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RESEARCH ARTICLE

Distributed Task-space Tracking for Multiple Manipulators with Uncertain Dynamics and Kinematics

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Abstract: This paper studies the task-space tracking problem for networked robot manipulators, while the dynamic and kinematic parameters of each manipulator are unknown. A velocity observer is first developed to estimate the task-space velocity, and reference sufficient conditions for observer parameters are also given to guarantee the convergence of observation error. Based on the proposed observer, an adaptive controller is first developed when the task-space velocity is measurable, then a modified controller is proposed considering the case when the task-space velocity is unavailable. Using graph theory and Lyapunov analysis, the proof of the system stability is given. Simulations are provided to demonstrate the effectiveness of the proposed control method.

Keywords: Adaptive Control, Task-space Tracking, Uncertain Kinematics.

1. INTRODUCTION

The multiple robot manipulators (MRMs) are widely used in modern manufacturing such as welding, painting, assembling, transporting, since many benefits can be obtained when a single complicated manipulator is replaced by multiple but simpler manipulators. However, the system model is nonlinear and highly coupled, and at the same time, the exact parameters of the system model are usually unknown, which makes it difficult to fulfill the precise control of MRMs.

In this endeavor, two approaches are used for controlling MRMs: the centralized approach [1 - 3] and the distributed approach [4 - 9]. Considering the inevitable physical constrains such as short wireless communication ranges, limited energy, the distributed approach is believed more promising [10]. The objective of distributed control for MRMs is to design a distributed protocol for MRMs based on only local information exchange, so that the MRMs track the desired trajectory, which might be either constant [4 - 9] or time-varying [11 - 15]. A distributed containment controller is explored in [16], while the consensus equilibrium is constant. Distributed tracking problem is studied in [17] for networked Euler-Lagrange systems with a dynamic leader, and then a model-independent sliding mode controller is proposed. In [18], a continuous distributed robust tracking protocol is introduced for multiple Euler-Lagrange systems with uncertain dynamics. A distributed leaderless consensus algorithm is proposed for the networked Euler-Lagrange system without considering the gravity effect. In [19], a neural network based distributed adaptive controller is discussed, which requires fixed communication topologies.

All the previous mentioned literature consider the joint-space control of MRMs with dynamic uncertainties, while in reality, the mission of MRMs is usually defined in the task-space, and MRMs achieve specific tasks by grasping tools. The key idea of task-space tracking problems is to convert the desired trajectory form task-space into joint space using inverse kinematics, which the kinematic parameters are essential. The uncertain kinematic parameters will generate error in creating reference joint-space trajectory, which may cause instability of the closed-loop system. Some

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researches on task-space tracking problem with kinematic uncertainties can be seen in [20 - 24]. In [23], task-space regulation problem is researched considering uncertain dynamics and kinematics, and in [24], a terminal robust controller is developed, but the end-effecter velocity and acceleration are assumed to be known. All the researches focus on single robot manipulator. In [25], a distributed task-space controller is designed considering dynamic and kinematic uncertainties, but the desired trajectory is constant. When the desired trajectory is time-varying, controllers are designed in [26] and [27], the tracking errors are proved to asymptotically converge to zero. However, the velocity and acceleration of the desired trajectory are assumed to be available to all manipulators. Considering the case that only a subset of the robots has access to the desired trajectory, these controllers are not suitable. Inspired from [29], a velocity observer is constructed for each robot, and the output of the observer is used for further controller design.

In this paper, we consider the task-space tracking problem for MRMs with uncertain dynamics and kinematics, while the desired trajectory is only available to a subset of the manipulators, which is difficult to fulfill task-space tracking. Inspired from [29 - 32], a velocity observer which is second order derivable is constructed using only local information exchange, and the observed velocity is proved to converge to the desired velocity. Based on the observer, we propose adaptive tracking controllers without measuring the joint-space acceleration. By selecting updating law of the dynamic and kinematic parameters, the controllers achieve globally asymptotic tracking. The stabilities of the closed-loop systems are proved *via* Lyapunov theory. The main contributions of this paper are: 1, using only local information exchange a velocity observer which is second order derivable is constructed. 2, based on the designed observer, a chattering free control strategy is proposed despite the desired trajectory is only available to a subset of the manipulators. 3, some improvements to the control strategy are also made, and a new adaptive tracking controller is developed without measuring the joint-space acceleration.

The remainder of this paper is organized as follows. Section 2 gives a brief introduction on some preliminary and the problem formulation. In section 3, controllers are designed for the cases when the task-space velocity is measurable and immeasurable, respectively. In section 4, examples and numeral simulations are provided to show the effectiveness of the proposed methods. Finally, conclusions are given in section 5.

2. PRELIMINARIES

2.1. Problem Formulation

Consider MRMs consisting of N robot manipulators, the dynamic model of each manipulator is given as:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i \tag{1}$$

where $i = 1,2,...N$, $q_i, \dot{q}_i, \ddot{q}_i \in R^p$ are the vector of generalized position, velocity and acceleration, respectively. $M_i(q_i) \in R^{p\times p}$ denotes the symmetric positive definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i \in R^p$ is the vector of centripetal-Coriolis force, $G_i(q_i) \in R^p$ is the vector of gravitational force, $\tau_i \in R^p$ is the vector of control input.

The dynamic model (1) satisfies the following properties.

Property 1. $M_i(q_i)$ and $C_i(q_i, q_i)$ satisfy the following skew symmetric relationship:

$$\xi^T(\frac{1}{2}\dot{M}_i(q_i) - C(q_i,\dot{q}_i))\xi = 0, \forall \xi \in R^p. \tag{2}$$

Property 2. The left hand side of Eq. (1) can be written linearly in a set of dynamic parameters $a_d^i \in R^m$ [24]:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = Y_d^i(q_i,\dot{q}_i,\dot{q}_i,\ddot{q}_i)a_d^i \tag{3}$$

where $Y_d^i(q_i,\dot{q}_i,\dot{q}_i,\ddot{q}_i) \in R^{p\times m}$ is the dynamic regression matrix.

Let $X_i \in R^i$ represent the position of the ith manipulator's end-effecter in the task space, the relationship between and can be described as:

$$X_i = h_i(q_i) \tag{4}$$

where $h_i(\bullet) \in R^l$ denotes the mapping between the joint space and the task space, the time derivative of is

$$\dot{X}_i = J_i(q_i)\dot{q}_i \tag{5}$$

where $J_i(q_i) = \partial h_i(q_i) / \partial q_i \in R^{l \times p}$ is called the Jacobian matrix, the right-hand side of Eq. (5) can be written linearly in a set of kinematic parameters $a_k^i \in R^n$ [24].

$$J_i(q_i)\dot{q}_i = Y_k^i(q_i, \dot{q}_i)a_k^i \tag{6}$$

where $Y_k^i(q_i, \dot{q}_i) \in R^{l \times n}$ is the kinematic regression matrix.

2.2. Graph Theory

In this paper, an undirected graph $G = \{V, E\}$ is used to describe the information exchange among N agents, where $V = \{1,2,...N\}$ and $E \in V$ X V and denote the set of agents and edges, respectively. An edge is an ordered pair $\{j,i\} \in E$ if the jth agent can get information from the ith agent, in undirected graph, if $\{i,j\} \in E$, then $\{j,i\} \in E$. The set of ith agent's neighbors is defined as $N_i = \{ j \mid \{ j, i \} \in E\}$. The adjacency matrix $A = [a_{ij}] \in R^{N \times N}$ with $\boldsymbol{G}$ is defined such that $a_{ij} = 1$ if $\{ j, i \} \in E\}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = [l_{ij}] \in R^{N \times N}$ with $\boldsymbol{G}$ is defined as $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. The access of agents to the desired trajectory is described by a matrix $B = diag\{a_{10}, a_{20}, \dots a_n\}$. If the ith agent has access to the desired trajectory, $a_i = 1$; otherwise, $a_i = 0$. To facilitate the subsequent analysis, we define a matrix $H = L + B$, which is named as the information-exchange matrix.

Some important lemmas that will be used in further analysis are given as follows.

Lemma 1 ([8]). All of the nonzero eigenvalues of L are real and positive for an undirected graph. Zero is a simple eigenvalue of L and the associated eigenvector is $\underline{1}$ if and only if the undirected graph is connected, where $\underline{1} = [1, 1, \dots 1]^T \in R^N$ is a unitary column vector.

Lemma 2 ([9]). If $\boldsymbol{G}$ is a connected undirected graph and at least one agent has access to the desired trajectory, then H is symmetric and positive definite.

To facilitate the subsequent controller design and analysis, the desired trajectory is defined as a virtual leader v, whose states are noted by X and $\dot{X}$.

2.3. Control Objective

This paper considers the task space tracking problem of multiple manipulators, the desired trajectory is denoted as X, and only a subset of the manipulators has access to X. The kinematic and dynamic parameters of the manipulator are unknown. The measurable states are the joint position q_i, the joint velocity $\dot{q}$, the task-space position X_i, it is notable that joint-space acceleration $\ddot{q}$ is unavailable. The control objective is described as follows.

Definition. Design distributed task-space tracking protocol $\tau_i(i = 1,...,N)$ without measuring the joint-space acceleration $\ddot{q}$, such that the states X_i and $\dot{X}_i$ of the robot manipulators governed by (1), (4) and (5) reach consensus and asymptotically follow the desired trajectory X and $\dot{X}$ in task space, in the sense that:

$$\begin{cases} \lim_{t\to\infty}(X_i(t) - X_j(t)) = 0, \\ \lim_{t\to\infty}(\dot{X}_i(t) - \dot{X}_j(t)) = 0 \end{cases} \tag{7}$$

3. MAIN RESULTS

3.1. Velocity Observer Design

Before designing the tracking controller, we make some assumptions as follows.

Assumption 1. The desired trajectory X is bounded up to its fourth derivative.

Assumption 2. The singularity of the manipulators is avoided. *i.e.*, the Jacobian matrix J_i is always invertible [21 - 28].

Remark: Assumption 1 can be realized by path planning, and continuous derivative of higher order would protect the robot by avoiding vibration. Similar assumptions can be also seen in reference [19, 20]. Assumption 2 is a common assumption in the field of task-space tracking, since singularity of the manipulators would result in inexistence of the reference trajectory in the joint space or the infinity of the control torque, which would cause instability of the closed-loop system.

As only a subset of the manipulators has access to the desired trajectory, an observer is used for each manipulator to observe the velocity of the desired trajectory. The observer is designed as:

$$\dot{\hat{v}}_i(t) = -(k_1+1)\hat{v}_i(t) - \int_0^t (k_2 \sum_{j=0}^{N} a_{ij}(\hat{v}_i(\iota) - \hat{v}_j(\iota)) \\ + k_3 \operatorname{sgn}(\sum_{j=0}^{N} a_{ij}(\hat{v}_i(t) - \hat{v}_j(t)))\mathrm{d}\iota \tag{8}$$

where k_1, k_2, $k_3 > 0$ are positive scalars $\hat{v}_i(t)$, $\hat{v}_j(t)$, i, $j = 1,...,N$ denote the ith and jth output of the observer, respectively, and $\hat{v}$ is defined as $\hat{v}(t) = \dot{X}(t)$.

Define the observation error as:

$$\overline{v}_i = \sum_{j=0}^{N} a_{ij}(\hat{v}_i - \hat{v}_j) \tag{9}$$

For simplicity, we write the concatenated form of $\hat{v}_i$ and $\overline{v}$ as follows:

$$\hat{v} = [\hat{v}_1^T, \hat{v}_2^T, ..., \hat{v}_N^T]^T, \overline{v} = [\overline{v}_1^T, \overline{v}_2^T, ..., \overline{v}_N^T]^T \tag{10}$$

Thus, Eq. (8) and (9) can be rewritten as:

$$\dot{\hat{v}} = -(k_1+1)\hat{v} - \int_0^t (k_2\overline{v}(\iota) + k_3 \operatorname{sgn}(\overline{v}(\iota)))\mathrm{d}\iota \tag{11}$$

$$\overline{v} = \mathcal{H}\hat{v} - \mathcal{B}\underline{\dot{X}}_0 \tag{12}$$

where $\mathcal{H} = H \otimes I_p$ and $\mathcal{B} = B \otimes I_p$, respectively. $\otimes$ denotes the Kronecker product.

Differentiating Eq. (12) with respect to time gives:

$$\dot{\overline{v}} = -\int_0^t (k_2\mathcal{H}\overline{v}(\iota) + k_3\mathcal{H} \operatorname{sgn}(\overline{v}(\iota)))\mathrm{d}\iota \\ -(k_1+1)\mathcal{H}\hat{v} \quad - \mathcal{B}\underline{\ddot{X}}_0 \tag{13}$$

Define the error function as:

$$Hs = \dot{\bar{v}} + \bar{v} \tag{14}$$

Differentiating Eq. (14) yields:

$$\begin{aligned} H\dot{s} &= \ddot{\bar{v}} + \dot{\bar{v}} \\ &= -(k_1+1)H\dot{\hat{v}} - k_2 H\bar{v} - k_3 H \operatorname{sgn}\bar{v} \\ &\quad + H\dot{\hat{v}} - B\dddot{\underline{q}}_0 - B\ddot{\underline{q}}_0 \\ &= -k_1 H\dot{\hat{v}} - k_2 H\bar{v} - k_3 H \operatorname{sgn}\bar{v} - B(\dddot{\underline{q}}_0 + \ddot{\underline{q}}_0) \\ &= -k_1 Hs - (k_2 H - k_1)\bar{v} - k_3 H \operatorname{sgn}\bar{v} \\ &\quad - B\dddot{\underline{q}}_0 - (k_1+1)B\ddot{\underline{q}}_0 \\ &= -k_1 Hs - (k_2 H - k_1)\bar{v} - k_3 H \operatorname{sgn}\bar{v} + H\phi_d \end{aligned} \tag{15}$$

where. $\phi_d = -\dddot{q}_0 - (k_1+1)\ddot{q}_0$ According to Assumption 1, there exist two positive constants satisfying.

$$\|\phi_d\| < c_1, \quad \|\dot{\phi}_d\| < c_2 \tag{16}$$

Lemma 3. Suppose that ***G*** is a connected undirected graph, and Assumption 1 holds, the observer described by (8) ensures $\hat{v}_i \to \dot{X}$ and $\dot{\hat{v}}_i \to \ddot{X}$ as $t \to \infty$, for all $i = 1,...,N$, provided that k_1 and k_2 are chosen according to the following sufficient conditions

$$\begin{cases} k_3 > c_1 + c_2 \\ k_1 < 4k_2\lambda_{\min}(H) \end{cases} \tag{17}$$

where $\lambda_{\min}(H)$ denotes the minimum eigenvalue of matrix H.

Proof. Consider a function as follows

$$V_0 = \frac{1}{2}s^T Hs + \frac{k_2}{2}v^T v + p(t) \tag{18}$$

Where

$$\begin{aligned} p(t) &= \bar{v}^T(0)k_3 \operatorname{sgn}(\bar{v}(0)) - \bar{v}^T(0)\phi_d(0) \\ &\quad - \int_0^t s^T H(\phi_d(\iota) - k_3 \operatorname{sgn}(\bar{v}(\iota)))\mathrm{d}\iota \end{aligned} \tag{19}$$

First, we will prove that V is a Lyapunov function candidate. It is obvious that $\frac{1}{2}s^T Hs \geq 0$ and $\frac{k_2}{2}v^T v \geq 0$. Some calculations on the integral part $p(t)$, we have

$$\begin{aligned}
&\int_0^t s^T H(\phi_d - k_3 \operatorname{sgn}\bar{v})\mathrm{d}\iota = \int_0^t (Hs)^T (\phi_d - k_3 \operatorname{sgn}\bar{v})\mathrm{d}\iota \\
&= \int_o^t \dot{\bar{v}}^T \phi_d \mathrm{d}\iota - \int_0^t \dot{\bar{v}}^T k_3 \operatorname{sgn}\bar{v}\mathrm{d}\iota + \int_0^t \bar{v}^T \phi_d \mathrm{d}\iota \\
&\qquad - \int_0^t \bar{v}^T k_3 \operatorname{sgn}\bar{v}\mathrm{d}\iota \\
&= \bar{v}^T \phi_d \mid_0^t - \int_0^t \bar{v}^T \dot{\phi}_d \mathrm{d}\iota - \bar{v}^T k_3 \operatorname{sgn}\bar{v} \mid_0^t \\
&\qquad + \int_0^t \bar{v}^T \phi_d \mathrm{d}\iota - \int_0^t \bar{v}^T k_3 \operatorname{sgn}\bar{v}\mathrm{d}\iota \\
&= \bar{v}^T(t)\phi_d(t) - \bar{v}^T(0)\phi_d(0) + \bar{v}^T(0)k_3 \operatorname{sgn}(\bar{v}(0)) \\
&\qquad - \bar{v}^T(t)k_3 \operatorname{sgn}(\bar{v}(t)) \\
&+ \int_0^t \bar{v}^T(\phi_d - \dot{\phi}_d - k_3 \operatorname{sgn}\bar{v})\mathrm{d}\iota \\
&\le \bar{v}^T(0)k_3 \operatorname{sgn}(\bar{v}(0)) + \int_0^t \|\bar{v}\|(\|\phi_d\| + \|\dot{\phi}_d\| - k_3)\mathrm{d}\iota \\
&\qquad - \bar{v}^T(0)\phi_d(0) \\
&+ \|\bar{v}\|(\|\phi_d\| - k_3) \\
&\le \bar{v}^T(0)k_3 \operatorname{sgn}(\bar{v}(0)) + \int_0^t \|\bar{v}\|(c_1 + c_2 - k_3)\mathrm{d}\iota \\
&+ \|\bar{v}\|(c_1 - k_3) - \bar{v}^T(0)\phi_d(0)
\end{aligned} \tag{20}$$

If k_2 is chosen according to (17), we have $k_3 > c_1 + c_2$, thus $p(t) \ge 0$. Then $V \ge 0$ is a Lyapunov function candidate. Differentiating Eq. (18) with respect to time, we have:

$$\begin{aligned}
\dot{V} &= s^T H\dot{s} + k_2 \bar{v}^T \dot{\bar{v}} + \dot{p}(t) \\
&= s^T(-k_1 Hs - (k_2 H - k_1)\bar{v} - k_3 H \operatorname{sgn}\bar{v} + H\phi_d) \\
&\quad + k_2 \bar{v}^T(Hs - \bar{v}) + s^T H(\phi_d - \operatorname{sgn}\bar{v}) \\
&= -k_1 Hs^T s + k_1 s^T v - k_2 \bar{v}^T \bar{v} \\
&= [\begin{array}{cc} s & \bar{v} \end{array}]^T K \begin{bmatrix} s \\ \bar{v} \end{bmatrix}
\end{aligned} \tag{21}$$

where $K = \begin{bmatrix} k_1 H & -\frac{k_2}{2} I \\ -\frac{k_2}{2} I & k_2 I \end{bmatrix}$, I, is an identity matrix:

with appropriate dimensions. Provided that k_1 and k_2 satisfy $k_1 k_2 H - k_1^2 / 4 \ge 0$, *i.e.*, $k_1 \le 4k_2\lambda_{\min}$, we have $K \ge 0$, therefore $V \le 0$ is negative semi-definite, which implies that s and $\bar{v}$ are bounded. From Eq. (12) and (14), $\dot{\bar{v}}$ and $\dot{\hat{v}}$ are also bounded. Based on Eq. (15) and Assumption 1, we obtain that is bounded. Differentiating Eq. (21) yields.

$$\ddot{V}_0 = -[s^T, \bar{v}^T]K\begin{bmatrix}\dot{s}\\ \dot{\bar{v}}\end{bmatrix} \tag{22}$$

Using Barbalat's Lemma, $s \rightarrow 0$ and $\bar{v} \rightarrow 0$ as $t \rightarrow \infty$. Since $H \rightarrow 0$, we have $\bar{v} \rightarrow 0$ and $\dot{\hat{v}} \rightarrow 0$ as $t \rightarrow \infty$, *i.e*,

$$\lim_{t\to\infty}(H\hat{v}(t) - B\underline{\dot{X}}_0(t)) = 0 \tag{23}$$

$$\lim_{t\to\infty}(H\dot{\hat{v}}(t) - B\underline{\ddot{X}}_0(t)) = 0 \tag{24}$$

According to Lemma 1, $L\underline{\dot{X}} = 0$, $L\underline{\ddot{X}} = 0$. Eq. (23) is rewritten as:

$$\begin{aligned}&\lim_{t\to\infty}(H\hat{v}(t) - B\underline{\dot{X}}_0(t))\\ &= \lim_{t\to\infty}(H\hat{v}(t) - B\underline{\dot{X}}_0(t) - L\underline{\dot{X}}_0)\\ &= \lim_{t\to\infty}(H(\hat{v}(t) - \underline{\dot{X}}_0(t))) = 0\end{aligned} \tag{25}$$

Pre-multiplying H^1 to both sides of Eq. (25), we have $\hat{v}(t)$ - $\underline{\dot{X}}(t) \rightarrow 0$ as $t \rightarrow \infty$. Using the similar method on Eq. (24) derives as.

In this part, we design a velocity observer for each follower agents, and the derivative of the observed velocity is guaranteed continuous. In the following sections, we will propose adaptive task-space tracking controllers using the designed velocity observer.

Remark: In this section, a velocity observer is proposed to estimate the task-space velocity, and the sufficient conditions are also given. Note that the given conditions are just sufficient but not necessary conditions, parameters which do not satisfy those conditions may also stabilize the system. When selecting the observer parameters, a relative small k_1 can be first chosen, and according to the sufficient conditions (17), the parameters k_2 and k_3 can be selected relative large by trial.

3.2. Controller Design Using Task-Space Velocity

Define a reference trajectory in the joint space as

$$\dot{q}_{ri} = \hat{J}_i^{-1}(\hat{v}_i - k_4\sum_{j=0}^{N} a_{ij}(X_i - X_j)) \tag{26}$$

where k_4 is a positive scalar, $\hat{v}_i$ is the output of the observer introduced in section 3.1, $\hat{J}_i$ is the estimated Jacobian matrix whose kinematic parameters a_k^i are replaced by its estimate $\hat{a}_k^i$. Differentiating Eq. (26) yields:

$$\ddot{q}_{ri} = \hat{J}_i^{-1}[\dot{\hat{v}}_i - k_4\sum_{j\ 0}^{N} a_{ij}(\dot{X}_i - \dot{X}_j) - \dot{\hat{J}}_i\dot{q}_{ri}] \tag{27}$$

When the task-space velocity $\dot{X}_i$, $\dot{X}_j$ are measurable $\dot{q}_{ri}$, is available, by defining the sliding surface.

$$r_i = \dot{q}_i - \dot{q}_{ri} \tag{28}$$

The distributed task-space tracking algorithm can be designed as:

$$\tau_i = \hat{M}_i(q_i)\ddot{q}_{ri} + \hat{C}_i(q_i, q_i)\dot{q}_{ri} + \hat{G}_i(q_i) - \alpha r_i \quad \textbf{(29)}$$

where $\hat{M}_i(q_i)$, $\hat{C}_i(q_i,q_i)$ and $\hat{G}_i(q_i)$ denote the estimates of $M_i(q_i)$, $C_i(q_i,q_i)$ and $G_i(q_i)$, respectively.

Theorem 1. Suppose that ***G*** is a connected undirected graph, and Assumption 1, 2 and 3 hold, the observer (8) and controller (29) ensure $X_i \to X$ and $\dot{X}_i \to \dot{X}$ as $t \to \infty$, for all $i = 1,...,N$, provided that k_1, k_2 and k_3 are chosen according to the following sufficient conditions:

$$\begin{cases} k_3 > c_1 + c_2 \\ k_1 < 4k_2\lambda_{\min}(H) \end{cases}$$

along with the updating law:

$$\begin{cases} \dot{\hat{a}}_d^i = -\Gamma_1 Y_d^{iT}(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) r_i \\ \dot{\hat{a}}_k^i = -\Gamma_2 Y_k^{iT}(q_i, \dot{q}_i)(\hat{\dot{X}}_i - \dot{X}_i) \end{cases} \quad \textbf{(30)}$$

Proof. According to property 2, and substituting Eq. (29) into the robot dynamics Eq. (1), we have:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = Y_d^i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})\hat{a}_d^i - \alpha r_i \quad \textbf{(31)}$$

where $\hat{a}_d^{\,i}$ is the estimate of the a_d^i. Adding $-Y_d^i(q_i,\ \dot{q}_i,\ \dot{q}_{ri},\ \ddot{q}_{ri})a_d^{\,i}$ to both sides of Eq. (31) and using Eq. (3), we have:

$$M_i(q_i)\dot{r}_i + C_i(q_i, \dot{q}_i) r_i = Y_d^i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{q}_{ri})\tilde{a}_d^i - \alpha r_i \quad \textbf{(32)}$$

where $\tilde{a}_d^{\,i} = \hat{a}_d^{\,i} - a_d^{\,i}$.

Consider the Lyapunov function candidate:

$$V_1 = \frac{1}{2}\sum_{i=1}^{N} r_i^T M_i r_i + \frac{1}{2}\sum_{i=1}^{N} \tilde{a}_d^{i\,T} \Gamma_1^{-1} \tilde{a}_d^i + \frac{1}{2}\sum_{i=1}^{N} \tilde{a}_k^{i\,T} \Gamma_2^{-1} \tilde{a}_2^i + V_0 \quad \textbf{(33)}$$

where Γ_1 and $\Gamma_2 > 0$ are adjustable variables. $\hat{\dot{X}}_i$ is the estimate velocity of the end-effecter, according to Eq. (5) and (6), we have:

$$\hat{\dot{X}}_i - \dot{X}_i = Y_k^i(q_i, \dot{q}_i)\hat{a}_k^i - Y_k^i(q_i, \dot{q}_i) a_k^i = Y_k^i(q_i, \dot{q}_i)\tilde{a}_k^i \quad \textbf{(34)}$$

Differentiating Eq. (33), and combining Eq. (32) and the updating law (31) yields:

$$
\begin{aligned}
\dot{V}_1 &= \sum_{i=1}^{N} r_i^T M_i \dot{r}_i + \frac{1}{2}\sum_{i=1}^{N} r_i^T \dot{M}_i r_i + \sum_{i=1}^{N} \tilde{a}_d^{iT} \Gamma_1^{-1} \dot{\hat{a}}_d^i + \sum_{i=1}^{N} \tilde{a}_k^{iT} \Gamma_2^{-1} \dot{\hat{a}}_k^i + \dot{V}_0 \\
&= \frac{1}{2}\sum_{i=1}^{N} r_i^T \dot{M}_i r_i + \sum_{i=1}^{N} r_i^T (Y_d^i(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri})\tilde{a}_d^i - \alpha r_i - C_i(q_i,\dot{q}_i) r) \\
&+ \sum_{i=1}^{N} \tilde{a}_d^{iT} \Gamma_1^{-1} \dot{\hat{a}}_d^i + \sum_{i=1}^{N} \tilde{a}_k^{iT} \Gamma_2^{-1} \dot{\hat{a}}_k^i + \dot{V}_0 \\
&= \frac{1}{2}\sum_{i=1}^{N} r_i^T \dot{M}_i r_i + \sum_{i=1}^{N} r_i^T (Y_d^i(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri})\tilde{a}_d^i - \alpha r_i - C_i(q_i,\dot{q}_i) r) \\
&+ \sum_{i=1}^{N} \tilde{a}_d^{iT} Y_d^{iT}(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}) r + \sum_{i=1}^{N} \tilde{a}_k^{iT} Y_k^{iT}(q_i,\dot{q}_i)(\dot{\hat{X}}_i - \dot{X}_i) + \dot{V}_0 \\
&= -\alpha \sum_{i=1}^{N} r_i^T r_i - \sum_{i=1}^{N} \tilde{a}_k^{iT} Y_k^{iT}(q_i,\dot{q}_i) Y_k^i(q_i,\dot{q}_i)\tilde{a}_k^i + \dot{V}_0 \\
&\le 0
\end{aligned}
\tag{35}
$$

Therefore r_i, $\tilde{a}_d^i$, $\tilde{a}_k^i$ are bounded, then is also bounded. Based on the proof of Lemma 3, and are both bounded.

As trigonometric functions of q_i, $X_i J_i$ and $\hat{J}_i$ are all bounded. According to Eq. (26) and (28), $\dot{q}_{ri}$ and $\dot{q}_i$ are bounded. Since $\dot{\hat{J}}_i$ is a trigonometric function of q_i and a_k^i, $\dot{\hat{J}}_i$ is bounded, furthermore, $\ddot{q}_{ri}$ is bounded, according to Eq. (29), τ_i is bounded, so $\ddot{q}_i$ and $\dot{r}$ are bounded. Differentiating $Y_k^i(q_i)\tilde{a}_k^i$ gives:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}(Y_k^i(q_i,\dot{q}_i)\tilde{a}_k^i) &= \frac{\partial Y_k^i(q_i,\dot{q}_i)}{\partial q_i}\dot{q}_i\tilde{a}_k^i \\
&+ \frac{\partial Y_k^i(q_i,\dot{q}_i)}{\partial \dot{q}_i}\ddot{q}_i\tilde{a}_k^i + Y_k^i(q_i)\dot{\tilde{a}}_k^i \\
&= \frac{\partial Y_k^i(q_i)}{\partial q_i}\dot{q}_i\tilde{a}_k^i + \frac{\partial Y_k^i(q_i,\dot{q}_i)}{\partial \dot{q}_i}\ddot{q}_i\tilde{a}_k^i \\
&- Y_k^i(q_i)\Gamma_2 Y_k^{iT}(q_i)(\dot{\hat{X}}_i - \dot{X}_i)
\end{aligned}
\tag{36}
$$

Since $Y_k^i(q_i, \dot{q}_i)$ is a trigonometric function q_i and $\dot{q}_i$, q_i, $\dot{q}_i$ and $\ddot{q}_i$ are all bounded, we have $\frac{\mathrm{d}}{\mathrm{d}t}(Y_k^i(q_i,\dot{q}_i)\tilde{a}_k^i)$ is bounded.

Using Barbalat's Lemma, we have $r_i \to 0$, $Y_k^i(q_i)\tilde{a}_k^i \to 0$ as $t \to \infty$. Substituting Eq. (28) into (26) gives:

$$
\dot{X}_i - \hat{v}_i + k_4 \sum_{j=0}^{N} a_{ij}(X_i - X_j) = \hat{J}_i r_i - Y_k^i(q_i)\tilde{a}_k^i \tag{37}
$$

According to Lemma 1, $\hat{v}_i \to \dot{X}$ as $t \to \infty$, we have:

$$
\lim_{t\to\infty}\{\dot{X}_i(t) - \dot{X}_0(t) + k_4 \sum_{j=0}^{N} a_{ij}(X_i(t) - X_j(t))\} = 0 \tag{38}
$$

Rewriting Eq. (38) in the concatenated form as:

$$\lim_{t\to\infty}\{\dot{X}(t)-\dot{\underline{X}}_0(t)+k_4H(X_i-X_j)\}=0 \tag{39}$$

Since H positive definite, we have $\dot{X}(t)$ - $\dot{\underline{X}}(t) \to 0$ and $X(t)$ - $\underline{X}(t) \to 0$ as $t \to \infty$ that is, $\dot{X}_i(t)$ - $\dot{X}(t) \to 0$ and $X_i(t)$ - $X(t) \to 0$ as $t \to \infty$.

3.3. Controller Design Without Task-Space Velocity

In section 3.2, we discussed the case when the task-space velocity is available, however, in practice, measuring the task-space velocity requires high-resolution camera, which will inevitably add huge costs. To this end, the case when the task-space velocity $\dot{X}_i$ is immeasurable is considered.

To avoid measuring the task-space velocity, a low-pass filter is designed as:

$$\dot{y}_i+\lambda_1 y_i=\lambda_1\dot{X}_i \tag{40}$$

where λ_1 is a positive constant, is the filtered output of the task-space velocity with zero initial value (*i.e.*, $y_i = 0$). In the frequency domain, Eq. (40) becomes:

$$y_i=\frac{\lambda_1}{(p+\lambda_1)}\dot{X}_i=W_k^i(t)a_k^i \tag{41}$$

where p is the Laplace variable and $W_k^i(t) = \lambda_1 / (p + \lambda_1)Y_k^i(q_i, \dot{q}_i)$, $W_k^i(0) = 0$.

Note that if $\dot{X}_i$ is immeasurable, the definitions of $\dot{q}_{ri}$, $\ddot{q}_{ri}$ and the updating law $\dot{\hat{a}}_k^i$ used in section 3.2 are no longer suitable.

Let $\dot{\hat{X}}_0=\dot{X}_0$ and define the modified reference trajectory $\dot{q}_{mri}$ and sliding surface r_{mi} as:

$$\begin{cases}\ddot{q}_{mri}=\hat{J}_i^{-1}(t)[\dot{\hat{v}}_i(t)+k_6\hat{v}_i(t)-k_5\sum_{j=0}^{N}a_{ij}(\hat{\dot{X}}_i(t)-\hat{\dot{X}}_j(t)) \\ \quad -\dot{\hat{J}}_i(t)\dot{q}_{mri}(t) \quad -k_5k_6\sum_{j=0}^{N}a_{ij}(X_i(t)-X_j(t))]-k_6\dot{q}_{mri}(t) \\ r_{mi}=\dot{q}_i-\dot{q}_{mri}\end{cases} \tag{42}$$

Where k_5 and k_6 are adjustable positive scalars.

The modified controller has the similar form with controller (29).

$$\tau_i=\hat{M}_i(q_i)\ddot{q}_{mri}+\hat{C}_i(q_i,q_i)\dot{q}_{mri}+\hat{G}_i(q_i)-\alpha r_{mi} \tag{43}$$

Theorem 2. Suppose that ***G*** is a connected undirected graph, assumption 1, 2 and 3 hold, the observer (8) and controller (44) ensure $X_i(t) \to X(t)$ and $\dot{X}_i(t) \to \dot{X}(t)$ as $t \to \infty$, for all $i = 1,...,N$, provided that k_1, k_2 and k_3 are chosen according to the following sufficient conditions

$$k_3 > c_1 + c_2$$

$$k_1 < 4k_2\lambda_{\min}(H\ \ )$$

along with the updating law:

$$\begin{cases} \dot{\hat{a}}_d^i = -\Gamma_1 Y_d^{iT}(q_i, \dot{q}_i, \dot{q}_{mri}, \ddot{q}_{mri}) r_{mi} \\ \dot{\hat{a}}_k^i = -\Gamma_3 W_k^{iT}(t) \Gamma_4 (W_k^{iT}(t) \hat{a}_k^i - y_i) \end{cases} \quad \textbf{(44)}$$

Proof. Substituting Eq. (43) into (1) and using Property 2, we have:

$$M_i(q_i)\dot{r}_{mi} + C_i(q_i, \dot{q}_i) r_{mi} = Y_d^i(q_i, \dot{q}_i, \dot{q}_{mri}, \ddot{q}_{mri}) \tilde{a}_d^i - \alpha r_{mi} \quad \textbf{(45)}$$

Consider the Lyapunov function candidate:

$$\begin{aligned} V_2 = \frac{1}{2}\sum_{i=1}^{N} r_{mi}{}^T M_i r_{mi} + \frac{1}{2}\sum_{i=1}^{N} \tilde{a}_d^{i\,T} \Gamma_1^{-1} \tilde{a}_d^i \\ + \frac{1}{2}\sum_{i=1}^{N} \tilde{a}_k^{i\,T} \Gamma_3^{-1} \tilde{a}_2^i + V_0 \end{aligned} \quad \textbf{(46)}$$

Differentiating Eq. (46) with respect to time yields:

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2}\sum_{i=1}^{N} r_{mi}{}^T \dot{M}_i r_{mi} + \sum_{i=1}^{N} r_{mi}{}^T M_i \dot{r}_{mi} + \sum_{i=1}^{N} \tilde{a}_d^{i\,T} \Gamma_1^{-1} \dot{\hat{a}}_d^i \\ &+ \sum_{i=1}^{N} \tilde{a}_k^{i\,T} \Gamma_3^{-1} \dot{\hat{a}}_2^i + V_0 \\ &= -\alpha \sum_{i=1}^{N} r_{mi}{}^T r_{mi} - \sum_{i=1}^{N} \tilde{a}_k^{i\,T} W_k^{iT}(t) W_k^i(t) \tilde{a}_k^i + \dot{V}_0 \\ &\le 0 \end{aligned} \quad \textbf{(47)}$$

This indicates that r_{mi}, $\tilde{a}_k^{\,i}$ and $\tilde{a}_d^{\,i}$ are bounded, then $\tilde{a}_k^{\,i}$ and $\hat{J}_i$ are also bounded. Pre-multiplying $\hat{J}_i$ to r_{mi}, we have:

$$\hat{J}_i r_{mi} = \dot{\hat{X}}_i - \hat{J}_i \dot{q}_{mri} \quad \textbf{(48)}$$

Differentiating Eq. (49) with respect with time, and using Eq. (42), we obtain:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}(\hat{J}_i r_{mi}) &= \frac{\mathrm{d}}{\mathrm{d}t}\dot{\hat{X}}_i - \dot{\hat{J}}_i \dot{q}_{mri} - \hat{J}_i \ddot{q}_{mri} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\dot{\hat{X}}_i - \dot{\hat{v}}_i + k_5 \sum_{j=0}^{N} a_{ij}(X_i - X_j) + k_6(\dot{\hat{X}}_i - \hat{v}_i) \\ &+ k_5 k_6 \sum_{j=0}^{N} a_{ij}(X_i - X_j) - \hat{J}_i r_{mi} \end{aligned} \quad \textbf{(49)}$$

Eq. (49) can be rewritten as:

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\hat{X}}_i - \hat{v}_i - \hat{J}_i r_{mi} + k_5 \sum_{j=0}^{N} a_{ij}(X_i - X_j)) + k_6(\dot{\hat{X}}_i - \hat{v}_i - \\ &\hat{J}_i r_{mi} + k_5 \sum_{j=0}^{N} a_{ij}(X_i - X_j)) = -k_5 \sum_{j=0}^{N} a_{ij}(Y_k^i \tilde{a}_k^i - Y_k^j \tilde{a}_k^j) \end{aligned} \quad \textbf{(50)}$$

As $Y_k^i \tilde{a}_k^i \to 0$, $i = 1,...,N$, we have as $t \to \infty$, $\dot{\hat{X}}_i - \dot{v}_i - \hat{J}_i r_{mi} + k_5 \sum_{j=0}^{N} a_{ij}(X_i - X_j) \to 0$. According to Lemma 1, $\hat{v}_i \to \dot{X}_0$ and $\hat{J}_i r_{mi}$ as $t \to \infty$, we have

$$\lim_{t\to\infty}\{\dot{X}_i(t) - \dot{X}_0(t) + k_5 \sum_{j=0}^{N} a_{ij}(X_i(t) - X_j(t))\} = 0 \tag{51}$$

The rest of proof is similar to section 3.2 and thus omitted here.

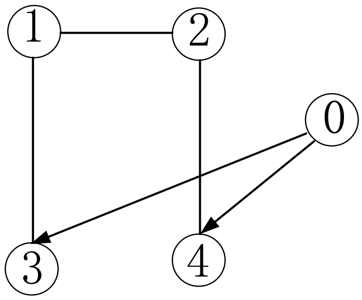

Fig. (1). Information-exchange graph (a).

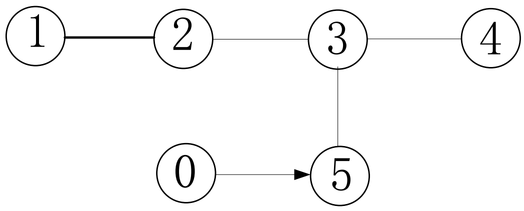

Fig. (2). Information-exchange graph (b).

4. SIMULATIONS

In this section, simulations are conducted to verify the effectiveness of the proposed controllers. The simulations are separated into three parts, in the first part, different communication topologies are simulated to verify the effectiveness of the velocity observer (8). In the second part, an adaptive controller using task-space velocity is researched, the third part studies the modified controller when the task-space velocity is immeasurable.

Simulation 1. Consider two multi-agent systems consisting of four and five manipulators respectively. The communication topologies (a) and (b) are illustrated in Figs. (**1** and **2**), in Fig. (**1**), the desired trajectory labeled as 0, is available to agent 3 and 4, while in Fig. (**2**), only agent 5 has access to

the desired trajectory. The desired trajectory X_d for task-space tracking, noted as the virtual leader agent 0, is selected as $X(t) = [1 + 0.5\cos(t), 1 + 0.5\sin(t)]^T$. The observer parameters are selected as $k_1 = 2$, $k_2 = 8$, $k_3 = 5$ and $k_1 = 2$, $k_2 = 10$, $k_3 = 5$, respectively.

The initial values of the observer are set to be $\hat{v}_1(0) = [\,-2\ -2\,]^T$, $\hat{v}_2(0) = [\,-4\ -4\,]^T$, $\hat{v}_3(0) = [\,2\ 2\,]^T$, $\hat{v}_4(0) = [\,3\ 3\,]^T$, $\hat{v}_5(0) = [\,4\ -4\,]^T$.

Figs. (**3** and **4**) show the observed output under graph (a), the output under graph (b) of X-axis and Y-axis are illustrated in Figs. (**5** and **6**). By selecting proper parameters according to the sufficient conditions (17), the observer output of each follower manipulators reach consensus asymptotically, the observation errors converge to zero.

Simulation 2. In this part, an adaptive controller is studied for networked robot manipulators, the communication topology is defined by graph (a), as shown in Fig. (**1**). For simplicity, manipulators are assumed to have the same dynamic model, the mechanical structure of the manipulators is given in Fig. (**7**) l_1 , l_2 , m_1 , m_2 , l_{g1} and l_{g2} are the lengths, masses and the centers of mass of the two links, respectively. and the gravitational forces are assumed to be zero. The physical parameters of manipulators are selected as $l_1 = 1m$, $l_2 = 1.2m$, $m_1 = 1kg$, $m_2 = 1kg$, $l_{g1} = 0.5m$, $l_{g2} = 0.6m$.

The kinematic and dynamic parameters are selected as $a_k^i = [l_1,l_2]^T$, $a_d^i = [m_1l^2g_1 + m_2l_1^2 + m_2l^2g_2 , m_2l_1l_2 , m_2l^2g_2]^T$. The regression matrixes and can be obtained according to Eq. (3) and (6), respectively. The initial states of the manipulators are given in Table **1**.

The initial estimates of and for the agents are selected as $a_k^i = [\ 1.1\ 0.9\ ]^T$ and $a_d^i = [\ 4\ 0.5\ 0.5\ ]^T$, $i = 1,2,3,4$. The control gains are selected as $k_4 = 5$, $k_5 = 10$, $k_6 = 10$, $\Gamma_1 = 3$, $\Gamma_2 = 0.05$, $\Gamma_3 = 1$, $\Gamma_4 = 1$.

Under the controller (29), manipulators track the desired trajectory as shown in Fig. (**8**), and the tracking errors of X-axis and Y-axis are given in Figs. (**9** and **10**). The end-effectors of the robots reach consensus asymptotically, and the tracking errors converge to zero. The control torque of the two joints by controller (29) is shown in Figs. (**11** and **12**). It is obvious that the control output is continuous. The controller avoids chattering phenomenon effectively.

Simulation 3. When the task-space velocity is immeasurable, the controller is (43), the tracking trajectories are illustrated in Fig. (**13**), the tracking errors of X-axis and Y-axis are shown in Figs. (**14** and **15**).

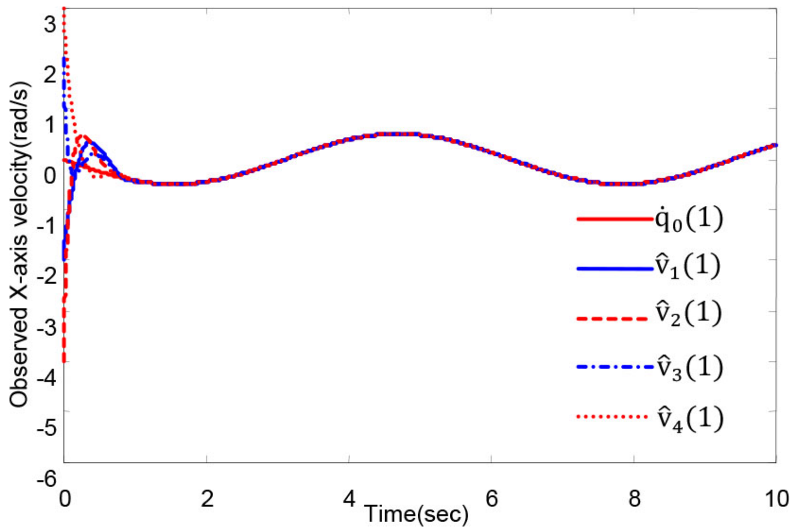

Fig. (3). Observed X-axis velocity under graph (a).

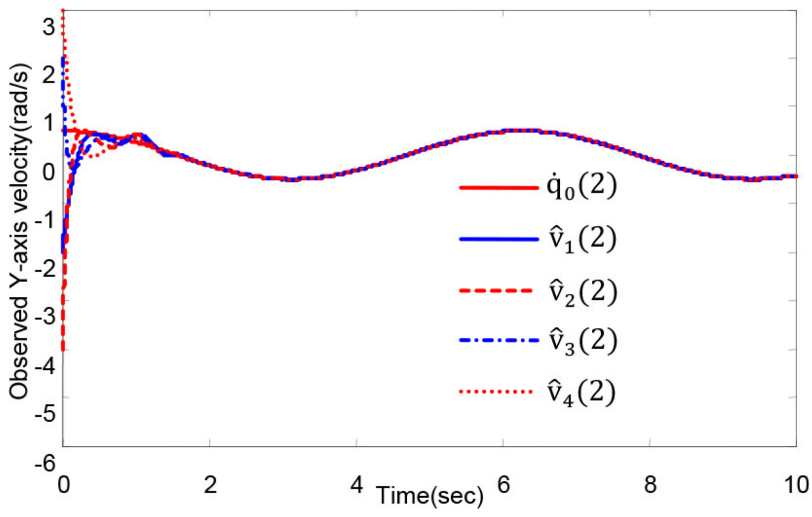

Fig. (4). Observed Y-axis velocity under graph (a).

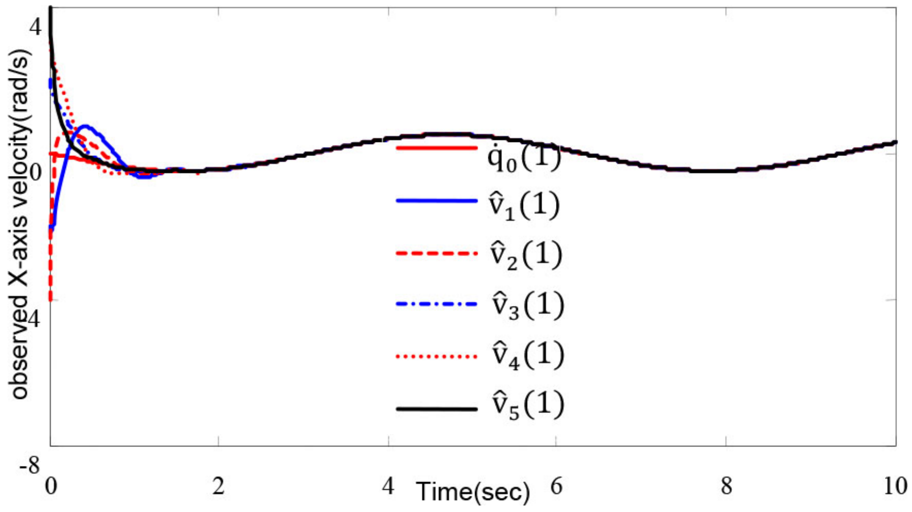

Fig. (5). Observed X-axis velocity under graph (b).

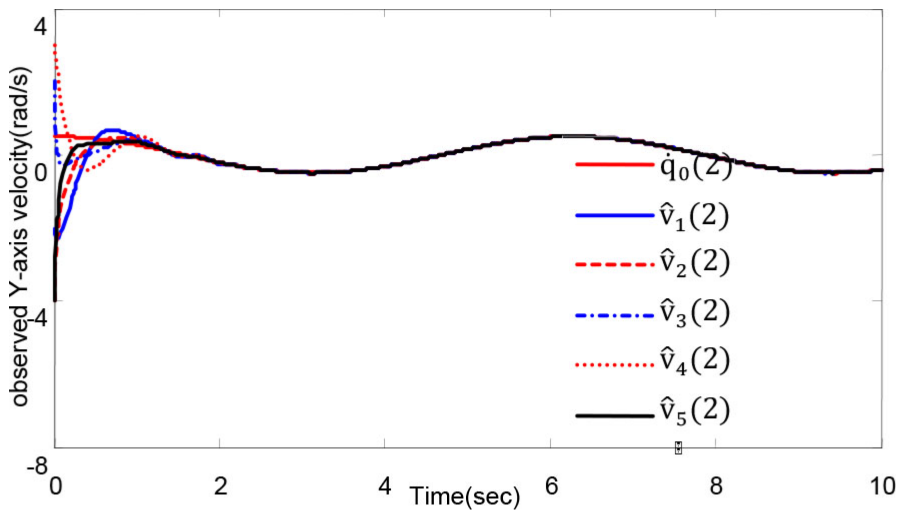

Fig. (6). Observed Y-axis velocity under graph (b).

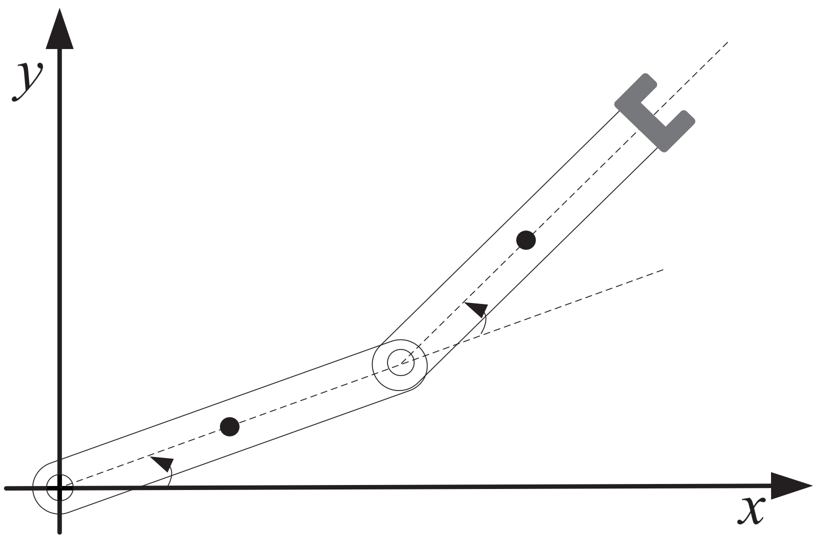

Fig. (7). Architecture of two rigid-link robot manipulator.

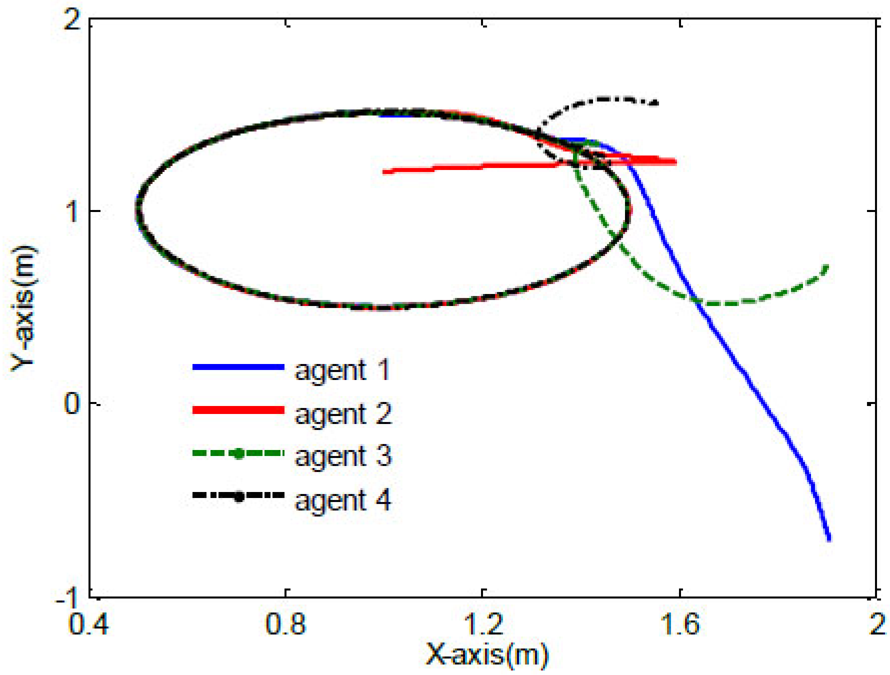

Fig. (8). Tracking trajectories of MRMs by controller (29).

Table 1. The physical manipulator parameters.

Agent	$q_1(rad)$	$q_2(rad)$	$\dot{q}_1(rad / s)$	$\dot{q}_1(rad / s)$
1	$-\pi / 4$	$\pi / 4$	0	0
2	0	$\pi / 2$	0	1
3	$\pi / 4$	$-\pi / 4$	1	0
4	$\pi / 4$	0	1	1

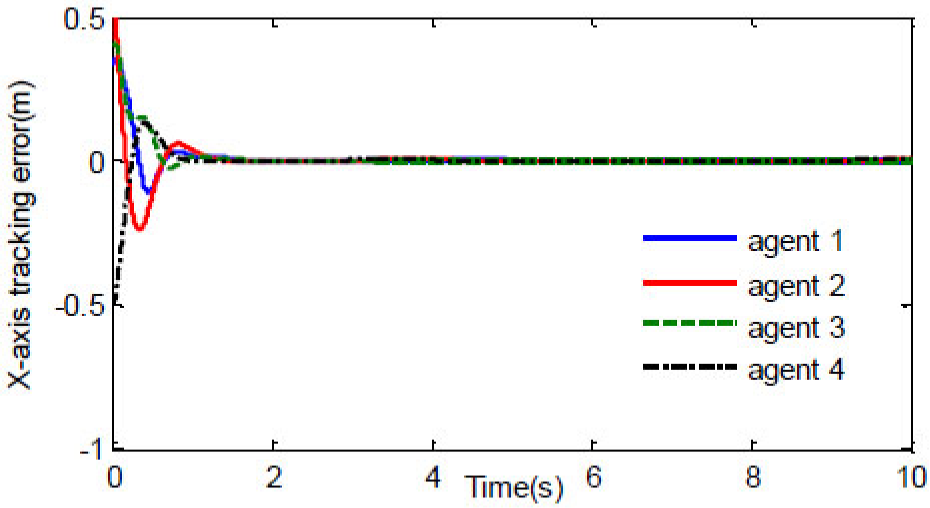

Fig. (9). X-axis tracking error by controller (29).

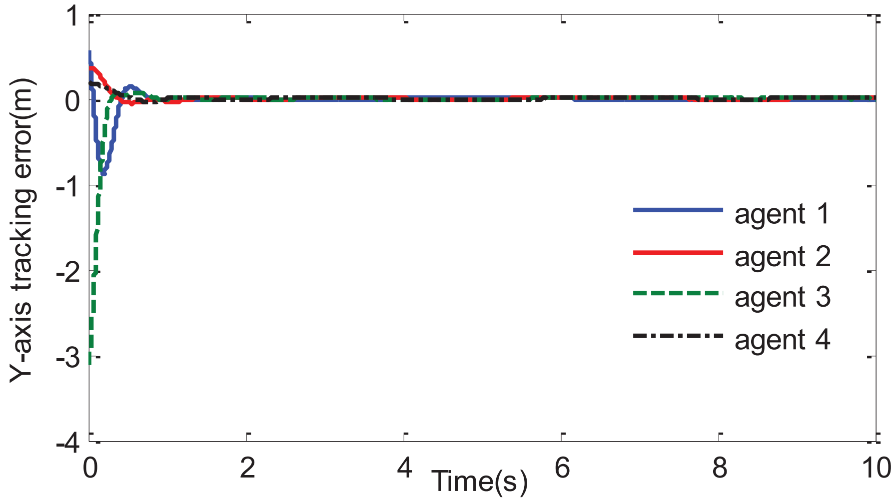

Fig. (10). Y-axis tracking error by controller (29).

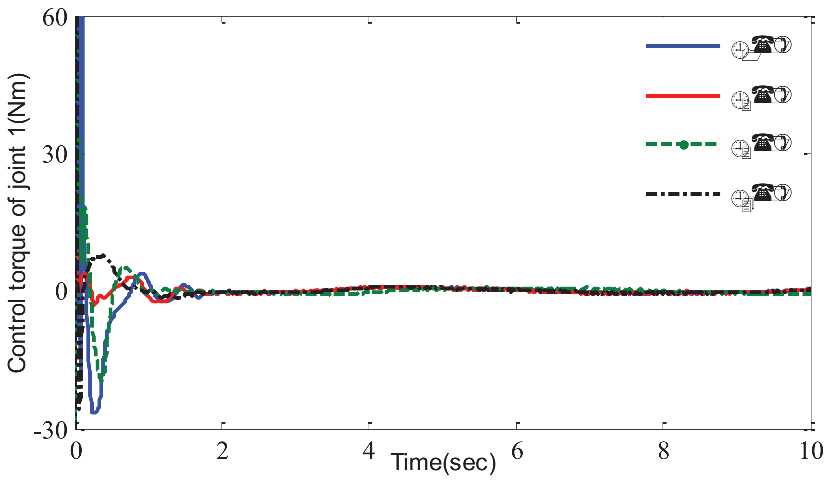

Fig. (11). Control torque of joint 1 by controller (29).

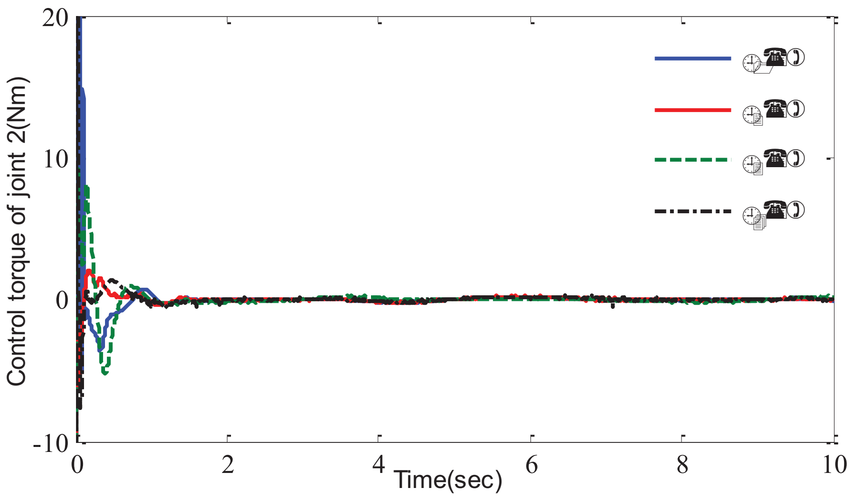

Fig. (12). Control torque of joint 2 by controller (29).

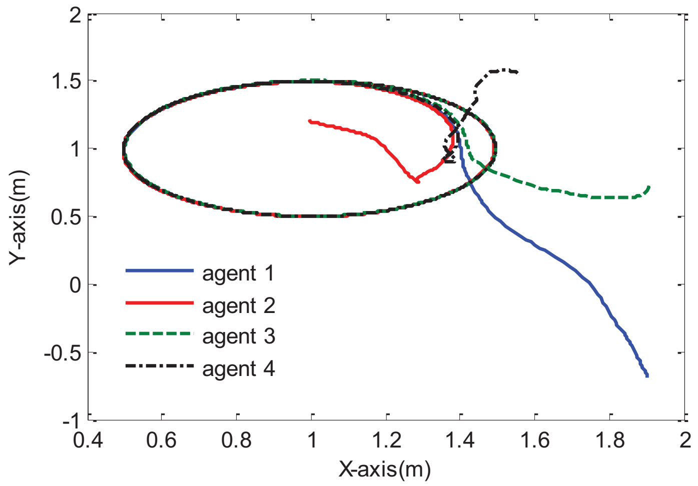

Fig. (13). Tracking trajectories of MRMs by controller (43).

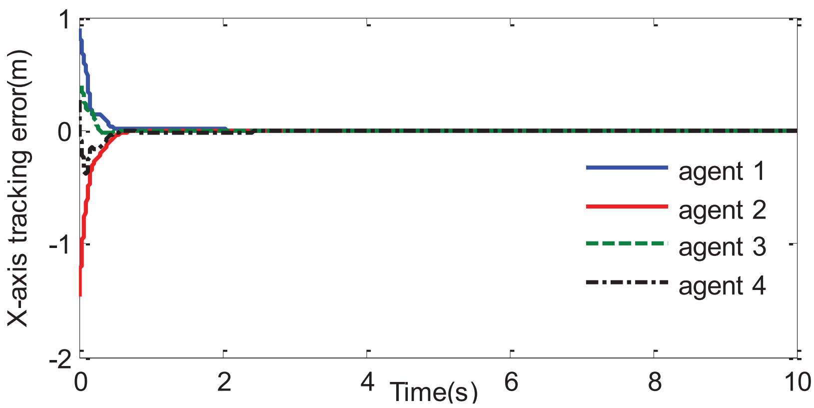

Fig. (14). Y-axis tracking error by controller (43).

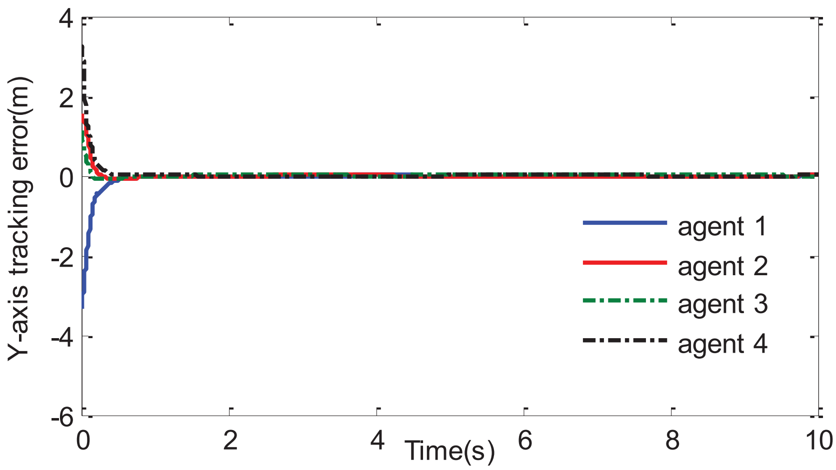

Fig. (15). Y-axis tracking error by controller (43).

With the definition of modified reference trajectory (42) and updating law (44), the MRMs can still track the desired trajectory asymptotically, the control torque are shown in Figs. (**16** and **17**). which shows the effectiveness of the proposed strategies. Parameter has close influence on the steady state error, and larger would make the faster convergence of the tracking error.

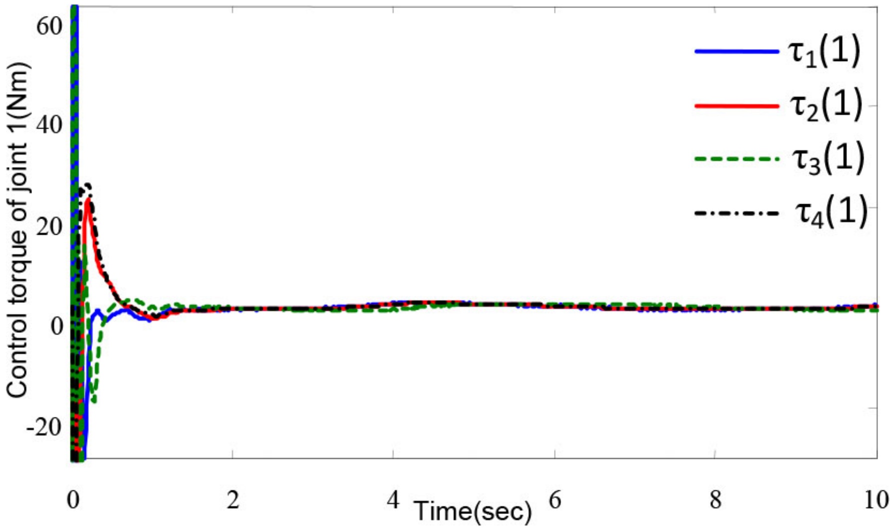

Fig. (16). Control torque of joint 1 by controller (43).

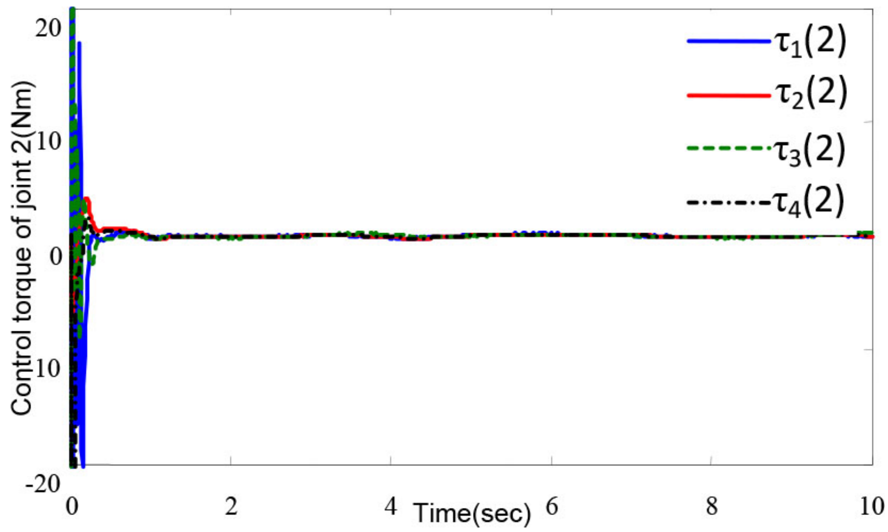

Fig. (17). Control torque of joint 2 by controller (43).

CONCLUSION

In this paper, the distributed task-space tracking problem for multiple robot manipulators is discussed. A continuous adaptive controller is designed to enable global asymptotic consensus tracking under undirected topology, the dynamic and kinematic uncertainties of the manipulators are also considered. Future research will expand to other topologies such as directed graph, time-delay, *etc*.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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