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Chaotic Particle Swarm Optimization Algorithm for Hub and Spoke Systems with Congestion

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Abstract: Considering the hub airports are traffic transfer points, the congestion is easily happened. The cost caused by congestion will rise significantly. The hub-and-spoke airline network optimization model with congestion cost is designed. In the actual operations of airlines, such problem is difficult to be solved by using the classical optimization methods. This paper presents a Particle Swarm Optimization (*PSO*) algorithm. To improve the performance of standard *PSO* algorithm and avoid trapping into local excellent result, a chaos *PSO* algorithm of traffic volume multi-path assignment is presented. Empirical analysis shows that optimization design with congestion cost can avoid the excessive congestion phenomenon in the hub nodes. The proposed algorithm can solve the non-linear network optimization problem efficiently.

Keywords: Chaos, congestion cost, hub-and-spoke airline network, particle swarm optimization.

1. INTRODUCTION

Airline network layout has important strategic significance for the long term operation and market competitiveness of a company. In air transportation planning, the two most commonly used transport network layouts are point-to-point (*PP*) and hub-and-spoke (*HS*) structure. The disadvantage of *PP* networks is that they cannot effectively use the economies of scale, and currently most researches mainly focus on *HS* airline network design [1].

The literature [2-5] respectively present single allocation or multiple allocation mixed integer programming network design model, in which the goal is generally considered operation cost and construction cost. The solution is Lagrange relaxation algorithm, dual-ascent algorithm, heuristic based on tabu search and branch and bound method.

Grove and O'Kelly [6] simulates the operation of single allocation *HS* system, and analyzes that aircraft delay is greatly influenced by the heavy flow in hub airports.

Some papers [7-12] have tackled the congestion effects restricting the amount of flow transiting through a hub by means of capacity constraints. However, in the study of network design, it is insufficient that we only consider that the flow of hub node cannot exceed its capacity limit, because with the traffic flow close to capacity, the airport begins to appear congestion and lower utilization of resources. So we must consider the congestion effect of hub airport. With the increasing traffic flow of hub airports, the congestion is increasing, operation cost will show non-linear increasing trend. The current airline network design model and its algorithm cannot effectively solve the congestion problem.

Based on the above analysis, this paper mainly studies on the *HS* network optimization design, considering the congestion cost in the objective function. We can use heuristic algorithms to handle non-linear objective function, such as neural network algorithm, genetic algorithm and ant algorithm. However, the shortcomings of these methods are to solve with low efficiency and poor stability, which cannot ensure the optimization effect of solution.

The particle swarm algorithm has a few individual numbers, simple calculation, good robustness and parallel computing advantage. In order to avoid falling into local optimal solution, it can combine with chaos and particle swarm optimization [13]. Thus, this paper constructs a chaotic particle swarm optimization algorithm (*CPSOA*) of multi path traffic assignment, the solving result can avoid the heavy traffic flow of any one hub node exceeding its capacity restriction, which has effective shunt effect. Also, it guarantees the efficiency for solving large nonlinear network optimization problems.

2. MODEL FORMULATION: *HS* NETWORK DESIGN WITH CONGESTION

Considering hub airport is gathering the large amount of traffic flow, congestion occurs mainly in the hub airport. Elhedhli and Hu [14] has considered the costs of the congestion effects explicitly on the objective function. Using a convex cost function that increases exponentially as more flows go through the hubs.

$$f(u) = au^b$$

where u is the flow at a hub; a and b are positive constants with $b \geq 1$.

Using the model of the uncapacitated multiple allocation p hub location problem (UMApHMP) [15], the flow through

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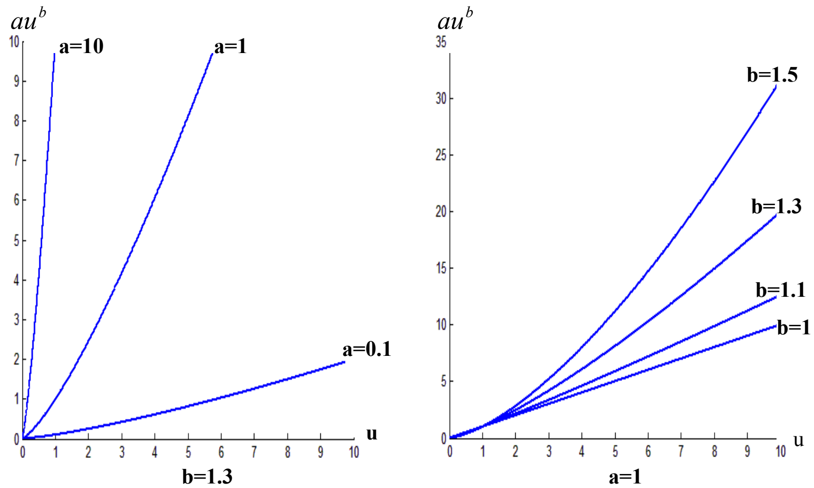

Fig. (1). The Congestion Cost Functions with Different Values of *a* and *b*.

hub k is $\sum_r \sum_{s>r} \sum_m W_{rs} X_{rskm}$. So, the congestion cost at hub k is $f(u)$.

$$f(u) = au^b = a(\sum_r \sum_{s>r} \sum_m W_{rs} X_{rskm})^b$$

Fig. (**1**) plots the congestion cost functions for different values of the parameters *a* and *b*.

Considering the congestion effect on network design, some improvement on UMA*p*HMP model is presented. The congestion convex cost function is added into the objective function. The model is stated as follows:

$$\min \quad Z = \sum_{r=1}^{n} \sum_{s=1}^{n} \sum_{k=1}^{n} \sum_{m=1}^{n} W_{rs} C_{rskm} x_{rskm}$$

$$+ \sum_k a(\sum_r \sum_{s>r} \sum_m W_{rs} X_{rskm})^b \tag{1}$$

$$s.t. \ \sum_{k=1}^{n} y_k = p \tag{2}$$

$$\sum_{k=1}^{n} \sum_{m=1}^{n} x_{rskm} = 1 \ ; \ r,s = 1,\cdots,n \tag{3}$$

$$\sum_{m=1}^{n} x_{rskm} \le y_k \ ; \ r,s,k = 1,\cdots,n \tag{4}$$

$$\sum_{k=1}^{n} x_{rskm} \le y_m \ ; \ r,s,m = 1,\cdots,n \tag{5}$$

$$y_k \in \{0,1\} \ ; k = 1,\cdots,n \ ;$$

$$0 \le x_{rskm} \le 1 \ ; \ r,s,k,m = 1,\cdots,n \tag{6}$$

The uncapacitated *HS* network design is defined on a graph $G=(N,A)$, where N is a set of nodes and A is the set of routes (r,s) (with origin node r and destination node s).

w_{rs} be the flow from node r to node s. We will generally assume that $w_{rs} = w_{sr}$, $r,s \in N$.

x_{rskm} be the fraction of flow from node r to node s that is routed *via* hubs at locations k and m in that order. If $k=m$, the variable x_{rskm} represents the one-hub-stop service through hub k.

c_{rskm} be the transportation cost per unit of flow from node r to node s routed *via* hubs k and m. k can be the same as m.

y_k be a binary variable defined as follows:

$$y_k = \begin{cases} 1 & \text{if a hub is loacated at node } k \\ 0 & \text{otherwise} \end{cases}.$$

The discount factor α represents the ecnomies of scale on the inter-hub connection, $0 < \alpha < 1$.

The objective function (1), Z calculates the sum of congestion cost and operating cost. Constraint (2) states that the number of hubs to be located is p. Constraints (3) assure that the flow for every pair r-s is routed *via* some hub pair. If only one hub is used, we have $k=m$. Constraints (4) and (5) express when a city is a non-hub city, there is no passengers transferring through it to other cities. Constraints (6) give the definition of decision variables.

The model is non-linear mixed integer planning problem. The papers [14, 16] solved it with Lagrangean heuristic algorithm and Benders algorithm. Considering the practicability of intelligent optimization algorithm, we use chaotic particle swarm optimization algorithm to solve this problem. *CPSO* algorithm has no limit for target function scale, which can be used to solve the nonlinear objective function model, and the complexity of the objective function has little impact on its convergence efficiency.

3. CHAOTIC PARTICLE SWARM OPTIMIZATION ALGORITHM

3.1. Algorithm Descriptions

Dedicated to computational intelligence research, Kennedy [17] first proposed *PSO* technique to replace the existing evolutionary techniques, such as genetic algorithm (*GA*) [18]. The algorithm simulates bird foraging for purpose of cluster flight sharing mechanism to make the group behavior to achieve optimal information in a group. Chaos is a kind of nonlinear phenomenon widely existing in nature. It is chaotic, and its internal structure is very delicate, and it is sensitive to initial conditions, in a certain range according to fixed rules, which are not repeated traversal of all state. Chaos have the characteristics of randomness, ergodicity and regularity. The chaos properties can be used to conduct search optimization, in order to integrate into particle swarm algorithm, namely, so-called *CPSOA*.

For an optimization problem, the decision variable X is n dimensional variable $X = [x_1, \cdots, x_n]$, and the variables are called the particles. Assumed in the *n*-dimensional solution space, each particle has two state description, position and velocity, respectively, $X_i = (X_{i1}, X_{i2}, \cdots, X_{in})$ and $V_i = (V_{i1}, V_{i2}, \cdots, V_{in})$. The position X_i represents the solution of problem.

The basic idea of *CPSOA* can be understood as: first initialized particle swarm and then in solution space, suppose in *t* times, and the optimal solution of particle *i* is $pbest_i(t)$, and this is individual extremum, and the optimal solution for the whole particle swarm is $gbest(t)$, which is global extremum. When iterate over the t+1 times, its update expression is:

$$V_i(t+1) = \omega V_i(t) + c_1 r_1 [pbest_i(t) - X_i(t)] + c_2 r_2 [gbest(t) - X_i(t)] \quad (7)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (8)$$

In the formula, *t* represents the current iteration times, c_1, c_2 represent two learning factors, c_1=c_2=2, And two random variables $r_1, r_2 \in (0,1)$. The inertia factor $\omega \in [\omega_{max}, \omega_{min}]$, t_{max} is the maximum number of iterations. In order to prevent the particle from leaving far away out of the searching space, the velocity of the particle is restricted in $[V_{min}, V_{max}]$, and the location of the particle also stays in permitted range, and finally the solution, *gbest*, is the global optimal.

According to the above mentioned basic particle swarm algorithm (*BPSO*), although it needs to confirm the parameters less, realizing process is simple, yet it is easy to trap into local optimum. Therefore chaotic dynamics is incorporated into the above *PSO*, the logistic reflection is defined as follows.

$$\beta_j^{k+1} = \mu \beta_j^k (1 - \beta_j^k) \ , \quad k = 1, 2 \cdots ,$$

$$\beta_j \in (0,1) \ , \quad \beta_j \neq 0.25, 0.5, 0.75 \quad (9)$$

3.2. Algorithm Implementation

The key point of *PSO* technique is to find a suitable expression to make the particle corresponding to the appropriate solution. One of the key problems of airline network optimization design is how to distribute the traffic flow in *O-D* path. Thus, this paper considers constructing a path number dimensional space, and passengers need to select a feasible path, and each path value is the distributed flow of this path, namely, the particle corresponding vector X_i represents a solution to the distribution of flow of each path.

The path flow f_k^{rs} and traffic demand q_{rs} of any *OD* path (*r*,*s*) need to meet the flow conservation principle, thus, we need to make corresponding revision in algorithm, The *CPSO* algorithm steps are listed as below.

Step 1, (initialization):

(1) initialize n, c_1, c_2, ω maximum number of iterations.

(2) initialize each dimension of X_i randomly between $[0, q_{rs}]$, that is, $f_k^{rs} = q_{rs} c_R$ ($c_R \in [0,1]$). In order to ensure the constant value q_{rs}, it needs to conduct normalized processing of f_k^{rs}.

$$\overline{f_k^{rs}} = q_{rs} f_k^{rs} / \sum_k f_k^{rs}$$

(3) Initialize each dimension of *V* randomly between [1-q_{rs},q_{rs}-1]

(4) Judge the fitness of all particles using objective function.

(5) Select the initial fitness values as the individual historical optimal solution P_i, then search for optimal solution within the swarm.

Step 2, Repeat until a stopping criterion or the maximal number of iteration is satisfied:

(1) For each particle, calculate particle swarm *V* and *X* from (7) and (8). When *V* and *X* exceed over the range, according to boundary value, and then normalized processing of *X*.

(2) Evaluate all the particles using objective function (1), when the current fitness function of a certain particle is better than its historical optimal fitness, the current fitness is remarked as historical optimal fitness, and the current position is the historical optimal location of this particle.

(3) Chaos optimized the optimal location P_g of current particle swarm using Logistic reflection (9), in order to obtain chaos optimization result.

(4) Seeking for the current optimal solution in the swarm. If it is better than the history optimal solution, then update P_g. If the plurality of individual have optimal solutions, the randomly selected one is the current optimal solution.

During the solution process of *CPSOA*, paralled distribute the flow for each feasible path, and flow distribution as the solution of the corresponding particle will adjust the

Table 1. 15 cities and their assigned number.

Num.	Cities	Num.	Cities	Num.	Cities
1	Beijing	6	Hangzhou	11	Wuhan
2	Changsha	7	Kunming	12	Urumqi
3	Chengdu	8	Nanjing	13	Xiamen
4	Guangzhou	9	Shanghai	14	Xi'an
5	Haikou	10	Shenyang	15	Zhenzhou

Table 2. Flow distribution ratio for 15 cities in China.

(*a*,*b*)	Flow Distribution Ratio			Max/min
	hub1	hub2	hub3	
a=0	42.8	22.7	34.5	1.89
(1,1.3)	39	27.2	33.8	1.43
(1,1.5)	37.5	29.5	33.0	1.27
(1,2.0)	35.2	32.2	32.6	1.09

traffic on the path using the historical experience of individual and groups, thus gradually close to optimal position, so as to obtain the optimal result.

4. COMPUTATIONAL EXPERIMENTS

HS topologies are an important class of network design that take full advantage of economies of scale on inter-hub connections. Hub location is very important and some airports having important region advantage become resources that airline companies use to compete. Considering the ranking order of cities, passenger traffic, geographical advantages and requirements of the Civil Aviation Administration of China, we can choose Beijing, Shanghai and Guangzhou as hub airports.

According to the airlines operational reports, the route cost is related to segment distance. In this computational example, we consider the route cost is the route distance corrected by empirical coefficient (airlines provided the cost of aircraft type operating the different routes). The passenger traffic between each pair of *OD* is obtained from the Civil Aviation Statistical Yearbook 2009. Suppose that an airline has established bases in Beijing, Shanghai and Guangzhou and prepares to construct HS network with 15 cities (See Table **1**).

The instance has been solved without congestion costs and then using b={1.3, 1.5, 2.0}. The flow distribution ratio of three fixed hubs are shown in Table **2**.

In Table **2**, the results of the first row don't consider the congestion effects. From the second column to the fourth column, the percentage of the total demand is calculated in each installed hub. The last column is the flow imbalance ratio, that is the ratio of the largest over the lowest percentage hub flow. For instance, on row one, the flow imbalance is taken as $\frac{42.8}{22.7}$=1.89. namely, the large flow imbalance would happen when no congestion effects are assumed. When the congestion cost is considered, the sorting of flow distribution of three hub airports is not varied, but the flow imbalance ratio is decreased. For instance, b=2, the flow imbalance ratio tends to its minimum value of 1. This can explained by the interference of congestion cost, the flow distribution of paths are very different from that of the uncapacitated model.

Figs. (**2**) and (**3**) are respectively flow distribution results of uncapacitated HS network and HS network with congestion cost. Although the two situations choose the same hub airports, their flow distribution path are different. In Fig. (**3**), the number of routes of network increases, the flow of *OD* is no longer simply distributed with the shortest path, such as, the *OD* flow from Xi'an to Nanjing, according to the shortest path, would select path as Xi'an-Beijing-Shanghai-Nanjing. Considering the congestion effect in hub airports, some passenger flow would choose Xi'an-Beijing-Nanjing, some other passenger flow would choose Xi'an-Shanghai-Nanjing. The *OD* flow from Shenyang to Shanghai, the shortest path is Shenyang-Beijing-Shanghai, considering the congestion cost in hub airports, some traffic flow would choose direct flight from Shenyang to Shanghai.

Table **3** represents the flow distribution of some paths with three groups of congestion parameters. The paths are randomly selected, and the number of each node corresponds with the city number in Table **1**. As a and b change, the distributed flow values of the same path are also different. The last three columns show that the flow distribution value of one path according to variations in a and b, and 0 represents no flow distribution of this path.

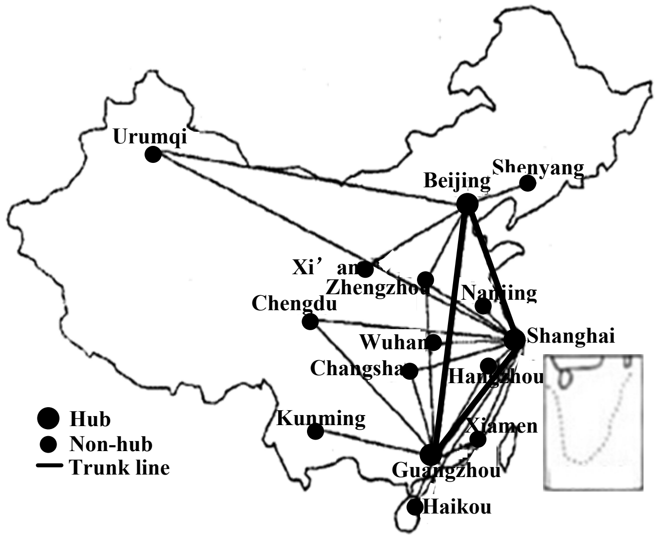

Fig. (2). Uncapacitated *HS* network ($p = 3,\ \alpha = 0.6,\ a = 0$).

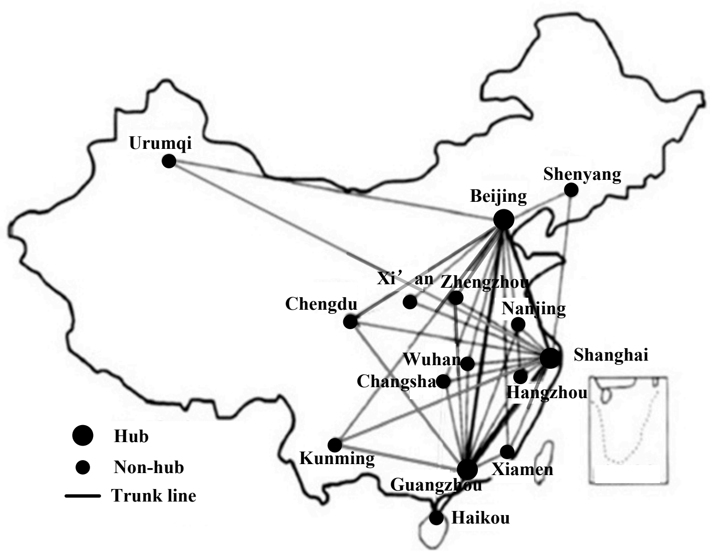

Fig. (3). *HS* network with congestion cost ($p = 3,\ \alpha = 0.6,\ a = 1,\ b = 2$).

Table 3. Flow distribution for some paths with three groups of congestion parameters.

Path Number	Some of Paths Set				Flow Distribution		
	r	*k*	*m*	*s*	*a*=1, *b*=1.1	*a*=1, *b*=1.5	*a*=1, *b*=2
1	1	1	1	3	199	103	96
2	1	4	4	10	0	0	74
3	2	1	1	3	0	59	101
4	2	4	9	10	0	0	0
5	3	4	4	6	0	42	109
6	3	9	9	2	134	98	57

Table 3. contd…

Path Number	Some of Paths Set				Flow Distribution		
	r	*k*	*m*	*s*	*a*=1, *b*=1.1	*a*=1, *b*=1.5	*a*=1, *b*=2
7	4	9	9	3	0	0	0
8	5	4	1	6	0	0	0
9	5	9	9	15	0	0	0
10	6	4	4	9	0	0	79
11	7	9	9	11	35	50	61
12	8	9	9	4	61	21	0.1
13	8	4	4	15	186	126	74
14	10	4	4	6	0	0	0
15	11	4	9	5	0	0	0
16	12	1	1	4	111	60	58
17	14	1	9	2	0	0	0
18	15	4	4	13	0	59	137

The results show that any hub airport can avoid overloaded traffic flow when considering non-linear congestion cost in the objective function, and the unbalanced flow distribution is reduced between hub airports. For example, As parameter *a* and *b* increase, the flow of some paths are changed from zero to a distributed value, such as the 2nd and 3rd path, also the distributed large amount of flow of some paths are diverted, such as 1st and 6th path. Therefore, some busy hub airports can reduce congestion in order to avoid the corresponding flight delay.

CONCLUSION

This paper proposes *CPSO* algorithm aimed to *HS* network design considering congestion cost at hub airports. The objective function is added non-linear congestion cost, when the flow value of hubs increases, the congestion cost is a convex function, which would increase exponentially, therefore, the flow of hub airports can be diverted effectively to minimize the total cost. Comparing with other algorithms of airline network design, *CPSO* can quickly get the traffic flow distribution of multi paths under specified hubs.

Given each *OD* demand, *CPSO* algorithm can get the results of the flow distribution. Before optimization, this algorithm must be determined the set of paths for each *OD* pair by the exhaustive procedure. Using the method of the mutative scale chaos mutation, which can be very good to accelerate the convergence speed and avoid trapping into local optimization.

Through example demonstration and comparison analysis of Chinese airline transport data, the results show that the algorithm has a good shunt effect on effectively solving hub node congestion problem, and it tries to alleviate the flight delay degree caused by hub node flow increase.

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CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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