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Generalized Projective Synchronization Based on Active Control Method

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Abstract: Based on active control method, this paper successfully realizes generalized projective synchronization for robust chaotic systems with exponential item that Bao Bocheng etc. proposed. The numerical simulation results verify validity of generalized projective synchronization controller. Meanwhile, the design of the controller circuit and its simulation results also show the circuit realization's feasibility and that the design of generalized projective synchronization controller is correct.

Keywords: Active control method, generalized projective synchronization, numerical and circuit simulation.

1. INTRODUCTION

Bao Bocheng *et al.* proposed a three-dimensional robust chaos system with exponential item [1], and realized with circuit [2]. This article is a study on the applicability of synchronization control of three-dimensional chaotic system, to achieve generalized projective synchronization for two chaotic systems using active control method. The generalized projective synchronization studied in this paper refers to link the generalized synchronization and projective synchronization together, and to achieve a chaotic signal having arbitrary ratio to the output of original driving chaotic system by a scale factor [3-5]. And the correctness of the method is verified through the theoretical analysis and numerical and circuit simulation.

2. ROBUST CHAOS SYSTEM WITH EXPONENTIAL

Bao Bocheng *et al.* [1] proposed a three-dimensional continuous autonomous dissipative chaotic system, the system equations contain 5 items on the right with the absence of constant controller, as a least item for three-dimensional system that generate chaotic stream, the mathematical model is as follows:

$$\begin{cases} \dot{x} = y - x \\ \dot{y} = xz + c \\ \dot{z} = \mu - \exp(xy) \end{cases} \quad (1)$$

Where μ is a real constant, c is a constant control parameters, x, y and z are state variables. The new system on the right equation contains an index square nonlinear term, a square nonlinear term, a constant parameter and two linear terms. When $\mu = 5$ and $c = 0$, the system (1) generates a typical double scroll chaos, as shown in Fig. (**1a** and **1b**); when $\mu = 5$ and $c = 2$, the system (1) generates a typical single

scroll chaotic attractor, as shown in Fig. (**1c**). Literature illustrated the dynamic behavior of the chaotic system specifically, and the results illustrate that the positive Lyapunov exponent area for system (1) is wide within parameter μ change range, which means the chaotic characteristics of the system are robust.

3. DESIGN OF GENERALIZED PROJECTIVE SYNCHRONIZATION CONTROLLER

The drive and response vectors achieve projective synchronization by scaling factor *k*, which also means the vector is adjustable. The system (1) is used as a driving system, which modified as is follows:

$$\begin{cases} \dot{x} = y_1 - x_1 \\ \dot{y}_1 = x_1 z_1 + c \\ \dot{z}_1 = \mu - \exp(x_1 y_1) \end{cases} \quad (2)$$

The controlled response system:

$$\begin{cases} \dot{x}_2 = y_2 - x_2 + u_1 \\ \dot{y}_2 = x_2 z_2 + c + u_2 \\ \dot{z}_2 = \mu - \exp(x_2 y_2) + u_3 \end{cases} \quad (3)$$

Where u_1, u_2, u_3 are synchronous controllers.

Suppose error signals $e_1 = x_1 - kx_2$, $e_2 = y_1 - ky_2$, $e_3 = z_1 - kz_2$, where the scaling factor is constant *k*, error equation as follows:

$$\begin{cases} \dot{e}_1 = e_2 - e_1 - ku_1 \\ \dot{e}_2 = x_1 z_1 + c - kx_2 z_2 - kc - ku_2 \\ \dot{e}_3 = \mu - \exp(x_1 y_1) - k\mu + k\exp(x_2 y_2) - ku_3 \end{cases} \quad (4)$$

Theorem 1: If the nonlinear feedback synchronization controller is

$$\begin{cases} u_1 = 0 \\ u_2 = \beta_1 (y_1/k - y_2) + x_1 z_1/k + (1/k-1)c - x_2 z_2 \\ u_3 = (1/k-1)\mu - \exp(x_1 y_1)/k + \exp(x_2 y_2) + \beta_2 (z_1/k - z_2) \end{cases} \quad (5)$$

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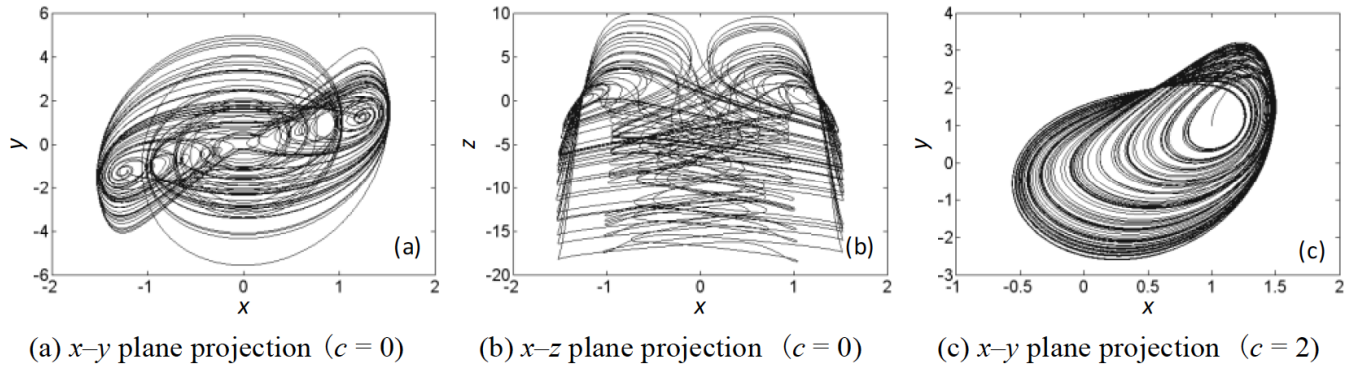
(a) x–y plane projection $(c = 0)$ (b) x–z plane projection $(c = 0)$ (c) x–y plane projection $(c = 2)$

Fig. (1). New chaotic attractors.

Where $\beta_1 > 0$ and $\beta_2 > 0$, then the chaotic systems (2) and (3) with different initial value, can achieve generalized projective synchronization.

Proof: according to the idea of active control, the control functions are designed as follows:

$$\begin{cases} \dot{e}_1 = e_2 - e_1 + v_1 \\ \dot{e}_2 = v_2 \\ \dot{e}_3 = v_3 \end{cases} \begin{cases} ku_1 = -v_1 \\ ku_2 = -v_2 + x_1 z_1 + c - kx_2 z_2 - kc \\ ku_3 = -v_3 + \mu - \exp(x_1 y_1) - k\mu + k\exp(x_2 y_2) \end{cases} \quad (6)$$

By substituting it into the error system (4) is acquired:

$$\begin{cases} \dot{e}_1 = e_2 - e_1 + v_1 \\ \dot{e}_2 = v_2 \\ \dot{e}_3 = v_3 \end{cases} \quad (7)$$

v_1, v_2, v_3 are error variables, e_1, e_2, e_3 are input control signals of the function. Set up

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = A\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\beta_1 & 0 \\ 0 & 0 & -\beta_2 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad (8)$$

By substituting into equation (7), is acquired:

$$\begin{cases} \dot{e}_1 = e_2 - e_1 \\ \dot{e}_2 = -\beta_1 e_2 \\ \dot{e}_3 = -\beta_2 e_3 \end{cases} \quad (9)$$

The corresponding characteristic equation of error system (9) is:

$$(\lambda + 1)(\lambda + \beta_1)(\lambda + \beta_2) = 0 \quad (10)$$

According to the linear system stability theory, as long as $\beta_1 > 0$ and $\beta_2 > 0$, all characteristic roots of the error system (10) will be negative, so that the error signals e_1, e_2, e_3 converge into zero, which means that the zero equilibrium point for the error system is asymptotically stable.

$\lim_{t \to \infty} E(t) = \lim_{t \to \infty}[X(t) - kY(t)] = 0$ shows that two three-dimensional chaotic systems obtain same structural generalized projective synchronization with arbitrary proportion. Formula (5) gives the designed controllers that are obtained by substituting formula (8) into (6).

The feature values of error system (10) are related to the speed that error tends to be zero. According to the system stability theory, the smaller the negative eigenvalues, the faster the response of the system. Therefore, from the characteristic equation (10), it is shown that the convergence speed of the error signals is related to the choice of β_1 and β_2. β_1 and β_2 are error convergence speed regulators.

4. NUMERICAL SIMULATION OF GENERALIZED PROJECTIVE SYNCHRONIZATION

Using Runge-Kutta method for numerical simulation, a group of parameters $\mu = 5$, $\beta_1 = 1$ were selected, $\beta_2 = 1$, so that the system is in chaotic state, initial values $(1, 1, 1)$ and $(2, -1, 0.5)$ were set for drive system and response system, respectively. And $c = 0$, when the scale factors for generalized projective synchronization k are 0.4 and -1.2 respectively, the generalized projective synchronization simulation results are shown in Figs. (**2**, **3**). When the control parameter c is 2 and k is -0.5, generalized projective synchronization
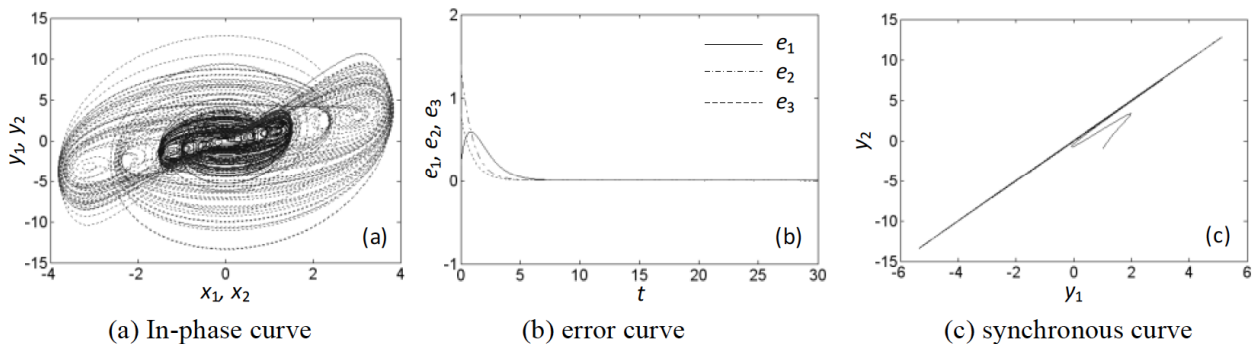

(a) In-phase curve   (b) error curve   (c) synchronous curve

Fig. (2). Generalized projective synchronization at K = 0.4.

(a) Anti-phase curve   (b) error curve   (c) synchronous curve
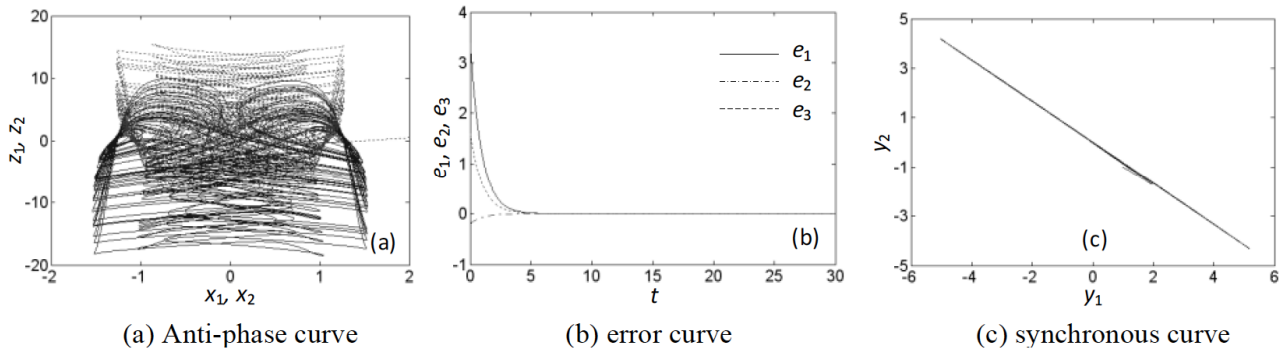
Fig. (3). Generalized projective synchronization at $K = -1.2$.

simulation results are shown in Fig. (**4**). In the picture, the projection phase orbit with solid and dashed lines are attractors of drive system and response system respectively.

Generalized projective synchronization of in-phase as shown in Fig. (**2a**) indicates the characteristics of the drive system and the response system. Fig. (**2b**) gives error signal evolution curve at $k = 0.4$, error initial values are $e_1(0) = 0.2$, $e_2(0) = 1.4$, $e_3(0) = 0.8$, we can see the error curve quickly decays to 0, indicating that state variables between two three-dimensional chaotic systems are controlled by the circuit.

5. CIRCUIT DESIGN AND SIMULATION

We realized generalized projective synchronization of robust chaotic systems with exponential item using the cir-

cuit designed in the literature [2]. The control circuit corresponding to the formula (5) is shown in Fig. (**5**), which achieved state variables in-phase or anti-phase generalized projective synchronization between the drive system and the response system respectively.

The control circuit method shows that in order to realize two systems generalized projective synchronization, it required only to adjust the linear resistor vector [R1c, R2c, R3c, R4c, R5c] in proportional action, and the corresponding feedback signal vector [S1c, S2c, S3c, S4c, S5c] is coupled into the response system. The feedback signals, in-phase and anti-phase signals, in the controller are derived from state variables and intermediate variables of drive system and response system respectively. With resistance vector in the controller, the feedback signal is in-
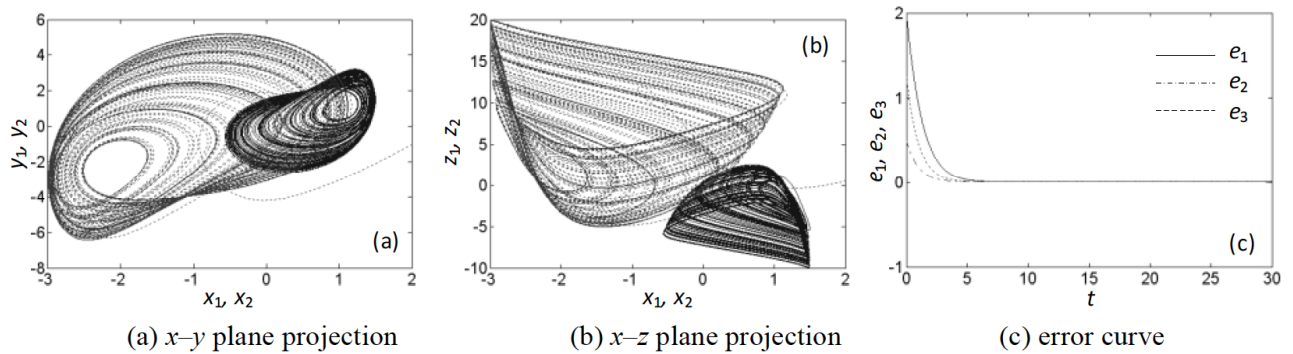

(a) x–y plane projection   (b) x–z plane projection   (c) error curve

Fig. (4). Anti-phase synchronization at $c = 2$.
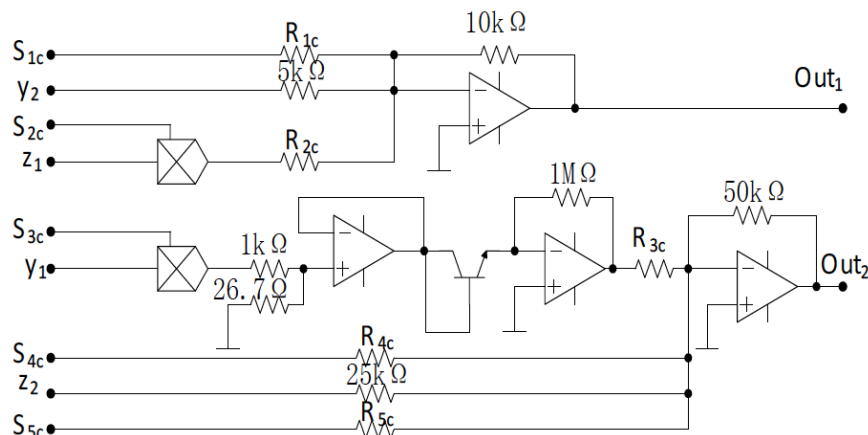
Fig. (5). Example of a ONE-COLUMN figure caption.

(a) y_1–$y_2(k = 0.4)$, 1/2   (b) y_1–y_2 (k =–1.2), 2/5   (c) y_1–$y_2(k = -0.5)$, 2/2
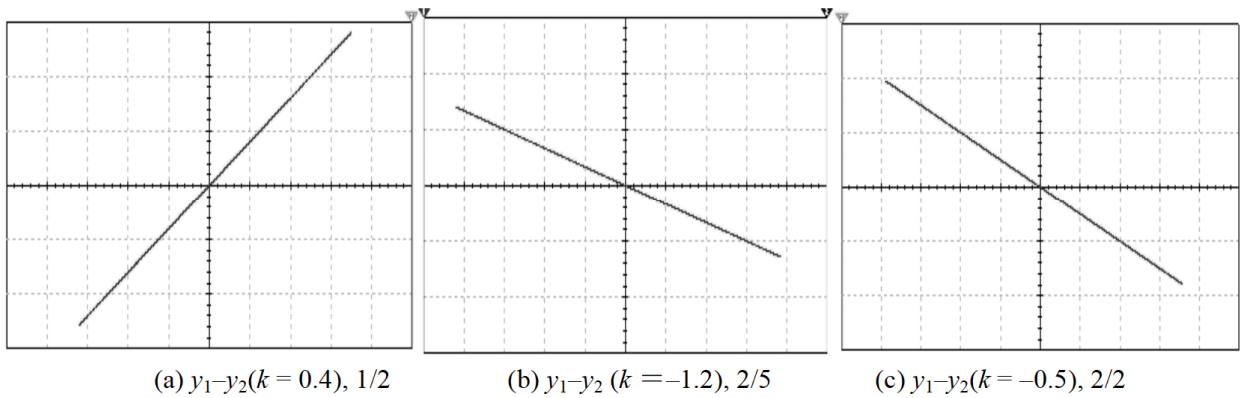
Fig. (6). Synchronization curves of simulation circuit.

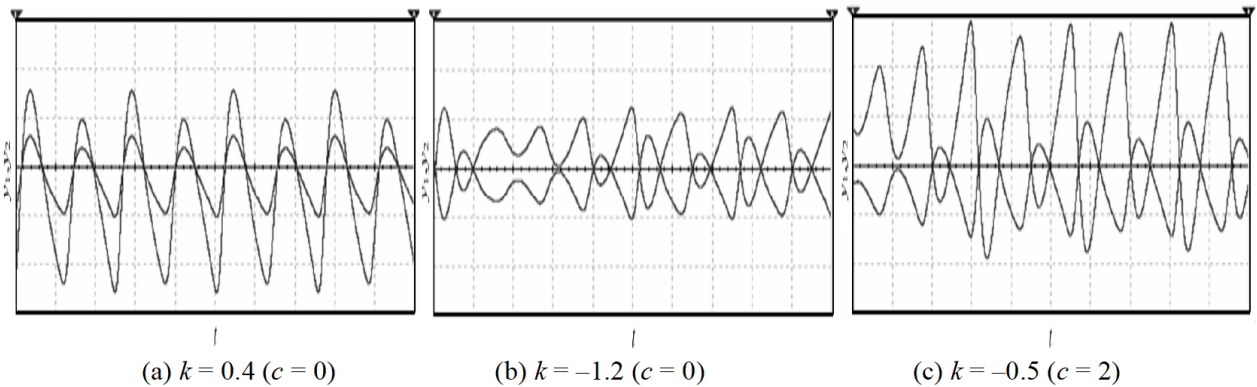(a) $k = 0.4$ ($c = 0$)   (b) $k = -1.2$ ($c = 0$)   (c) $k = -0.5$ ($c = 2$)

Fig. (7). State output signals of the drive system and the response system.

corporated into the addition operation module of the response system as an added signal. In Fig. (**5**), at k=0.4, the feedback signal vectors [S_{1c}, S_{2c}, S_{3c}, S_{4c}, S_{5c}] and the controller resistance vectors [R_{1c}, R_{2c}, R_{3c}, R_{4c}, R_{5c}] are [−y_1, −x_1, x_1, −z_1, −v_c] and [$2k\Omega$, $4k\Omega$, $20k\Omega$, $10k\Omega$, $4k\Omega$], respectively. At k=−1.2, the feedback signal vectors [S_{1c}, S_{2c}, S_{3c}, S_{4c}, S_{5c}] and the controller resistance vectors [R_{1c}, R_{2c}, R_{3c}, R_{4c}, R_{5c}] are [y_1, x_1, x_1, z_1, v_c] and [$12k\Omega$, $15k\Omega$, $60k\Omega$, $60k\Omega$, $50k\Omega$], respectively.

Circuit simulation was performed according to system parameter values in the fourth part of numerical emulation, the synchronization status between drive system and response system is shown in figure 6. In the graph, y_1–y_2 is the phase trajectory map, coordinate scale unit is V/div, such as 1/2 means that transverse coordinate of each grid is 1V, ordinate per cell is 2V. In Figs. (**6a, b** and **c**), numerical values can be obtained by oscilloscope, the ratio of state variables y_1 and y_2 is between 0.4, -1.2, -0.5, which illustrates the response system and drive system achieved a certain proportion of the generalized projective synchronization.

As shown in Fig. (**7**), the nonlinear controller designed in same structure generalized projective synchronization can connect the drive system and the response system, through proportional adjustment of the linear resistor vector, and the corresponding feedback signal vector coupled to the response system, so that the two chaotic attractors phase can be same or opposite. The output amplitude when the scale

factor is 0.4,–1.2,–0.5(c=2), verifies the possibility to achieve generalized projective synchronization between the drive system and the response system with the scale factor.

6. CONCLUSION

Analysis of the structural characteristics of chaotic attractor, the generalized projective synchronization controller, and the generalized projective synchronization of chaotic attractor can be verified by numerical simulation and circuit simulation.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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