

Local Government Debt Credit risk and Safe Debt Scale Based on the KMV Model

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Abstract: Based on the idea of KMV model to build China's local government debt credit risk model, and associate the credit risk with borrowing scale to put forward the moderate debt scale of local government. Studies show that: The credit risks of local government debt is very sensitive to debt scale, When debt scale increases to a certain extent, the government's default probability will rise sharply, the safe debt scale should be controlled before the default probability increases rapidly; The actual distribution of fiscal revenue has higher kurtosis compared to the lognormal distribution assumption, for the same debt scale, the default probability of local government debt based on the actual distribution is higher than the default probability under theoretical distribution. Suggest to develop the local government bond market, to improve the local government credit rating system, information disclosure system and bond insurance system.

Keywords: Credit risk, KMV model, local government debt, safe debt scale.

1. INTRODUCTION

Affected by the financial crisis in 2008, governments take expansionary fiscal policy to stimulate the economy with super strength, leading to the world's major economies fiscal risks rise sharply since 2010, the European debt crisis, the U.S. debt crisis broke out one after another have caused the social from all walks of life to pay attention to the local government debt risk in China. 2014 is the peak year of local government debt repayment, the national government debt audit results show that by the end of June 2013, the local government debt balance is nearly 18 trillion yuan, of which, the local governments have responsibility to repay is about 10.89 trillion yuan, account for 60.85%, have the debts of guarantee responsibility and may assume assistance responsibility respectively account for 14.9% and 24.25%. In recent years, the local government debt shows the trend of large scale, rapid growth and unreasonable structure, and the new borrowing subjects and borrowing ways become more complex and concealed, which has laid a huge hidden trouble for the healthy development of the economy.

This paper aims to study the local government debts' credit risk and safety scale from the quantitative point of view, based on the idea of internationally renowned KMV model, combined with the study of Han Liyan *et al.* [1] to build the local government debt credit risk measurement model which is suitable for China, and associate the credit risk with borrowing scale to put forward the moderate debt scale of local government, to further strengthen the risk management and scale control of local government debt.

2. LOCAL GOVERNMENT DEBT CREDIT RISK MODEL

The KMV model is Moody's KMV developed a credit monitoring model based on the option pricing theory, the model can be used for a single asset risk measurement, and in the use of the KMV model could assume that the enterprise assets' value subject to normal or lognormal distribution, make full use of capital market information rather than the historical data to estimate default probability, it can better reflect the current credit status of listed companies [2]. In this paper, the credit risk measurement of local government debt is only relates to the probability of default and the corresponding government debt scale, it doesn't involve the bond pricing and default loss, but stresses the forward-looking, therefore, the KMV model is the most suitable for this study. It's basic idea is to regard equity as option [3], apply this idea to the local government debt credit risk measure can be understood as the local government "transfer" the authority of local fiscal revenue to the investor, when the local debts expire, if local finance income can be used to repay exceed the debt scale, the local government repay debt and "redeem" the right of local fiscal revenue; Otherwise, the local government defaults.

Without loss of generality, assume that local fiscal revenue after deducting the necessary expenses that can be used to repay debt obey the following stochastic process:

$$M_t = f(Z_t) \quad (1)$$

There into, M_t is the part of local fiscal revenue after deducting the necessary expenses that can be used to repay debt, hereinafter referred to as the available guarantee revenue, $f(g)$ is an undetermined function, Z_t is random variable.

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When the local government debts expire (t=T), if the available guarantee revenue M_T is less than the sum of debt principal and interest due B_T , that is when $M_T < B_T$, the local government will default. Use p to express the probability of default, then:

$$p = P[M_T < B_T] = P[f(Z_T) < B_T] = P[Z_T < f^{-1}(B_T)] \tag{2}$$

Thus, introduce the Distance to Default (DD) of the KMV model, according to the definition we can get the Distance to Default of local government debt at maturity [4]:

$$DD = \frac{E(M) - B_T}{E(M)\sigma_M} = -f^{-1}(B_T) \tag{3}$$

Thereinto, σ_M is volatility of local fiscal revenue, B_T is the sum of debt principal and interest due.

Studies have found that the local fiscal revenue to meet the Markov random process (Li Jingfeng, 2011)[5], so it can be assumed that it follows the Geometric Brownian Motion:

$$dM_t = gM_t dt + \sigma_M M_t dz_t \tag{4}$$

Thereinto: dM_t —Increment of local fiscal revenue

dz_t —Increment of Wiener Process,

$$dz_t = \varepsilon\sqrt{dt}, \varepsilon \sim N(0,1)$$

g —growth rate of local fiscal revenue

σ_M —volatility of local fiscal revenue

By formula 4, we know that at t moment local fiscal revenue can be expressed as:

$$M_t = M_0 \exp\left\{\left(g - \frac{\sigma_M^2}{2}\right)t + \sigma_M \varepsilon \sqrt{t}\right\} \tag{5}$$

Thereinto, M_0 is the beginning available guarantee revenue, $\varepsilon \sim N(0,1)$, at this point the available guarantee revenue obey lognormal distribution.

According to the *Itô's Lemma*, we can get:

$$\ln M_t = \ln M_0 + \left(g - \frac{\sigma_M^2}{2}\right)t + \sigma_M \varepsilon \sqrt{t} \tag{6}$$

Therefore, we can obtain t moment the available guarantee revenue's logarithmic mean and variance:

$$E[\ln M_t] = \ln M_0 + \left(g - \frac{\sigma_M^2}{2}\right)t \tag{7}$$

$$Var[\ln M_t] = \sigma_M^2 t \tag{8}$$

Because of the available guarantee revenue M is a flow data, its growth rate g can't be replaced by the risk-free interest rate, thus it can be derived by formula 7 and formula 8.

We might as well suppose t=1 to make inferences in the empirical study, that is to calculate the default probability of local government debt one year later, we can get:

$$g = \frac{1}{n} \sum_{t=0}^{n-1} \ln \frac{M_{t+1}}{M_t} + \frac{\sigma_M^2}{2} \tag{9}$$

$$\sigma_M = \sqrt{\frac{1}{n-1} \sum_{t=0}^{n-1} \left(\ln \frac{M_{t+1}}{M_t} - \frac{1}{n} \sum_{t=0}^{n-1} \ln \frac{M_{t+1}}{M_t} \right)^2} \tag{10}$$

According to the method of KMV model, we can get the expected default frequency (EDF) of local government debt:

$$\begin{aligned} &= P\left[\ln M_0 - \ln B_T + \left(g - \frac{\sigma_M^2}{2}\right)t < -\sigma_M \varepsilon \sqrt{t} \right] \\ &= P\left[\frac{\ln \frac{M_0}{B_T} + \left(g - \frac{\sigma_M^2}{2}\right)t}{\sigma_M \sqrt{t}} > \varepsilon \right] \end{aligned} \tag{11}$$

For $\varepsilon \sim (0,1)$, so the formula 12 can be expressed as the cumulative normal distribution:

$$p = N\left[\frac{\ln \frac{M_0}{B_T} + \left(g - \frac{\sigma_M^2}{2}\right)t}{\sigma_M \sqrt{t}} \right] \tag{12}$$

The distance to default is:

$$DD = \frac{\ln \frac{M_0}{B_T} + \left(g - \frac{\sigma_M^2}{2}\right)t}{\sigma_M \sqrt{t}} \tag{13}$$

Accordingly, the EDF can be expressed as:

$$p = N(-DD) = 1 - N(DD) \tag{14}$$

It should be noted that, in practice the assets value of the enterprise generally show the statistical characteristics of non-normal, and actual default situation of enterprises are more complex, which makes the EDF under theoretical assumption is not entirely equivalent to the actual EDF [6]. Due to the current credit system in China is not perfect, and lack of a large number of historical data of local government debt defaults, so we can just get the theoretical EDF of local government debt. Therefore, this paper introduces the real probability distribution of local fiscal revenue, and uses non-parametric method to estimate the distribution, take it as the computational basis of the corresponding relationship between default probability and debt scale, so as to solve the application of local government debt credit risk model lack of historical default data.

3. EMPIRICAL STUDY BASED ON THE DATA OF ZHEJIANG PROVINCE

3.1. Research Thought

This paper takes Zhejiang Province as the research object, using the local government debt credit risk model estab-

lished above to measure the credit risk of local government debt in Zhejiang Province, to calculate the probability of default at different levels of government debt scale, associate the credit risk with debt scale, by set a limit to default probability to put forward the moderate debt scale of Zhejiang Province.

Considering the local financial revenue has significant time trend, and the local government debt credit risk model has considered the volatility of local fiscal revenue, this paper selects the first-order autoregressive model to predict the local fiscal revenue next year. Since 1994 China has carried out the tax reform, it makes the statistical caliber of local financial revenue obviously difference after 1994, so the sample capacity is quite limited if use local fiscal revenue data to make regression forecasting, it will affect the prediction effect of auto-regression. In order to solve the problem this paper adopts the following methods: From Fig. (1) we can see that the proportion of local fiscal revenue account for fiscal revenue is relatively stable, since 2004 it basically stable at around 50%.

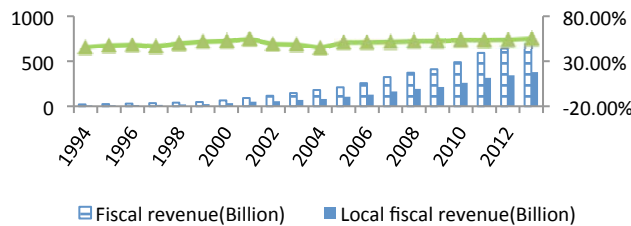


Fig. (1). The proportion of local fiscal revenue account for fiscal revenue in Zhejiang Province.

Therefore, this paper use the fiscal revenue data of Zhejiang province from 1978 to 2013, through auto-regression method to predict the 2014 fiscal revenue of Zhejiang Province, according to the proportion of local fiscal revenue account for fiscal revenue 50% to calculate the local fiscal revenue of Zhejiang Province in 2014. Particularly, we need to consider in the practical application local government must ensure some necessary expenditure, as the local debt repayment guarantee is only the balance after deducting the necessary expenses.

4. KEY PARAMETERS CALCULATION

4.1. Available guarantee revenue M

This paper based on the 1978-2013 fiscal revenue and GDP data of Zhejiang Province, choose the first-order autoregressive model to predict 2014 fiscal revenue of Zhejiang Province. Take the logarithm of each index and then make first-order autoregressive.

Upon testing, $\ln M$ and $\ln GDP$ are integrated of order sequences, and exist co-integration relationship. According to the regression results, we can get a balanced relational expression about $\ln M$ and $\ln GDP$:

$$\ln M_t = -0.2077 + 0.1212 \ln GDP_t + 0.8957 \ln M_{t-1} \quad (16)$$

2013 GDP of Zhejiang Province is 3756.85 billion, according to IMF, World Bank and Chinese Academy of So-

cial Sciences' prediction, we can calculate the GDP in 2014 is 4155.9 billion. Take into the formula 16, can calculate the 2014 fiscal revenue of Zhejiang Province is 810.58 billion, and then we can get the 2014 local fiscal revenue of Zhejiang Province is about 405.29 billion.

By learning from the existing literatures and in recent years education, health care, social security and other necessary local fiscal expenditures of Zhejiang Province account for about 60% of the local fiscal revenue, so the paper sets guarantee ratio u at 40%. Therefore, we can get the available guarantee revenue M_{2014} of Zhejiang Province in 2014 is 162.12 billion.

4.2. Growth Rate g and Volatility σ_M of Available Guarantee Revenue

Take M_{2014} into the formula 9 and formula 10, we can get: $g = 0.1912$ $\sigma_M = 0.0815$

4.3. Sum of Debt Principal and Interest Due B_T

According to incomplete statistics, the infrastructure investment of Zhejiang Province in 2013 is 386.2 billion, this study assumes that the infrastructure construction funds of Zhejiang Province in 2013 partially or entirely from local debt financing, according to different financing ratio 10%, 20%, 30% to set local government debt scale. Zhejiang Province as one of the first batch pilot governments which can issue bonds autonomously, has issued six local government bonds since 2011, we select the 5-year government bonds' average interest rate 3.57% as the interest rate of Zhejiang Province local government debt in the research, so we can get the sum of debt principal and interest Zhejiang Provincial government requires to repay in 2014 under each financing scale.

4.4. EDF Calculation

Take matured debts scale under different financing ratio into the formula 14 and formula 15 can get the DD and the EDF under various financing scale, as shown in Table 1.

In reality, the local fiscal revenue is not entirely obey the lognormal distribution, which will lead to changes in the distribution of credit risk. And due to the current lack of large amounts of historical data of local government debt defaults in China, thus unable to find out the mapping relationship between the DD and EDF. Therefore, this paper uses kernel density estimation method to fit the real distribution of logarithmic available guarantee revenue of Zhejiang Province, to calculate the default probability by the real distribution in order to be closer to the actual.

The real probability distribution curve of logarithmic available guarantee revenue of Zhejiang Province has higher kurtosis than lognormal distribution curve in Fig. (2). That is to say, for the same debt scale, the default probability of local government debt under the true distribution is higher than the default probability under the lognormal distribution. The results are shown in Table 2.

Table 1. DD and EDF.

Financing Ratio	Financing Amount (Billion)	The Ratio Account for 2013 Zhejiang Local Fiscal Revenue	Sum of Debt Principal and Interest Require to Repay in 2014 (Billion)	The Ratio Account for 2014 Zhejiang Local Fiscal Revenue	DD	EDF
10%	38.62	10.17%	39.999	9.87%	19.468	0
20%	77.24	20.34%	79.998	19.74%	10.967	0
30%	115.86	30.51%	119.996	29.61%	5.994	0
35%	135.17	35.60%	139.996	34.54%	4.103	0
40%	154.48	40.68%	159.995	39.48%	2.466	0.68446%
45%	173.79	45.77%	179.994	44.41%	1.021	15.36685%
50%	193.10	50.86%	199.994	49.35%	-0.271	60.68882%
55%	212.41	55.94%	219.993	54.28%	-1.440	92.51285%
60%	231.72	61.03%	239.992	59.22%	-2.507	99.39475%
70%	270.34	71.20%	279.991	69.08%	-4.398	100%
80%	308.96	81.37%	319.990	78.95%	-6.036	100%

Table 2. The EDF of theoretical distribution and true distribution.

Financing Ratio	20%	30%	35%	40%	45%	50%	55%	60%	70%
EDF of theoretical distribution (%)	0	0	0	0.6845	15.3669	60.6889	92.5129	99.3948	100
EDF of true distribution (%)	0	0	0.0299	4.0657	30.4645	73.8666	92.5685	98.9676	100

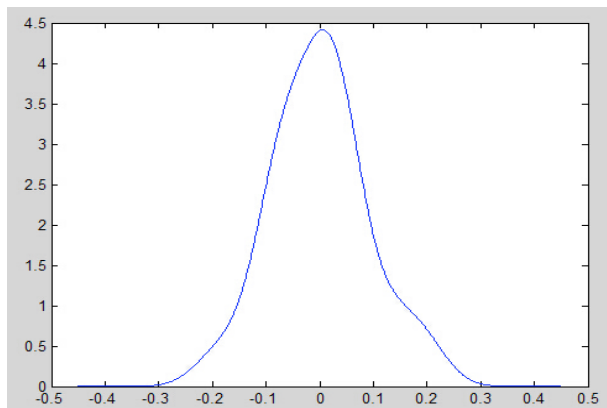


Fig. (2). The real distribution of logarithmic available guarantee revenue of Zhejiang Province.

4.5. Empirical Result Analysis

According to Table 1, under the assumption that Zhejiang Province local fiscal revenue obey the lognormal distribution, when local government offers less than 40% of infrastructure investment amount by debt financing, that is the scale of debt less than 154.48 billion, the default probability of government debt is very low, close to zero, and rise slowly, when the debt scale exceed 45% of infrastructure investment amount, the default probability presents nonlinear trend of accelerated growth. According to Table 2, it is not difficult to find that for the same debt scale, the default probabili-

ty of local government debt based on the actual distribution is higher than the default probability under the lognormal distribution. When the debt scale exceed 40% of infrastructure investment amount, the default probability begins to rise sharply. Visibly, the credit risk of local government debt in Zhejiang Province is sensitive to the debt scale, so strictly control the debt scale of Zhejiang Provincial government can effectively control the credit risk.

Moody's KMV obtained that credit rating of corporate bond upon Standard & Poor's BBB- or Moody's Baa3 belong to safe bonds based on historical data. And the credit risk of local government debt is lower than corporate bond, it means that its credit rating should be higher than the safe corporate bonds. Han Liyan (2003) pointed out in the study, the credit rating of municipal bond at least up to Standard & Poor's BBB + or Moody's Baa1, its EDF should be within 0.4%. Therefore, for the Zhejiang local government debt, requiring its EDF less than 0.4%, the borrowing should not exceed about 139.464 billion. According to the Zhejiang provincial government debt audit results, the government debt need to repay in 2014 is a total of 173.06 billion, of which government bears the responsibility to repay is 137.904 billion, it has reached the upper limit of safe debt scale.

CONCLUSION

According to the empirical study, we can get the following basic conclusions:

Firstly, the credit risk of local government debt is very sensitive to the debt scale, when debt scale increased to a certain extent, the government's default probability will rise sharply, the moderate borrowing scale should be controlled before the default probability rise quickly.

Secondly, the actual probability distribution of available guarantee revenue has higher kurtosis than lognormal distribution, that is "Leptokurtosis and Fat-Tail". Specific performance is that for the same debt scale, the default probability of local government debt based on the actual distribution is higher than the default probability under theoretical distribution. Thus based on prudent principles, local government should take the true distribution of the fiscal revenues as the calculated on the basis of the correspondence between default probability and debt scale.

POLICY RECOMMENDATIONS

Current local government debt in China presents the trend of large-scale, fast-growing, and exist the problems such as multiple borrowing, inefficient use of debt funds. To effectively prevent the credit risk of local government debt, governments at all levels should strengthen the control of debt scale, to resolve the stock of debt and strictly control the incremental debt at the same time [7]. Meanwhile, China should vigorously develop the local government debt market. Because of the lack of perfect debt information disclosure system, local government bond credit rating system and bond insurance system, it affects the development of local government bond market at the present stage [8]. Therefore, to develop local government bond market, improve the three system will be the emphasis of China's capital market development in the near future, it is conducive to establish local government's long-term financing mechanisms, regulate local government's financing behavior, strengthen local gov-

ernment debt information transparency, so as to effectively control the risks of local government debt.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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