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Research of Strip Flatness Control Based on Legendre Polynomial Decomposition

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Abstract: A traditional least squares has negatively a clear physical significance in the process of strip flatness pattern recognition and it is too complex in calculation to be suitable for industrial controller to realize. In this paper we deduced the basic model of strip flatness pattern recognition based on Legendre polynomial. Under this mode, there are three outstanding advantages: The integral value along the width direction of strip shape is 0, which satisfies the condition of equilibrium in the residual stress. Each component of polynomial corresponds to steer roller, bending roller and step cooling control respectively, which makes the physical meaning clear. Operational calculation is suitable for industrial controller. The simulation shows that calculation time-consuming dropped from average 0.012 s to average 0.002 s. Also practice in the real world had proved that this method is effective and applicable.

Keywords: Legendre polynomial, Least squares, Flatness recognition, Flatness control.

1. INTRODUCTION

Flatness quality is an important index of plate and strip products. Under the condition that the thickness control technology is almost mature, flatness control has already become the key part of process on high quality plate and strip. Fine flatness can not only improve coating thickness and uniformity, but also ensure the plate and strip deep drawing performance and the follow-up processes. Furthermore, it can reduce the probability of roller change and belt broken, and increase run speed and production efficiency. However, there are many factors which can influence the flatness, so the strip flatness control is the core and hard technology in the wide strip mill.

The basic principle of flatness control is to ensure the similarity between the section shape of entrance and delivery [1]. The basic control methods are steer roller, bending roller and step cooling control [2]. Steer is used to adjust first order of strip section shape, and bending is used to adjust the second order of strip section shape. The rest of high orders are performed by step cooling control [3]. In these components, first order and second order which are weighted heavily are the foundation of the strip shape quality. Though the weight of high orders is smaller, it is important for high precision control on strip shape. Therefore, we need to perform pattern recognition based on the features of strip shape control devices to decompose each order precisely, so that we can get high quality strip shape.

Currently, the widely used method of flatness pattern recognition and flatness signal processing is polynomial regression analysis based on least squares [4]. This method uses the simplest least-squares method, which regresses the deviation detected by flatness gauge into the form of a polynomial. The algorithm is simpler, yet there are a lot of defects. The main disadvantage is that the algorithm is complex in calculation, and the physical meaning is not clear. Besides, the polynomial model cannot be converted to the regulating variables of the flatness control device, which makes it difficult to be used for control directly. In addition, in order to enhance the accuracy, the orders of polynomial have to be increased, which will produce more polynomials, further weakening the physical meaning. The extra parameters go against the analysis and control [5, 6]. Recent years, people have put forward to use Legendre polynomial as the basic model of pattern recognition to fit several common strip shapes in the industry. The orthogonal Legendre polynomial model overcomes the defects of complex computing and unclear physical meaning [7].

In this paper, we mainly discuss the following problem: (1) Deduce the basic model of flatness pattern recognition based on Legendre polynomial. (2) Analysis of shape control algorithm by means of Legendre polynomial. (3) Discuss the comparison results between least squares and modified least squares method based on Legendre polynomial.

2. DERIVATION OF LEGENDRE POLYNOMIALS

Legendre equation as well as its solution function is named by the French mathematician Adrien-Marie Legendre. Legendre equation is a kind of ordinary differential equation which physics and other technical fields often encounter [8, 9]. When trying to solve three-dimensional Laplace equations in spherical coordinates (or related partial differen-

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tial equations), the problems will be down to solve Legendre equation.

Legendre equation is as follows:

$$(1-x^2)y''-2xy'+n(n+1)y=0 \tag{1}$$

Which is

$$(1-x^2)\frac{d^2p(x)}{dx^2}-2x\frac{dp(x)}{dx}+n(n+1)p(x)=0 \tag{2}$$

Among them, n is any real number.

Set solution as the form of series, we get:

$$y=p(x)=x^c(a_0+a_1x+a_2x^2+\cdots+a_kx^k+\cdots)=\sum_{k=0}^{\infty}a_kx^{k+c} \tag{3}$$

Differential termwise, we can obtain:

$$\frac{dy}{dx}=\sum_{k=0}^{\infty}(k+c)a_kx^{k+c-1}$$

$$\frac{d^2y}{dx^2}=\sum_{k=0}^{\infty}(k+c)(k+c-1)a_kx^{k+c-2}$$

Bring the two differential results into the Legendre equation:

$$\begin{aligned}&(1-x^2)\sum_{k=0}^{\infty}(k+c)(k+c-1)a_kx^{k+c-2}\\&-2x\sum_{k=0}^{\infty}(k+c)a_kx^{k+c-1}+n(n+1)\sum_{k=0}^{\infty}a_kx^{k+c}=0\\&=\sum_{k=0}^{\infty}(k+c)(k+c-1)a_kx^{k+c-2}\\&-x^2\sum_{k=0}^{\infty}(k+c)(k+c-1)a_kx^{k+c-2}\\&-2x\sum_{k=0}^{\infty}(k+c)a_kx^{k+c-1}+n(n+1)\sum_{k=0}^{\infty}a_kx^{k+c}\\&=\sum_{k=0}^{\infty}(k+c)(k+c-1)a_kx^{k+c-2}\\&-\sum_{k=0}^{\infty}(k+c)(k+c+1)a_kx^{k+c}+n(n+1)\sum_{k=0}^{\infty}a_kx^{k+c}\end{aligned}$$

Then we can come to the conclusion:

$$\begin{aligned}&\sum_{k=0}^{\infty}(k+c)(k+c-1)a_kx^{k+c-2}\\&-\sum_{k=0}^{\infty}[(k+c)(k+c+1)-n(n+1)]a_kx^{k+c}=0\end{aligned} \tag{4}$$

Separate the items before k=0 and the items after k=1 in (4), after finishing reduction we get the following formula:

$$\begin{aligned}&c(c-1)a_0x^{c-2}+c(1+c)a_1x^{c-1}+\{\sum_{k=0}^{\infty}(k+c+2)(k+c+1)a_{k+2}\\&-[(k+c)(k+c+1)-n(n+1)]a_k\}x^{k+c}=0\end{aligned} \tag{5}$$

Obviously:

$$c(c-1)a_0=0$$

$$c(c+1)a_1=0$$

$$\begin{aligned}&(k+c)(k+c+1)a_{k+2}\\&-[(k+c)(k+c+1)-n(n+1)]a_k=0\end{aligned}$$

Then we can get recursive formula:

$$a_{k+2}=\frac{(k-n)(k+n+1)}{(k+2)(k+1)}a_k \tag{6}$$

Therefore, we can represent Legendre polynomial solutions with the lowest terms.

$$y(x)=a_0y_0+a_1y_1$$

$$a_0y_0=a_0[1-\frac{n(n+1)}{2!}x^2+\frac{n(n-2)(n+1)(n+3)}{4!}\cdots]$$

$$\begin{aligned}&a_1y_1=a_1[x-\frac{(n-1)(n+2)}{3!}x^3\\&+\frac{(n-1)(n-3)(n+2)(n+4)}{5!}x^5\cdots]\end{aligned}$$

Obviously, the solution is composed of odd and even number terms.

After finishing recursive formula, we get:

$$a_k=\frac{(k+2)(k+1)}{(k-n)(k+n+1)}a_{k+2} \tag{7}$$

Through equation (7), when K = n - 2 as the first item, bring it into the recursive formula

$$a_{n-2}=-\frac{n(n-1)}{2(2n-1)}a_n$$

Then, we set the highest item

$$a_n=\frac{1\cdot 3\cdot 5\cdots(2n-1)}{n!}=\frac{(2n)!}{2^n(n!)^2}\quad \text{(n=0, 1, 2 …)}$$

We can get the following expression:

$$a_{n-2}=\frac{n(n-1)}{-2(2n-1)}\cdot\frac{(2n)!}{2^n(n!)^2}=(-1)\frac{(2n-2)!}{2^n(n-1)!(n-2)!}$$

$$a_{n-4}=(-1)^2\frac{(2n-4)!}{2^n2!(n-2)!(n-4)!}$$

$$\cdots$$

$$a_{n-2s}=(-1)^s\frac{(2n-2s)!}{2^ns!(n-s)!(n-2s)!}$$

$s = 0,1,2\cdots[\frac{n}{2}]$

$[\frac{n}{2}]$ is the greatest integer less than $\frac{n}{2}$.

When n is an even number, we can calculate that:

$$a_0 y_0 = \sum_{s=0}^{\frac{n}{2}} (-1)^s \frac{(2n-2s)!}{2^n s!(n-s)!(n-2s)!} s^{n-2s}$$

When n is an odd number, we can calculate that:

$$a_1 y_1 = \sum_{s=0}^{\frac{n-1}{2}} (-1)^s \frac{(2n-2s)!}{2^n s!(n-s)!(n-2s)!} s^{n-2s}$$

After combining the two formulas, it can be expressed as:

$$y = p(x) = \sum_{s=0}^{[\frac{n}{2}]} \frac{(2n-2s)!}{2^n s!(n-s)!(n-2s)!} x^{n-2s} \tag{8}$$

Then, according to the general term formula, we can solve the Legendre polynomial.

3. THE MODIFIED ALGORITHM METHOD

There are six kinds of common engineering strip shapes. They are left waves, right waves, middle waves, bilateral waves, quartered waves and edge waves. When establishing the mathematical model, we need to satisfy the condition of equilibrium in the residual stress. The so-called equilibrium in the residual stress is that the integral value along the width direction of strip shape is 0 after rolling. The formula is as follows.

$$\int_{-1}^{1} \delta(x)dx = 0 \tag{9}$$

The Legendre polynomial will converge to infinite series when the integral is from -1 to 1. In order to make some certain items of Legendre polynomial become the basic model of flatness pattern recognition, we choose the first, second and fourth as the basic model, they are:

Left waves: $y_1 = p_1(x) = x$

Right waves: $y_2 = -p_1(x) = -x$

Middle waves: $y_3 = p_2(x) = \frac{3x^2-1}{2}$

Bilateral waves: $y_4 = -p_2(x) = -\frac{3x^2-1}{2}$

Quartered waves: $y_5 = p_4(x) = \frac{35x^4-30x^2+3}{8}$

Edge waves: $y_6 = -p_4(x) = -\frac{35x^4-30x^2+3}{8}$

The stress distribution curve of strip flatness is shown below in Fig. (**1**). Both X-axis and Y-axis units are non-unitized.

X-axis expresses

(length from center to edge)/ (total length)

Y-axis expresses

(center thickness - edge thickness)/ (center thickness)

The linear combination of flatness basic model is:

$$\delta(x) = a_1 p_1(x) + a_2 p_2(x) + a_4 p_4(x)$$

We detect a set of discrete data (y_i, σ_i) through measuring roller, $i = 1,2,\cdots,n$. n is the number of points we measure. y_i is the abscissa, and σ_i is model deviation value in y_i .

Based on the least squares method, we can get the following formula:

$$\min \sum_{i=1}^{n} [p(x_i) - \delta_i]^2 \tag{10}$$

Convert into the multivariate function:

$$\phi(a_1, a_2, a_4) = \sum_{i=1}^{n} [\sum_{j=1}^{1,2,4} a_j p_j(x_i) - \delta_i]^2 \tag{11}$$

Differentiate the above formula:

$$\frac{\partial \phi}{\partial a_k} = 2\sum_{i=1}^{n} [\sum_{j=1}^{1,2,4} a_j p_j(x_i) - \sigma_i] p_k(x_i) = 0 \text{ (k=1, 2, 4)} \tag{12}$$

Set $c_{jk} = \sum_{i=1}^{n} p_j(x_i) p_k(x_i)$, $b_k = \sum_{i=1}^{n} p_k(x_i) \sigma_i$ (k=1,2,4)

Bring the above formula into formula (12):

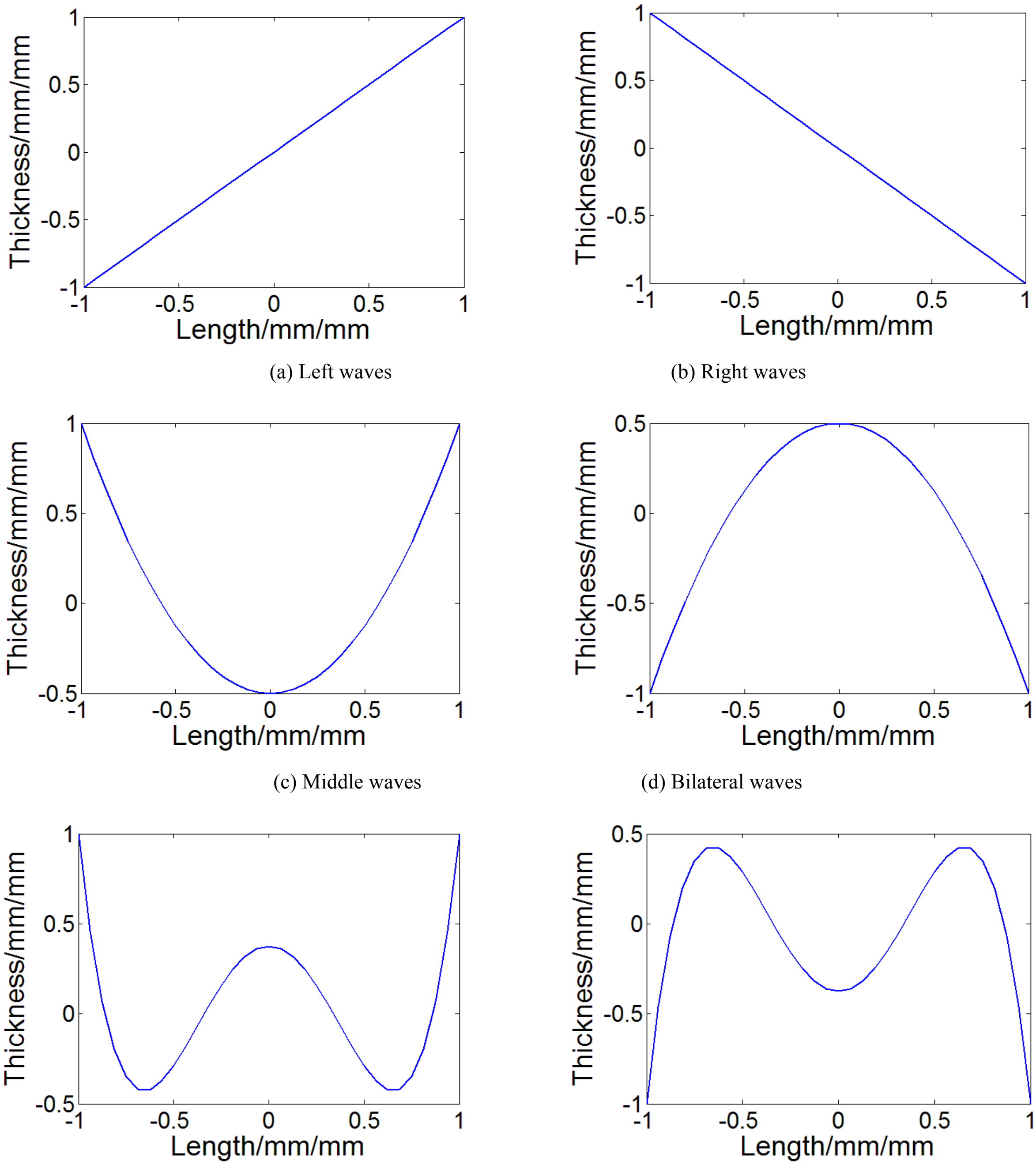
$$\sum_{j=1}^{1,2,4} c_{jk} . a_j = b_k \text{ (k=1, 2, 4)} \tag{13}$$

Written in matrix form:

$$C \bullet A = B \tag{14}$$

And, $A = \begin{pmatrix} a_1 \\ a_2 \\ a_4 \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ b_4 \end{pmatrix}$, $C = \begin{pmatrix} c_{11} & c_{12} & c_{14} \\ c_{21} & c_{22} & c_{24} \\ c_{41} & c_{42} & c_{44} \end{pmatrix}$

Calculate the coefficients of matrix C and matrix B.



(a) Left waves (b) Right waves

(c) Middle waves (d) Bilateral waves

(e) Quartered waves (f) Edge waves

Fig. (1). The stress distribution curve of strip flatness.

$$C=\begin{pmatrix} \sum_{i=1}^{n} p_1(x_i)p_1(x_i) & \sum_{i=1}^{n} p_1(x_i)p_2(x_i) & \sum_{i=1}^{n} p_1(x_i)p_4(x_i) \\ \sum_{i=1}^{n} p_2(x_i)p_1(x_i) & \sum_{i=1}^{n} p_2(x_i)p_2(x_i) & \sum_{i=1}^{n} p_2(x_i)p_4(x_i) \\ \sum_{i=1}^{n} p_4(x_i)p_1(x_i) & \sum_{i=1}^{n} p_4(x_i)p_2(x_i) & \sum_{i=1}^{n} p_4(x_i)p_4(x_i) \end{pmatrix} \qquad B=\begin{pmatrix} \sum_{i=1}^{n} p_1(x_i)\sigma_i \\ \sum_{i=1}^{n} p_2(x_i)\sigma_i \\ \sum_{i=1}^{n} p_4(x_i)\sigma_i \end{pmatrix}$$

Using Matlab, We can calculate the results directly by left except.

Table 1. The first group of test data (unit: mm).

1	2	3	4	5
-0.31	-0.25	-0.197	-0.147	-0.101
6	7	8	9	10
-0.06	-0.02	0.012	-0.042	0.069
11	12	13	14	15
0.091	0.111	0.126	0.138	0.147
16	17	18	19	20
0.152	0.153	0.151	0.146	0.137
21	22	23	24	25
0.124	0.108	0.088	0.133	0.065
26	27	28	29	30
0.038	0.007	-0.028	-0.066	-0.109
31	32	33		
-0.155	-0.206	-0.32		

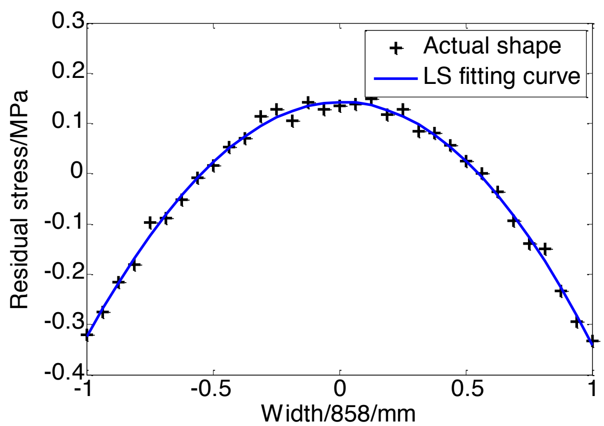


Fig. (2). Comparison of the actual shape curve and the least squares fitting curve (Table **1**).

$$A = C \setminus B \tag{15}$$

We can get dispensable mold, quadratic form, biquadratic form so as to achieve the purpose of controlling strip shape.

4. SIMULATION

In this section, we discuss the comparison results between least squares and modified least squares method based on Legendre polynomial. The modified LS can reflect the physical meaning of the control variables better and it has faster computing speed in theory. So we compare the two algorithms through some data obtained from industrial production. We mainly complete the following tasks: (1) Test accuracy of the modified method. (2) Compare operation speed of these two kinds of algorithms.

The simulation platform we used is Intel Core-2 due 2.2GHz CPU and Matlab 7.14.

Two groups of data are obtained from the real world. The first group of data are shown in the Table **1**, a total of 33 group detection units.

Comparison of the actual shape and the least squares fitting curve are shown in Fig. (**2**). Comparison of the actual shape and the modified least squares fitting curve are shown in Fig. (**3**). Comparison of the least squares fitting curve and the modified least squares fitting curve are shown in Fig. (**4**).

The second group of data are shown in the following Table **2**, a total of 33 group detection units.

Comparison of the actual shape and the least squares fitting curve are shown in Fig. (**5**). Comparison of the actual shape and the modified least squares fitting curve are shown in Fig. (**6**). Comparison of the least squares fitting curve and the modified least squares fitting curve are shown in Fig. (**7**).

From the results of these two groups' data, we can come to a conclusion that the modified least squares curve fits more closely to the actual shape. Accuracy is similar to the least squares.

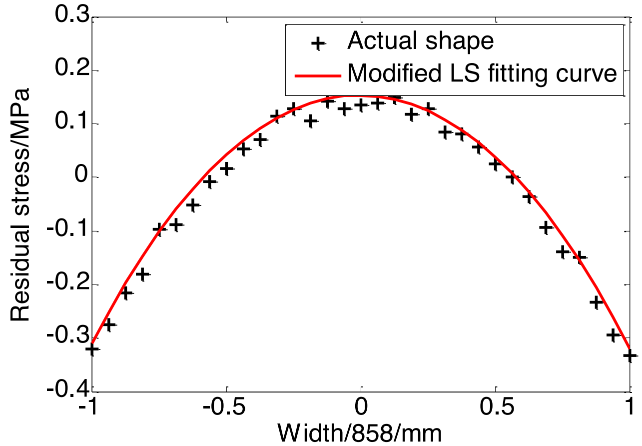


Fig. (3). Comparison of the actual shape and the modified least squares fitting curve (Table **1**).

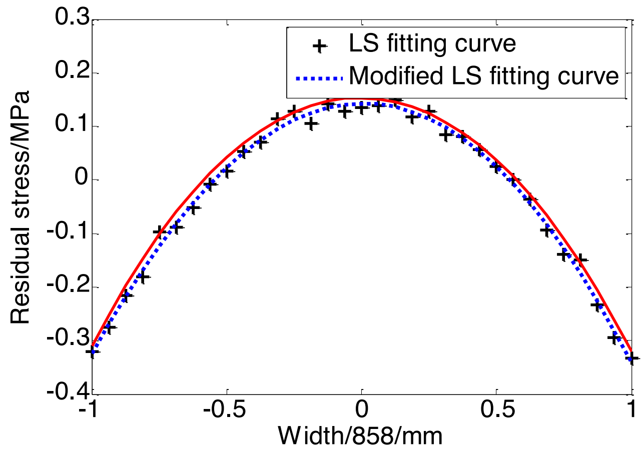


Fig. (4). Comparison of the least squares fitting curve and the modified least squares fitting curve (Table **1**).

Table 2. The second set of test data (unit: mm).

1	2	3	4	5
0.162	0.137	0.112	0.085	0.058
6	7	8	9	10
0.032	0.006	-0.018	-0.04	-0.061
11	12	13	14	15
-0.079	-0.094	-0.107	-0.116	-0.122
16	17	18	19	20
-0.125	-0.124	-0.12	-0.112	-0.101
21	22	23	24	25
-0.087	-0.068	-0.049	-0.026	-0.001
26	27	28	29	30

Table 2. contd…

1	2	3	4	5
0.026	0.056	0.086	0.117	0.149
31	32	33		
0.181	0.211	0.241		

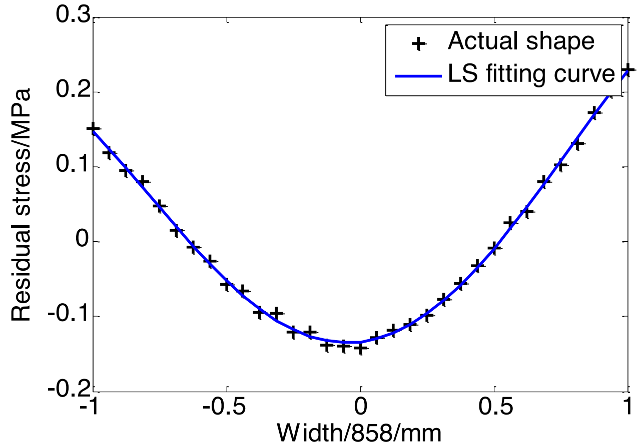


Fig. (5). Comparison of the actual shape and the least squares fitting curve (Table **2**).

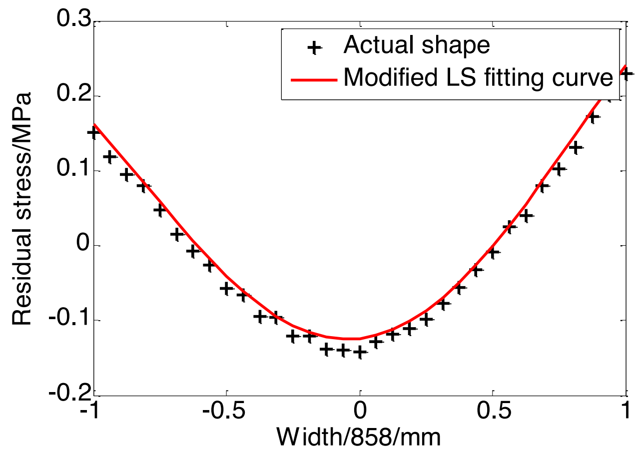


Fig. (6). Comparison of the actual shape and the modified least squares fitting curve (Table **2**).

Table 3. The performance comparison of two kinds of algorithm.

		Run time(s)	Mean Square Error
LS	Group 1	0.013	0.0094
	Group 2	0.011	0.0042
	average	0.012	0.0068
Modified LS	Group 1	0.002	0.0185
	Group 2	0.004	0.0089
	average	0.003	0.0137

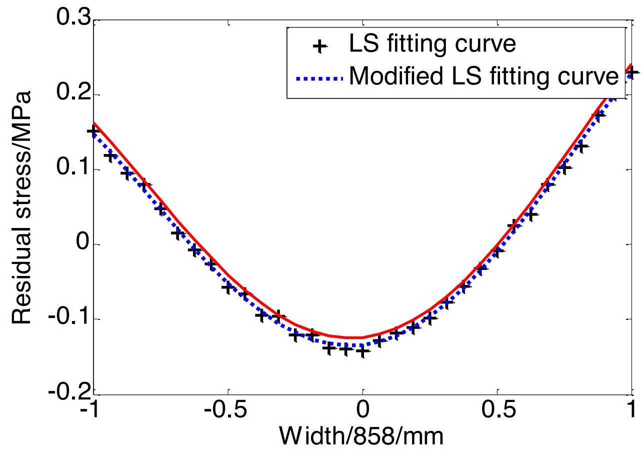


Fig. (7). Comparison of the least squares fitting curve and the modified least squares fitting curve (Table **2**).

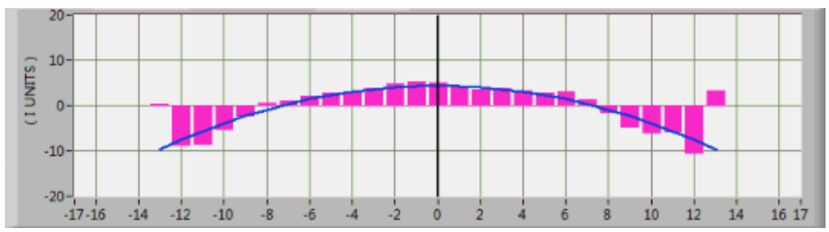


Fig. (8). Flatness detection and control results.

The performance comparison of two kinds of algorithms is shown in Table **3**.

Through the above simulation results, we can see, the modified LS improves a lot in speed and avoids complex calculation effectively. Although the mean square error compared with the ordinary method is a little poor, it has little impact in engineering application.

The research results have been successfully applied in an 1850mm production line in an aluminum foil factory, achieving fine control effect. Its flatness detection and control results are shown in Fig. (**8**).

Fig. (**8**) is a real control interface. The continuous curve expressed setting flatness, which is decided by supplied strip. Each bar is value of flatness detecting roller.

If every bar's summit is coincidence on curve, it means entrance section shape is the same with delivery section shape. Fine flatness will be gotten.

CONCLUSIONS

(1) The modified least squares method based on Legendre polynomial satisfies the condition of equilibrium in the residual stress, so it is more suitable than LS as the basic model of flatness pattern recognition.

(2) The modified least squares method based on Legendre polynomial has obvious improvement in speed, and avoids complex calculation effectively, which is more suitable in industrial field. Although Mean square error compared with the ordinary method is a little poor, the step cooling method is generally used to eliminate the high order error, so it doesn't affect the final control results.

(3) The modified least squares method based on Legendre polynomial can obtain three kinds of control quantity of strip shapes directly, and the physical meaning is clear. This is the most prominent place compared to LS. LS can only stay on the theoretical level; however, modified LS can be used in industrial production.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interests regarding the publication of this article.

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