Model after Embedding Watermark

Process the inverse orthogonal transformation to each team of the modified coefficient matrix: $R''' = \{r_i'\}$. At last, transform the spherical coordinates to the rectangular coordinate V_T'.

$$\begin{cases} V_{Tix}' = r_i' \sin(\varphi_i)\cos(\theta_i) \\ V_{Tiy}' = r_i' \sin(\varphi_i)\sin(\theta_i) \\ V_{Tiz}' = r_i' \cos(\varphi_i) \end{cases} \tag{13}$$

Translate to the original position and get the vertex of mesh model that has watermark information, namely: $V_w' = V_T' + V_c$.

6. EXTRACTION OF WATERMARK

First of all, through affine transformation can we calculate the barycentric coordinates of 3D mesh model in which watermark has been embedded. Then translate the watermark-embedded mesh model to move the barycenter of mesh model to the origin. Next, transfer the vertex coordinates of 3D mesh model into spherical coordinates. At last, orderly select 64 × S elements according to the sequenced collection in watermark embedding algorithm and group it as 64 elements each group. Orderly organize a 8×8 matrix with the 64 elements of each group and make it go through the previously generated complete system of orthogonal function.

(**a**) original 3D

(**b**) 3D mesh model

(**c**) 3D model after model embedding watermark

Fig. (3). 3D model notation.

After that, extract the watermark from the low-frequency coefficient which watermark has been embedded into [26]. If the coefficient is close to even times of the given step-size, the watermark should be "0". And if it is close to odd times of the given step-size, the watermark should be "1". In this way can the binary sequence of watermark be achieved according to which the watermark information can be reconstructed. Hereto, the extraction of watermark is completed.

7. THE EXPERIMENTAL RESULTS AND CONCLUSION

To verify the feasibility of the above algorithm, here separately select a glider model to process the watermark embedding and extraction, of which the glider model of is consist of 2156 vertexes and 1239 patches. The test applies binary watermarking whose size is 16×16 and thus generates the watermark sequence whose length is 256.

The results of the test are as Fig. (**3**). (a) in Fig. (**3**) is the original model and (b) in it is 3D model. (c) is the watermark-embedded model. The results show the algorithm in this text is completely feasible and has strong robustness.

From the view of the visibility of watermark, the algorithm here has strong invisibility, which ensures the consistency and security of the appearance of 3D model. Meanwhile, the algorithm use normalized correlated coefficient to evaluate for the attacks as rotation, scaling, translating and

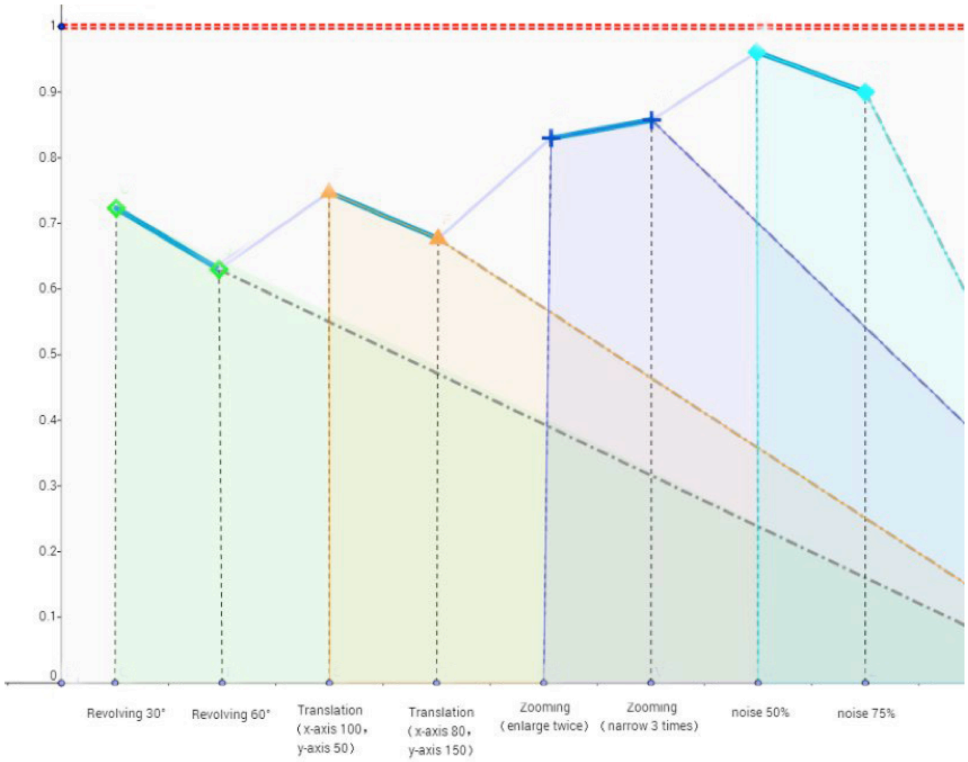

Fig. (4). The chart shows the normalized hamming similarity comparison between the attacks of watermark to 3D model and the original model.

noise. Namely, the existence of watermark can be judged by comparing the similarity (relevance) between the extracted watermark information and the original watermark information W_i. The results are demonstrated as follows:

$$N = \left[\sum_{i=1}^{M}\left(W_i^{'} - \overline{W_i^{'}}\right)\times\left(D_i^{w} - \overline{D_i^{w}}\right)\right] / \left[\sqrt{\sum_{i=1}^{M}\left(W_i^{'} - \overline{W_i^{'}}\right)^2} \times \sqrt{\sum_{i=1}^{M}\left(D_i^{w} - \overline{D_i^{w}}\right)^2}\right] \quad (14)$$

CONCLUSION

From the view of the complete system of Legendre orthogonal functions, this text emphatically discusses a new type of 3D digital watermarking algorithm. Process the spread spectrum code of the embedded binary watermark information and process the affine transformation on 3D model simultaneously. Also the primitive is embedded into watermark according to a set of complete system of Legendre orthogonal function--the transformation for the set of distance from vertex of 3D model to its barycenter. Then embed the watermark date in it. According to the even quality and recursion of the complete orthogonal Legendre polynomial, the watermark information will be embedded in the distance from vertex to barycenter which is transformed through the complete system of Legendre orthogonal functions. The test proves the strong robustness and invisibility of the algorithm. As the technology of 3d printing and digital watermarking develop their mature, we can make sure that the technology will be the key point in protecting the intellectual property rights and the orderly progress of a fair market in the near future.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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