Reliability Analysis Method Study of Multi-state System Based on Fuzzy Bayesian Network

Zhai Sheng[*,1], Shi Wuxi[2] and Xiu Chunbo[2]

[1]*School of Mechanical Engineering, Tianjin Polytechnic University, Tianjin, 300387, P.R. China*

[2]*School of Electrical Engineering and Automation, Tianjin Polytechnic University, Tianjin, 300387, P.R. China*

Abstract: To deal with fuzzy and uncertain information in reliability analysis of multi-state systems, an approach based on fuzzy Bayesian networks is proposed. In this approach the fuzzy set theory and Bayesian networks methodology are applied. The fuzzy assessment information from different experts is described by triangular fuzzy numbers, which denotes the fuzzy probabilities of different states of root nodes. The depended uncertain ordered weighted averaging (DUOWA) operator is used to aggregate assessments information. Through defuzzification calculation, the crisp probabilities are obtained as the prior probabilities of the root nodes. Bayesian network (BN) is used to handle the uncertainty of logical relationship between nodes and to compute system reliability. A case study concerning the reliability analysis in cell production line is provided and conducted using the proposed approach to illustrate its effectivity and practicability.

Keywords: Bayesian networks, fuzzy number, multi-state system, reliability analysis.

1. INTRODUCTION

The traditional reliability theories including Reliability Block Diagram (RBD), Binary Decision Diagram (BDD), Fault Tree Analysis (FTA), etc. have been applied extensively in reliability analysis of systems [1-3]. Faced with increasingly complex systems and work environments, the requirements for systems reliability analysis are getting stringent, which makes the application of these traditional reliability theories restricted a lot [4]. The uncertainty and fuzziness are inherent to complex systems.

In recent years, originated in the field of artificial intelligence, Bayesian Network (BN) as the representative of describing uncertain knowledge and advanced inference technology has been widely applied in reliability analysis [5, 6], risk analysis [7, 8] and fault diagnosis [9, 10]. Due to its representation ability of multi-state characteristic and uncertainty of logical relationship between nodes, Mahadevan [11] and Wilson [12] established reliability analysis models of two-state and multi-state system using BN technique. And through BN bidirectional inference, the marginal probability and posterior probability were obtained to analyze the weaknesses in the system, which provided decision-making basis of design and maintenance of the system.

BN model above is based on the condition that the elementary events probabilities are the exact values, but in practical cases, due to the lack of historical data, systems and environments complexity, the exact values of the elementary events probabilities are difficult to obtain. To this

end, in the risk forecast of subway fire Lu [13] used fuzzy numbers in fuzzy set theory to represent the fuzzy probabilities of elementary events, by defuzzification, the crisp values of fuzzy probabilities of elementary events were obtained, but failed to establish a reliability analysis modeling of a multi-state system. Chang *et al.* [14] considered both fuzzy states and fuzzy information. Simon [15] and Weber developed a Bayesian network model for determining reliability of the system with fuzzy states and imprecise probabilities.

In the process of synthesizing multiple expert judgments, Lu [13] integrated the multiple expert assessments using arithmetic average method, ignoring the different experts' weights. And in many cases, due to the policy-makers' limited expertise in related problem areas, it is difficult to specify the experts' weights, which makes experts' weights uncertain. And determining the experts' weights is mostly subjective, which may also cause a large deviation and affect the final evaluation.

A reliability analysis method of multi-state system based on fuzzy Bayesian networks is proposed to handle the fuzziness of data and uncertainty of logical relationship between events. The fuzzy uncertainty of the elementary events probabilities is described using fuzzy numbers in fuzzy set theory, the uncertainty of experts' weights is determined using depended uncertain ordered weighted averaging (DUOWA) [16] operator, the BN model is used to express the uncertain causal relationship between events and to compute system reliability.

The rest of this paper is organized as follows: Section 2 introduces the representation and integration of the experts' assessments. Section 3 briefly reviews the Bayesian network and introduces how to model multi-state system reliability based on BN. Section 4 illustrates the proposed approach
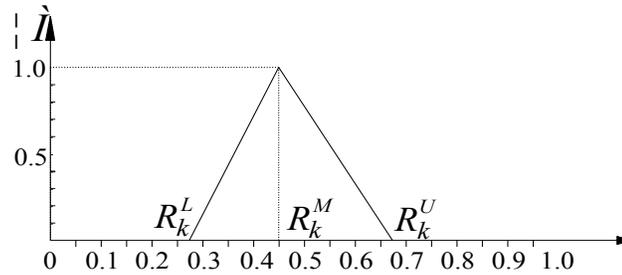
Fig. (1). Expert assessment in the form of triangular fuzzy number.

application in a cell production line. Section 5 draws the conclusion.

2. REPRESENTATION AND AGGREGATION OF THE EXPERTS' ASSESSMENTS

2.1. Representation of the Experts' Assessments

Under the conditions of being difficult to obtain exact events probabilities, the results are drawn from the experts' experience through the method of group decision-making. As the experts' assessments are often fuzzy information, in order to objectively describe the experts' assessments, in this paper the fuzzy numbers in fuzzy set theory is used to represent the experts' assessments, i.e. the fuzzy probabilities of events. Using fuzzy numbers to describe the occurrence probabilities of events may reduce the difficulty of obtaining the exact values.

There are many forms of fuzzy number, such as the interval fuzzy number, triangular fuzzy number, trapezoidal fuzzy number, normal fuzzy number and LR fuzzy number. This paper takes the form of triangular fuzzy number to represent the experts' assessments. The fuzzy probability may be expressed in the form of fuzzy set, shown in Fig. (**1**).

The triangular fuzzy number may be denoted as $\tilde{R}_k = (R_k^L , R_k^M , R_k^U)$. The triangle membership function $\mu_{\tilde{R}}(x)$ is expressed as:

$$\mu_{\tilde{R}}(x) = \begin{cases} \dfrac{x - R_k^L}{R_k^M - R_k^L}, & R_k^L \le x \le R_k^M \\ \dfrac{R_k^U - x}{R_k^U - R_k^M}, & R_k^M \le x \le R_k^U \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

2.2. Aggregation of the Experts' Assessments

In the case of being difficult to determine the experts' weights, in order to achieve the objective evaluation of the results, this paper relies on DUOWA operator to integrate the experts' assessments, that is, by calculating the similarity degree between individual values and the average assessed value to determine the experts' weights. The more similar it is, the bigger the weight is. The less similar it is, the smaller the weight is.

Suppose an expert panel composed of *n* experts makes events probabilities assessments. The triangular fuzzy number of assessment value of *k*-th expert is $\tilde{R}_k = (R_k^L , R_k^M , R_k^U)$, the average value is $\tilde{R}_a = (R_a^L , R_a^M , R_a^U)$, and

$$R_a^L = \frac{1}{n}\sum_{k=1}^{n} R_k^L , \quad R_a^M = \frac{1}{n}\sum_{k=1}^{n} R_k^M , \quad R_a^U = \frac{1}{n}\sum_{k=1}^{n} R_k^U ,$$

$$k = 1, 2, \cdots, n.$$

The distance $d(\tilde{R}_k, \tilde{R}_a)$ between $\tilde{R}_k$ and $\tilde{R}_a$ is:

$$d(\tilde{R}_k, \tilde{R}_a) = \frac{1}{3}\left(| R_k^L - R_a^L | + | R_k^M - R_a^M | + | R_k^U - R_a^U | \right) \qquad (2)$$

The similarity degree $s(R_k, R_a)$ between $\tilde{R}_k$ and $\tilde{R}_a$ is:

$$s(\tilde{R}_k, \tilde{R}_a) = 1 - \frac{d(\tilde{R}_k, \tilde{R}_a)}{\sum_{k=1}^{n} d(\tilde{R}_k, \tilde{R}_a)} \qquad (3)$$

Using DUOWA operators to integrate the experts' assessments, the group assessment value $\tilde{R}$ is obtained as:

$$\tilde{R} = (R^L, R^M, R^U) = DUOWA(\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_n)$$

$$= \sum_{k=1}^{n} \frac{s(\tilde{R}_k, \tilde{R}_a) \times \tilde{R}_k}{\sum_{k=1}^{n} s(\tilde{R}_k, \tilde{R}_a)} \qquad (4)$$

Using the mean area method to defuzzify, the group assessment value $\tilde{R}$ is transformed into the crisp group assessment value R. The defuzzification formula of the mean area method is:

$$R = \frac{R^L + 2R^M + R^U}{4} \qquad (5)$$

3. MULTI-STATE SYSTEMS RELIABILITY MODELING BASED ON BN

3.1. Bayesian Network

BN is a Directed Acyclic Graph (DAG) of statistical dependence between random variables, which can be expressed as $B = \{G, P\} = \{X, E, P\}$, where G is a DAG representing variable domain, P represents a group of the corresponding

set of conditional probabilities, X is a random variable set, $X=\{x_1, x_2,\ldots, x_l\}$, each node in G represents a random variable, and each node corresponds to a conditional probability table (CPT), E is the set of directed edges, each edge in G represents the dependency between variables. Each node x_i in a BN predetermined graph is independent of any child nodes consisted of x_i non descendant nodes given by x_i parent node.

If $A(x_i)$ represents a subset of any node consisted of non-descendant nodes, indicating the direct parent node of x_i with $\pi(x_i)$, then according to the conditional independence assumptions, there is:

$$P(x_i|A(x_i),\pi(x_i))=P(x_i|\pi(x_i)) \qquad (6)$$

Assuming $x_1, x_2\ldots x_l$ are the nodes of BN, the joint probability of all nodes in the network as follows:

$$P(x_1,x_2,\ldots,x_l)=\prod_{i=1}^{k}P(x_i|\pi(x_i)) \qquad (7)$$

So, BN can express joint probability distribution of the variables, and greatly simplify the solving of joint probability of variables.

3.2. Multi-state Systems Reliability Modeling Based on BN

Procedure for modeling multi-state system reliability based on BN is as follows:

1. Determining the nodes of BN, the root node of network representing the elementary events in the system, leaf nodes representing the system.

2. According to the causal relationship between the nodes determining the parent node, the child node, and establishing DAG which expresses the interdependence relationship between the nodes.

3. Determining the distribution parameters of priori probability of the root node and CPT.

In this paper, the BN model is established using method of Fault Tree (FT) mapping. FT model is established based on logical relationships between variables, and then the FT model is mapped into a BN model. A mapping algorithm includes graphical and numerical tasks. In graphical mapping, primary events, intermediate events, and the top event of the FT are represented as root nodes, intermediate nodes, and the leaf node in the corresponding BN, respectively. In numerical mapping, the occurrence probabilities of the primary events are assigned to the corresponding root nodes as prior probabilities [11].

In BN model, the root node variable of BN is x_i ($i = 1, 2, \ldots, l$), its state is v_i ($v_i= 0, 1, \ldots, r-1$); intermediate node variable is y_j ($j = 1, 2, \ldots, m$), its state is w_j ($w_j= 0, 1, \ldots, s-1$); the leaf node variable is T, and its state is Q_T ($Q_T = 0, 1, \ldots, n-1$), Where in the normal state is state 0. Given occurrence probability of various fault states of each root is $P(x_i= v_i)$, using the bucket elimination method, the occurrence probability $P(T=Q_T)$ is obtained when the leaf node $T=Q_T$:

$$P(T=Q_T)$$

$$= \sum_{x_1,x_2,\mathrm{L}\ ,x_l,y_1,y_2,\mathrm{L}\ ,y_m} P(x_1,x_2,\mathrm{L}\ ,x_l,y_1,y_2,\mathrm{L}\ ,y_m,T=Q_T)$$

$$= \sum_{\pi(T)} P(T=Q_T|\pi(T))\times \sum_{\pi(y_1)} P(y_1|\pi(y_1))\cdots\times \sum_{\pi(y_j)} P(y_j|\pi(y_j))\cdots\times$$

$$\sum_{\pi(y_m)} P(y_m|\pi(y_m))P(x_1=v_1)\cdots P(x_i=v_i)\cdots P(x_l=v_l) \qquad (8)$$

When the root node $x_i= v_i$, the probability of leaf node $T=Q_T$ is:

$$P(T=Q_T|x_i=v_i) = \frac{P(T=Q_T,x_i=v_i)}{P(x_i=v_i)}$$

$$= \frac{\sum\limits_{x_1,x_2,\cdots,x_l,y_1,y_2,\cdots,y_m} P(x_1,x_2,\cdots,x_l,y_1,y_2,\cdots, y_m,T=Q_T,x_i=v_i)}{P(x_i=v_i)} \qquad (9)$$

When a leaf node $T=Q_T$, the state probability (posterior probability) of root node $x_i= v_i$ is:

$$P(x_i=v_i|T=Q_T) = \frac{P(x_i=v_i,T=Q_T)}{P(T=Q_T)}$$

$$= \frac{\sum\limits_{x_1,x_2,\cdots,x_l,y_1,y_2,\cdots,y_m} P(x_1,x_2,\cdots,x_l,y_1,y_2,\cdots,y_m,T=Q_T,x_i=v_i)}{P(T=Q_T)} \qquad (10)$$

According to Bayes' theorem:

$$P(x_i=v_i|T=Q_T) = \frac{P(x_i=v_i)P(T=Q_T|x_i=v_i)}{P(T=Q_T)} \qquad (11)$$

In order for the reliability analysis of multi-state system, importance analysis is indispensable. Next, the state importance and the probability importance of the node will be defined.

Definition 1: When the root node x_i is in the fault state v_i, the state importance $I_{Q_T}^{\mathrm{Pr}}(x_i=v_i)$ of the leaf node $T=Q_T$ in this state is defined as:

$$I_{Q_T}^{\mathrm{Pr}}(x_i=v_i) = P(T=Q_T|x_i=v_i) - P(T=Q_T|x_i=0) \qquad (12)$$

It represents the effective degree on the leaf nodes $T=Q_T$ when root node $x_i= v_i$.

Definition 2: when the leaf node T is in the state Q_T, the probability importance $I_{Q_T}^{\mathrm{Pr}}(x_i)$ of root node x_i is defined as:

$$I_{Q_T}^{\mathrm{Pr}}(x_i) = \frac{1}{r-1}\sum_{v_i=1}^{r-1} I_{Q_T}^{\mathrm{Pr}}(x_i=v_i)$$

$$= \frac{1}{r-1}\sum_{v_i=1}^{r-1}[P(T=Q_T|x_i=v_i) - P(T=Q_T|x_i=0)] \qquad (13)$$
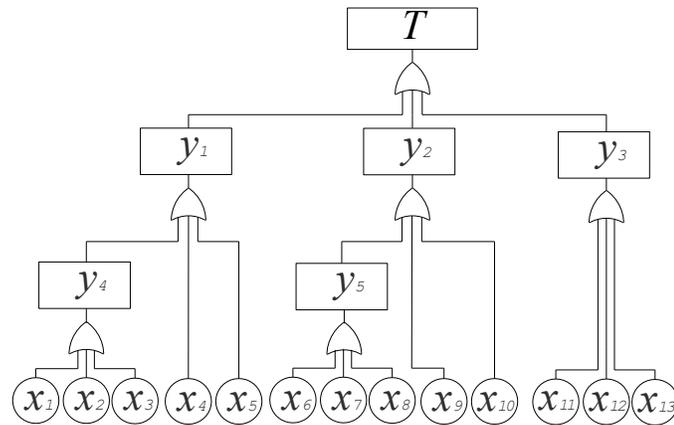
Fig. (2). Fault Tree Model of Feeding MnO$_2$ Subsystem.
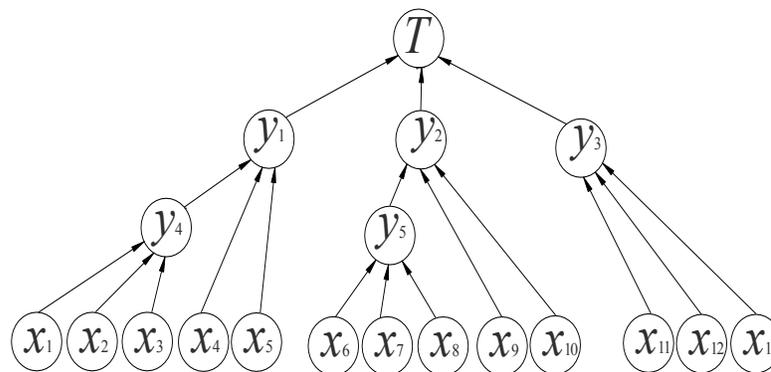
Fig. (3). BN model of subsystem feeding MnO2.

It represents the effective degree of the root node x_i on the leaf node $T = Q_T$.

Thus calculating the state importance and probability importance of each root node in the different fault states of system, we can know their impact on the system and find out the weak link of system. Calculating the posterior probabilities, we can identify the occurrence probability of each root node when the system is in fault state, and find out the cause of system fault. Finally, we provide reliable decision support for the design and maintenance of the system.

4. CASE STUDY

The following will be the reliability analysis of a cell production line for an enterprise. The main task of the production line is: putting the lithium-chip, the separator paper and the manganese cathode soaked by electrolyte into the negative electrode with set nets, and supplementing with volatile electrolyte in cathode film. Under the premise to ensure complete penetration of electrolyte, there are some other processes such as stamping cathode casing, sealing etc.

4.1. Establishment of BN Reliability Modeling

According to the function of the system, the system is divided into five working subsystems, namely the subsystem

of feeding negative shell and lithium, subsystem of dividing the paper into negative shell, subsystem of feeding MnO2, subsystem of assembling positive steel shell, sealing subsystem. Among them the subsystem of feeding MnO2 is selected for analysis.

According to the logical relationship between events, FT model which top event is subsystem fault of feeding MnO$_2$ is established, shown in Fig. (**2**).

Specification between the events and the notations is shown in Table **1**.

The FT model being built is mapped to the BN model (omitted CPT due to space reason) as shown in Fig. (**3**), so the reliability model based on BN is established.

According to the practical situation, the components and the system are set in three states 0, 1, 2, that is, Q_T = 0, 1, 2 and v_i = 0, 1, 2. Where 0 is normal operation state, 1 is a state of slight fault state, 2 is serious fault state.

4.2. Reliability Analysis System

1) Experts' assessments.

First, a panel of 4 experts is established, according to their experience the experts assess the occurrence probability of each fault mode and the state, and then give the corresponding triangular fuzzy numbers. Using DUOWA operator

Table 1. Nodes description.

Node	Event	Node	Event
T	Subsystem feeding MnO2 fault	x_5	Sensor fault
y_1	Device feeding negative shell fault	x_6	Cylinder feeding MnO2 jam
y_2	Device feeding MnO2 fault	x_7	Defective MnO2
y_3	Out-feeding device fault	x_8	Solenoid valve fault
y_4	Cylinder feeding negative shell fault	x_9	Conveyor belt fault
y_5	Cylinder feeding MnO2 fault	x_{10}	Sensor fault
x_1	Cylinder feeding negative shell jam	x_{11}	Solenoid valve fault
x_2	Negative shell deformation	x_{12}	Out-feeding cylinder jam
x_3	Solenoid valve fault	x_{13}	Sensor fault
x_4	Conveyor belt fault		

Table 2. Expert assessment value and the exact value of the fault probability of the root node.

(2-1)

Title	x1=1	x2=1	(x3, x8, x11) =2	x4=1	x4=2	(x5, x10, x13) =1
Expert 1	(0.15,0.22,0.3)	(0.1,0.16,0.22)	(0.38,0.43,0.52)	(0.22,0.26,0.32)	(0.1,0.15,0.2)	(0.05,0.1,0.15)
Expert 2	(0.18,0.25,0.32)	(0.08,0.18,0.25)	(0.35,0.45,0.55)	(0.24,0.28,0.32)	(0.08,0.16,0.22)	(0.08,0.15,0.2)
Expert 3	(0.2,0.25,0.3)	(0.1,0.18,0.22)	(0.4,0.45,0.5)	(0.2,0.25,0.3)	(0.09,0.14,0.19)	(0.06,0.1,0.17)
Expert 4	(0.16,0.2,0.25)	(0.08,0.13,0.20)	(0.37,0.42,0.47)	(0.22,0.28,0.33)	(0.08,0.13,0.18)	(0.1,0.15,0.2)
Group	(0.173,0.231,0.295)	(0.092,0.164,0.223)	(0.377,0.437,0.509)	(0.220,0.268,0.323)	(0.089,0.145,0.197)	(0.072,0.125,0.180)
Exact value	0.232	0.161	0.44	0.270	0.144	0.126

(2-2)

Title	x6=1	x7=1	x9 =1	x9 =2	x12=1
Expert 1	(0.18,0.24,0.3)	(0.12,0.18,0.24)	(0.24,0.28,0.34)	(0.1,0.15,0.2)	(0.28,0.35,0.4)
Expert 2	(0.2,0.25,0.3)	(0.1,0.18,0.25)	(0.26,0.3,0.34)	(0.08,0.16,0.22)	(0.25,0.3,0.35)
Expert 3	(0.22,0.26,0.32)	(0.1,0.15,0.22)	(0.23,0.28,0.35)	(0.09,0.14,0.19)	(0.26,0.32,0.36)
Expert 4	(0.2,0.26,0.3)	(0.12,0.18,0.25)	(0.24,0.3,0.25)	(0.08,0.13,0.18)	(0.27,0.33,0.38)
Group	(0.200,0.253,0.304)	(0.111,0.174,0.241)	(0.242,0.290,0.345)	(0.089,0.145,0.197)	(0.265,0.325,0.372)
Exact value	0.252	0.175	0.292	0.144	0.322

method, according to Eq. (2-4) the estimation information of experts are integrated, the results of group assessment are obtained, after defuzzification by Eq. (5) the exact value of occurrence probability of each fault mode in different states is obtained, as shown in Table **2**.

2) Calculation of the importance.

The exact values gained after defuzzification as priori probabilities are used to conduct inference calculation. Next we take node x_4 for example.

As the fault state of node x_4 is 1, i.e. v_4=1, the state importance $I_1^{\text{Pr}}(x_4 = 1)$ when T =1 can be obtained by Eq. (12) as follows:

Table 3. State Importance of Nodes.

Node	State Importance				Node	State Importance			
	$I_1^{\mathrm{Pr}}(x_i=1)$	$I_2^{\mathrm{Pr}}(x_i=1)$	$I_1^{\mathrm{Pr}}(x_i=2)$	$I_2^{\mathrm{Pr}}(x_i=2)$		$I_1^{\mathrm{Pr}}(x_i=1)$	$I_2^{\mathrm{Pr}}(x_i=1)$	$I_1^{\mathrm{Pr}}(x_i=2)$	$I_2^{\mathrm{Pr}}(x_i=2)$
x_1	0.99353	0	0	0	x_8	0	0	0	0.99883
x_2	0.9949	0	0	0	x_9	0.99374	0	0	0.99520
x_3	0	0	0	0.99883	x_{10}	0.99343	0	0	0
x_4	0.99372	0	0	0.99854	x_{11}	0	0	0	0.99883
x_5	0.99343	0	0	0	x_{12}	0.99362	0	0	0
x_6	0.99355	0	0	0	x_{13}	0.99343	0	0	0
x_7	0.99504	0	0	0					

Table 4. Probability Importance of Nodes.

Node	Probability Importance		Node	Probability Importance	
	$I_1^{\mathrm{Pr}}(x_i)$	$I_2^{\mathrm{Pr}}(x_i)$		$I_1^{\mathrm{Pr}}(x_i)$	$I_2^{\mathrm{Pr}}(x_i)$
x_1	0.49677	0	x_8	0	0.49942
x_2	0.49745	0	x_9	0.49687	0.49760
x_3	0	0.49942	x_{10}	0.49672	0
x_4	0.49686	0.49927	x_{11}	0	0.49942
x_5	0.49672	0	x_{12}	0.49681	0
x_6	0.49678	0	x_{13}	0.49672	0
x_7	0.49752	0			

$$I_1^{\mathrm{Pr}}(x_4=1) = P(T=1|x_4=1) - P(T=1|x_4=0)$$

$$= 0.99372$$

The state importance of node x_4 $I_1^{\mathrm{Pr}}(x_4=1)$ when $T=1$ can be obtained by Eq. (13) as follows:

$$I_1^{\mathrm{Pr}}(x_4) = \frac{1}{2}\left(I_1^{\mathrm{Pr}}(x_4=1) + I_1^{\mathrm{Pr}}(x_4=2)\right)$$

$$= \frac{1}{2}\sum_{v_i=1}^{2}\left[P(T=1|x_4=v_i) - P(T=1|x_4=0)\right]$$

$$= 0.49686$$

The state importance and the probability importance of other nodes can be obtained as shown in Table **3** and Table **4**.

3) The posterior probability calculation.

Taking the node x_4 for example, the posterior probability of node x_4 can be obtained using Eq. (10) when system fault state **T=1**:

$$P(x_4=1|T=1) = \frac{P(x_i=1,T=1)}{P(T=1)}$$

$$= \frac{\sum\limits_{x_1,x_2,\cdots,x_{13},y_1,y_2,\cdots,y_5} P(x_1,x_2,\cdots,x_{13},y_1,y_2,\cdots,y_5,T=1,x_4=1)}{P(T=1)}$$

$$= 53.0$$

4) Reliability analysis.

The weaknesses of the system can be identified through the importance of the root node, and by increasing the reliability of links the system reliability can also be improved.

Table **3** shows that the effect of the current fault status of the root node on the system is as follows: in the state of slight fault, the node x_7 has the greatest impact; in the state of serious fault, the nodes x_3, x_8, x_{11} have the greatest impact.

Table **4** shows that the effect of the root node on the system is as follows: in the state of slight fault, the node x_7 has the greatest impact; in the state of serious fault, the nodes x_3, x_8, x_{11} have the greatest impact.

Table 5. Priori & Posterior Probability of Nodes (10^{-3}/h).

Node	Priori Probability	Posterior Probability T= 1	Posterior Probability T=2	Node	Priori Probability	Posterior Probability T= 1	Posterior Probability T= 2
	$v_i =1, v_i =2$	$v_i =1, v_i =2$	$v_i =1, v_i =2$		$v_i =1, v_i =2$	$v_i =1, v_i =2$	$v_i =1, v_i =2$
x_1	0.232, 0	45.5, 0	0.232, 0	x_8	0, 0.44	0, 0	0, 273.8
x_2	1.61, 0	315.9, 0	1.61, 0	x_9	0.292, 0.144	57.3, 0	0.26, 89.6
x_3	0, 0.44	0, 0	0, 273.8	x_{10}	0.126, 0	24.7, 0	0.126, 0
x_4	0.270, 0.144	53.0, 0	0.24, 89.6	x_{11}	0, 0.44	0, 0	0, 273.8
x_5	0.126, 0	24.7, 0	0.126, 0	x_{12}	0.322, 0	63.2, 0	0.322, 0
x_6	0.252, 0	49.4, 0	0.252, 0	x_{13}	0.126, 0	24.7, 0	0.126, 0
x_7	1.75, 0	343.4, 0	1.75, 0				

These nodes mentioned above are needed to be paid attention to and to be improved.

According to the posterior probability of each root node, fault probability of each node can be gained after a system fault, and thus fault diagnosis for system can be made, that is according to the posterior probability, the corresponding root nodes are detected top down.

Table **5** summarizes when the slight system fault occurs, the order of checking slight fault nodes is x_7, x_2, x_{12}, x_9, x_4, x_6, x_1, (x_5, x_{10}, x_{13}); when serious system fault occurs, the order of checking serious fault nodes is (x_5, x_{10}, x_{13}) , x_4, x_9, the order of checking slight fault nodes is x_7, x_2, x_{12}, x_9, x_6, x_4, x_1, (x_5, x_{10}, x_{13}).

5. CONCLUSION

The proposed method combines the advantages of fuzzy set theory and BN technology to solve the problem of the uncertainty and fuzziness in reliability analysis of complex system.

In this paper, the triangular fuzzy number is adopted to describe the fuzzy probability of the event, which solves the problem of being difficult to obtain precise data; through the method of group decision, using DUOWA operator, the assessments of experts with uncertain weights are integrated, which solves the problem of the influence of uncertain weights to the results of assessment.

Bayesian network model is used to handle the uncertainty of logical relationship between events and to compute system reliability. Through the calculation of importance of node and posterior probabilities, a more effective decision support is offered for the design and maintenance of complex systems.

The analysis of cases has proved that this method provides a powerful analytical tool for the reliability analysis of complex systems in a complex environment.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

REFERENCES

[1]   S. P. Wang, *Engineering reliability*, Beihang University Pess: Beijing, 2000.

[2]   L. D. Xing, and G. Levitin, "BDD-based reliability evaluation of phased-mission systems with internal/external common-cause failure", *Reliability Engineering and System Safety*, vol. 84, no. 2, pp. 145-153, 2013.

[3]   R. X. Duan, J. L. Tu, and D. C. Dong, "A novel hybrid approach of fault tree and Bayesian networks for fault diagnosis", *Journal of Computational Information System*, vol. 6, no. 11, pp. 3605-3612, 2010.

[4]   H. Langseth, and L. Portinale, "Bayesian networks in reliability", *Reliability Engineering System Safety*, vol. 92, no.1, pp. 92-108, 2007.

[5]   Y. Jiang, H. Zhang, X. Y. Song, X. Jiao, H. N. N. Hung, M. Gu, and J. Sun, "Bayesian-network-based reliability analysis of plc systems", *IEEE transactions on industrial electronics*, vol. 60, no. 11, pp. 5325-5336, 2013.

[6]   B. P. Cai, Y. Liu, Z. Liu, X. Tian, X. Dong, and S. Yu, "Using Bayesian networks in reliability evaluation for subsea blowout preventer control system", *Reliability Engineering and System Safety*, vol. 108, no. 2, pp. 32-41, 2012.

[7]   K. Nima, K. Faisal, and A. Paul, "Safety analysis in process facilities: Comparison of fault tree and Bayesian network approaches", *Reliability Engineering System Safety*, vol. 96, no. 8, pp. 925-932, 2011.

[8]   M. R. Martins, and M. C. Maturana, "Application of Bayesian Belief networks to the human reliability analysis of an oil tanker operation focusing on collision accidents", *Reliability Engineering and System Safety*, vol. 110, pp. 89-109, 2013.

[9]   R. X. Duan, J. L. Tu, and D. C. Dong, "A novel hybrid approach of fault tree and Bayesian networks for fault diagnosis", *Journal of Computational Information System*, vol. 6, no.11, pp. 3605-3612, 2010.

[10]   X. Y. Wang, and Y. H. Qiu, "Software dependability evaluation based on linguistic Bayesian network and OWA operator", *Journal*

of Computational Information System, vol. 9, no. 7, pp. 2889-2896, 2013.

[11]   S. Mahadevan, R. Zhang, and N. Smith, "Bayesian networks for system reliability reassessment", *Structural Safety*, vol. 23, pp. 231-251, 2001.

[12]   A. G. Wilson, and A. V. Huzurbazar, "Bayesian networks for multilevel system reliability", *Reliability Engineering and System Safety*, vol. 92, no. 10, pp. 1413-1420, 2007.

[13]   Y. Lu, Q. M. Li, and Z. P. Zhou, "Safety risk prediction of subway operation based on fuzzy Bayesian network", *Chinese Journal of Southeast University*, vol. 40, no. 5, pp. 1110-1114, 2010.

[14]   C. C. Chang, R. S. Chen, and P. R. Sun, "Applying fuzzy Bayesian maximum entropy to extrapolating deterioration in repairable systems", (Y. Zhaf, ed.), *Machine learning,* Intech, 2010, pp. 217-238.

[15]   C. Simon, and P. Weber, "Evidential networks for reliability analysis and performance evaluation of systems with imprecise knowledge", *IEEE Transactions on Reliability*, vol. 58, no.1, pp. 69-87, 2009.

[16]   Z. S. XU, "Dependent uncertain ordered weighted aggregation operators", *Information Fusion*, vol. 9, no. 2, pp. 310-316, 2008.