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The Method of Design Sine Grating based on Suboptimal PWM

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Abstract: In the structured light 3 d optical measurement field, the sinusoidal grating design is one of the research focuses. The traditional focus methods of generating sine pattern is not widely accepted because of many well known defects. While another traditional binary pattern generated from Ronchi grating needs defocusing processing, its application has also been a certain limit because of its projected pattern contrast ratio is not high.In this paper, we have proposed a designing sine grating method with the spatial modulation theory based on Suboptimal PWM in electronic engineering. Compared with Ronchi grating, this method of generating pattern has a better fringe contrast, and more simple defocusing requirements, so this method has a good practical economic value.

Keywords: 3d profile measurement, PWM, sine grating, the fourier transform.

1. INTRODUCTION

Non-contact optical three-dimensional contour measurement technology [1] is widely used in CAD/CAM, reverse engineering, rapid prototyping and virtual reality, etc. Its main research methods are moire profilometry, phase measuring profilometry, Fourier transform profilometry and spatial phase detection, etc. The outline of the measuring methods need to project sinusoidal pattern to the surface of the object to be tested, and there are two ways of generating sine pattern [2], one is the traditional focusing method of generating sine patterns (FSP) [3], namely, the projector directly projects out the sinusoidal fringe pattern. Although this approach has been widely used at present, there are some obvious defects of FSP:

1. The projector nonlinear gamma problem.

2. the camera and the projector must be precisely synchronized.

3. the camera exposure time demands

Although the TI [4] (Texas Instruments) can solve the above problems, its high equipment cost prevents its wide application; the other one is DBP(defocusing binary patterns),namely the method of getting sinusoidal pattern by defocusing the binary pattern. It doesn't have a projector's nonlinear gamma problem, does not need complex devices to generate sinusoidal pattern, and he no synchronization problems and the demanding of exposure time between FSP cameras and projectors does not exist. With the help of general commercial projector, DBP can project sinusoidal pattern, but its key to success is to design a good binary pattern [5] to make the fringe contrast best. At present, the common adoption of DBP is Ronchi grating method [6], but it has the problem of pattern projected to the measured object as it has a low contrast, and the problem of requiring high defocusing degree.

This paper first presents the principle and method of Pulse Width Modulation (PWM). Further we put forward one design method of sine grating by using sub-optimal space width modulation, then compare the pattern‘s frequency spectrum and filtering performance between this method and the traditional Ronchi grating and finally indicate this design is better than Ronchi grating both on grating fringe contrast and defocusing requirement.

2. DESIGN OF SINE GRATING

Sampling control theory has an important conclusion: equal narrow impulses with different shapes have basically the same effect when they are added on to the inertial link. PWM control technology [7] is based on the theory of the conclusion, and we have designed two kinds of sinusoidal grating methods based on the above PWM control technology.

Designing sine grating based on space suboptimal width modulation is shown in Fig. (**1**), namely using high frequency triangle wave as the carrier, and the modulation wave is

$$g(x) = M \cdot [\sin(wx) + \frac{1}{4}\sin(f_c wx)]$$

(In the formula, M is the carrier amplitude normalization's modulation depth, f_c is the injection of harmonic, and it is used to eliminate the low times harmonics), the modulation wave's cycle is X, and divide one cycle into 2N intervals, interval Numbers are $0,1,2,\cdots,2N-1$. The point of intersection between each interval and x axis is:

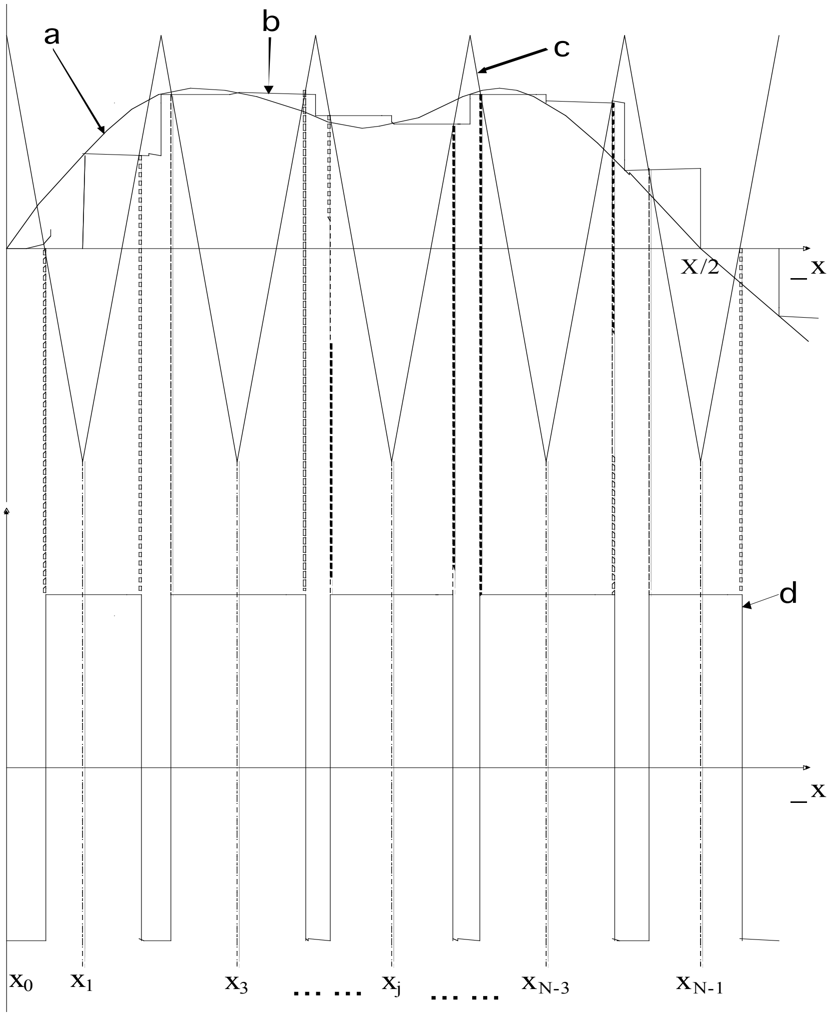

Fig. (1). - - - - - - As the switch time, - · - · - · - · - As the Sampling time.

x_0£-x_1£-x_2£¬$\ldots, x_{2N-1}, x_{2N}$, and the $0 \le j \le 2N-1$ interval can be expressed as: $[x_j, x_{j+1}]$, and sets the frequency of the triangular wave carrier $N \cdot \omega$, the carrier period is $\overline{X} = 2\pi / (N \cdot \omega)$。

The j-th(we may assume that j is even) pulse width is:

a-The modulation wave g(x) b-Sampling waveform c-carrier wave d-PWM wave

$$\delta_j = \frac{\overline{X}}{4}\{1 - M \cdot [\sin(\omega \cdot x_j) + \frac{1}{4}\sin(f_c \omega \cdot x_j)]\} \tag{1}$$

The (j+1)th pulse-width interval is:

$$\delta_{j+1} = \frac{\overline{X}}{4}\{1 + M \cdot [\sin(w \cdot x_{j+1})) + \frac{1}{4}(f_c w \cdot x_{j+1})]\} \tag{2}$$

In each cycle of the triangular wave, there is a low-amplitude pulse width X_{ne}, and a high-amplitude pulse width X_{po},

$X_{ine} = \delta_{2i}, X_{ipo} = \frac{\overline{X}}{2} + \delta_{2i+1} - \delta_{2i}$ (i is a positive integer between interval [0, N-1]).

In the spatial axis, the starting point is r_i, the ending point is e_i and waveform duration can be expressed as:

$r_i = i \cdot \overline{X} + X_{ine}$; $e_i = r_i + X_{ipo}$

With the help of the above formula of r_i and e_i. When, $\overline{X} = 2\pi$, N=17, M=1 and fc=17. We can design the grating pattern as shown in Fig. (**2**).

3. THE PARAMETERS DESIGNS OF THE GRATING AND SPECTRUM ANALYSIS

The grating pattern must defocus to obtain sinusoidal pattern with continuous variation of gray level, namely binary grating pattern through a circular aperture of defocused imaging projection optical system.

The optical transfer function of projection system is:

$$H(f_x, f_y) = \frac{J_1(2\pi\beta\sqrt{f_x^2 + f_y^2} / 2f_0)}{\pi\beta\sqrt{f_x^2 + f_y^2} / 2f_0} \tag{3}$$

In the formula, J_1 is a first order Bessel function, and β is the degree of defocusing.

As a result, the sine grating spectrum after filtering is:

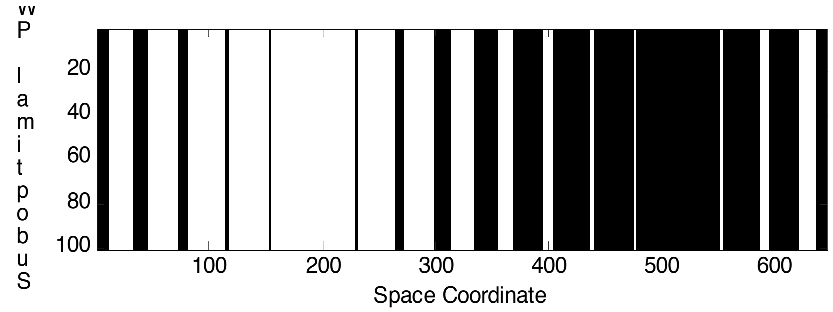

Fig. (2). Suboptimal PWM Pattern.

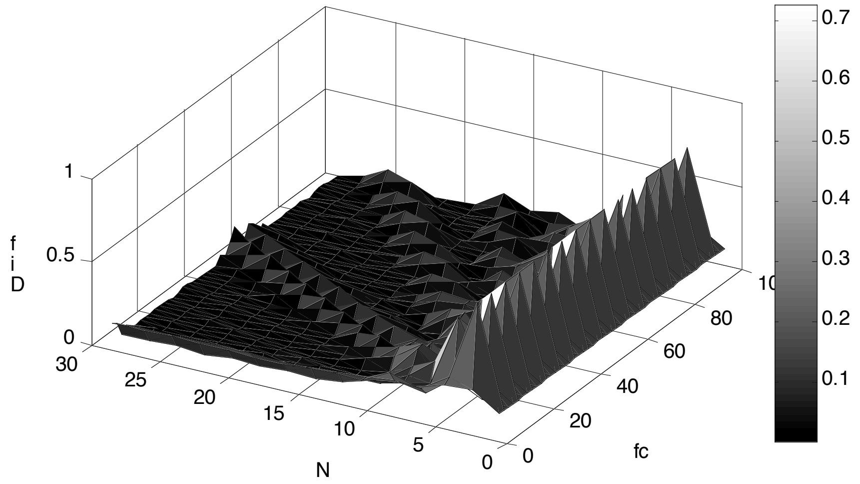

Fig. (3). Dif relationship diagram based on N and fc.

$$Y(f_x, f_y) = G_r(f_x, f_y)H(f_x, f_y) \tag{4}$$

In the above formula, $G_r(f_{x,}f_y)$ are kinds of grating spectrum components, and $Y(f_x, f_y)$ is the pattern of the spectrum after filtering. The defocusing operation is similar to pass the binary pattern through a gaussian low-pass filter, and filtering the sinusoidal fringe is $y(x)$.

The filtering grating stripe $y(x)$ is related to the modulation wave

$$g(x) = M \cdot [\sin(wx) + \frac{1}{4}\sin(f_c wx)] \cdot$$

Whether the grating's design in two indicatorsis good or not, , one is that low harmonics of the binary stripe before filtering occupy small proportion, the other one is that the light and shade contrast of filtering stripes must be high.

The former indicator can be described as $Dif = \frac{\sum_{i=2}^{10} G_i}{G_1}$, in the formula, Gi represent the amplitude of binary stripes in the i-th frequency component, and G_1 is the fundamental component; the latter indicator can be described as $K = \frac{y_{\max} - y_{\min}}{y_{\max} + y_{\min}}$, $y_{\max}, y_{\min}$ are the maximum and the minimum of the filtering stripe's amplitude respectively.

Both indicators are related to the three parameters M, N, f_c of the modulation wave, Where M is the modulation depth, N is the carrier ratio, and f_c is the number of harmonics injected.

First of all, we do analysis of the carrier ratio N, the larger the N means the larger the number of single cycles needed to characterize the fringe, and the more resource intensive. So we limit the odd N within 29. After changing the parameter M and f_c and after many simulations, we can get a three-dimensional surface wave of Dif as shown in Fig. (**3**). When N is between 15-29, Dif can be a smaller value, taken together, we set N to 17. When N is 17, we can obtain three-dimensional surface map Dif and K as shown in Figs. (**4-6**) by changing the value M, and value f_c.

In Fig. (**4**), we can draw that f_c has a larger influence on Dif, Dif values were approximated with cyclical changes (the period is about 37) in the value of f_c, and M makes Dif change little; Fig. (**5**) is a front view of fc, It can be found that Dif varies generally not more than 0.02 with the value of M under a particular f_c. In a single cycle, we can point out several frequencies corresponding to the small Dif, for example, Dif is not more than 0.02. Average corresponds to the maximum value when f_c can be 15, 17, 23, 33 and 35. Namely, the proportion of low-order harmonics of the fundamental wave does not exceed 0.02 times the fundamental

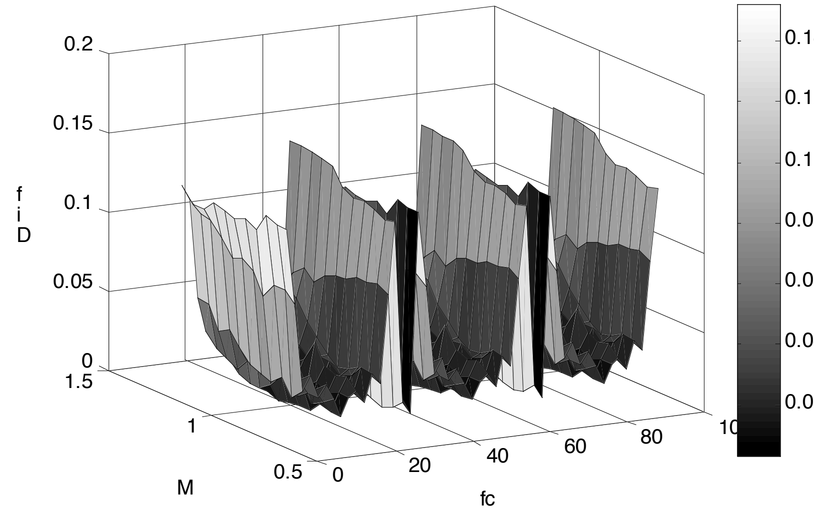

Fig. (4). Dif relationship diagram based on M and fc.

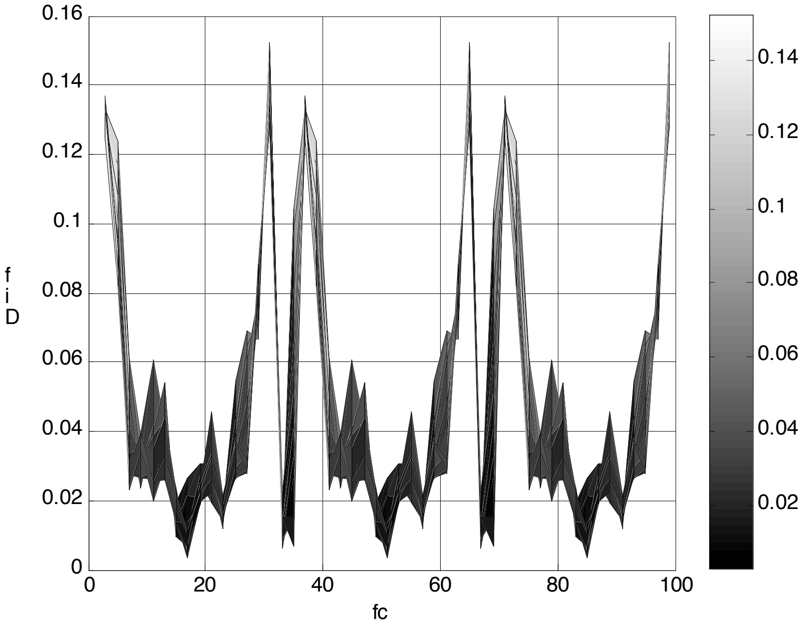

Fig. (5). Dif relationship diagram based on fc

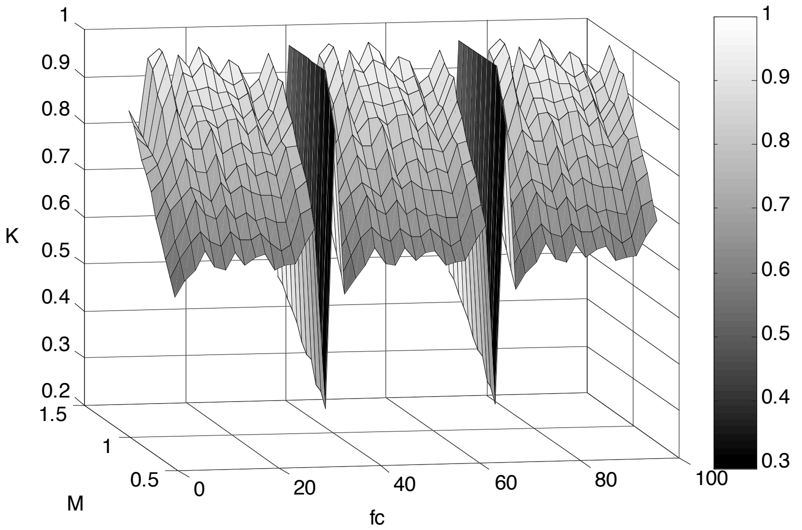

Fig. (6). K relationship diagram based on M and fc.

Table 1. Dif's relationship with M and fc.

fc M	0.7	0.75	0.8	0.85	0.9	0.95	1.0	1.05	1.1	1.15	1.2
15	**0.0092**	0.0114	0.0132	0.0131	0.0118	0.0145	**0.0084**	**0.0061**	0.0111	0.0183	0.0231
17	0.0110	0.0106	0.0132	**0.0065**	**0.0061**	**0.0055**	**0.0034**	**0.0070**	0.0116	0.0213	0.0264
23	0.0107	**0.0079**	**0.0088**	0.0110	0.0102	0.0109	**0.0087**	0.0115	0.0125	0.0121	0.0149
33	0.0159	0.0115	0.0115	0.0119	0.0155	0.0115	0.0106	**0.0092**	**0.0069**	**0.0088**	**0.0061**
35	**0.0088**	**0.0021**	**0.0034**	**0.0077**	0.0153	0.0252	0.0315	0.0404	0.0435	0.0532	0.0594

Table 2. K's relationship with M and fc.

fc M	0.7	0.75	0.8	0.85	0.9	0.95	1.0	1.05	1.1	1.15	1.2
15	0.6034	0.6463	0.6993	0.7432	0.7773	0.8225	0.8753	0.8946	0.9247	0.9498	0.9776
17	0.6049	0.6500	0.6898	0.7373	0.7578	0.8079	**0.8541**	0.8860	0.9295	0.9724	1.0000
23	0.6131	0.6583	0.6833	0.7122	0.7594	0.8082	0.8532	0.9043	0.9289	0.9549	0.9776
33	0.4559	0.4559	0.5086	0.5560	0.5822	0.6084	0.6500	0.6607	0.7027	0.7553	0.7578
35	0.7553	0.8019	**0.8541**	0.9050	0.9422	0.9851	1.0000	1.0000	1.0000	1.0000	1.0000

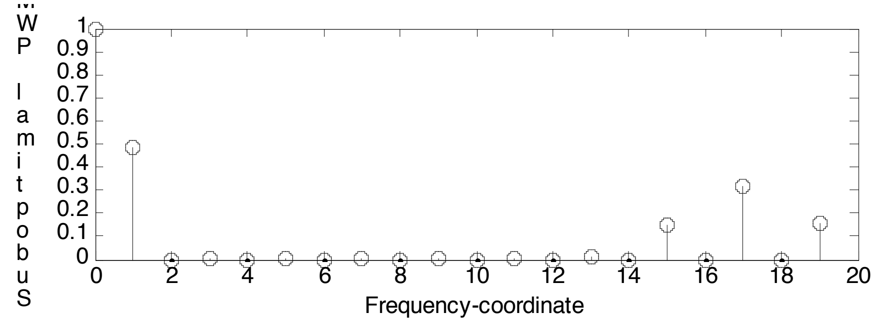

Fig. (7). The sinusoidal gratings' spectrum based on Suboptimal PWM.

wave component. Thus the generated pattern satisfies the condition of the low harmonic component with lower amplitude. Specific numerical values are shown in Tables **1** and **2**.

Table **1** shows Dif varies with M when f_c occupies five frequencies; Table **2** shows the changes situation of K with M.

Bold numbers in Table **1** means the value of Dif is less than 0.001. By comparing these bold values with the K in Table **2**, two Dif and K can be obtained which means the relative position correspond to the optimum value. As shown in Table **2**. When M takes 1.0, fc takes 17, and M takes 0.8, fc takes the value of 35, the K values are 0.8541. Fig. (**2**) is a binary pattern of these optimum values for the previous case. Fig. (**7**) shows the spectrum analysis of patterns generated from Suboptimal PWM generation, as a comparison, the Ronchi grating spectrum is shown Fig. (**8**). Table **3** lists two grating frequency components. DBP grating must take defocusing processing operations, harmonic components within 10 times are likely to affect the filtering effect, so Table **3** lists front 10 frequency component amplitudes corresponding to (Figs. **7** and **8**).

By the results shown in Fig. (**7**) to Fig. (**8**) and Table **3**, for the suboptimal method in Fig. (**7**), its frequency is mainly on the fundamental wave and harmonic 15, 17 and the greater values, there is almost no amplitude in other low harmonics.By the contrast of Ronchi grating spectrum, it is easy to find that various odd harmonic components are big. Therefore, the conditions of low-pass filtering for suboptimal grating is much easier than the Ronchi grating's.

4. PERFORMANCE ANALYSIS OF SINE GRATING AFTER DEFOCUSING

From Figs. (**7 and 8**), and the spectrum in Table **3**, the traditional Ronchi grating needs higher defocusing demand, namely, it needs a narrow band low pass filter. The Suboptimal PWM grating in this paper, needs broadband low pass filter, its filtering requirements is far better than Ronchi grating's.

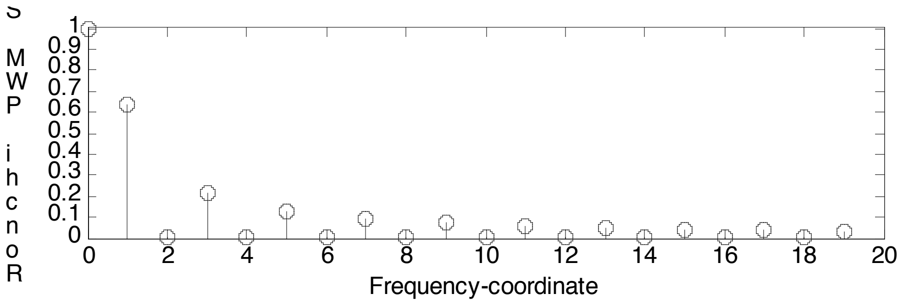

Fig. (8). The Ronchi's spectrum.

Table 3. Frequency components of two grating pattern.

Grating Amplitud Frequency	1	2	3	4	5	6	7	8	9	10
Suboptimal	0.4870	0.0000	0.0011	0.0000	0.0052	0.0000	0.0049	0.0000	0.0051	0.0000
Ronchi	0.6366	0.0016	0.2122	0.0016	0.1273	0.0016	0.0910	0.0016	0.0707	0.0016

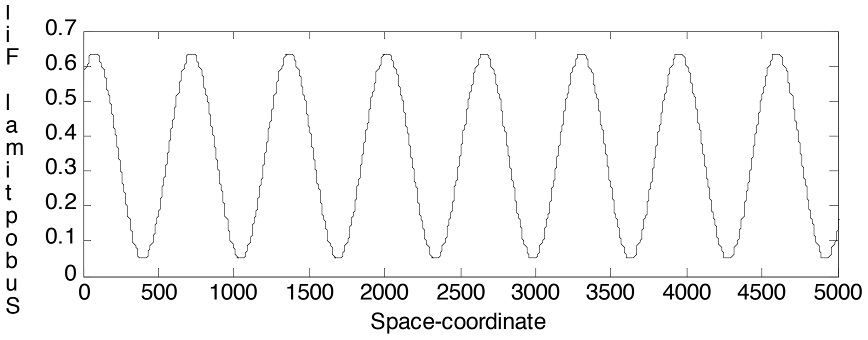

Fig. (9). Suboptimal PWM pattern after filtering.

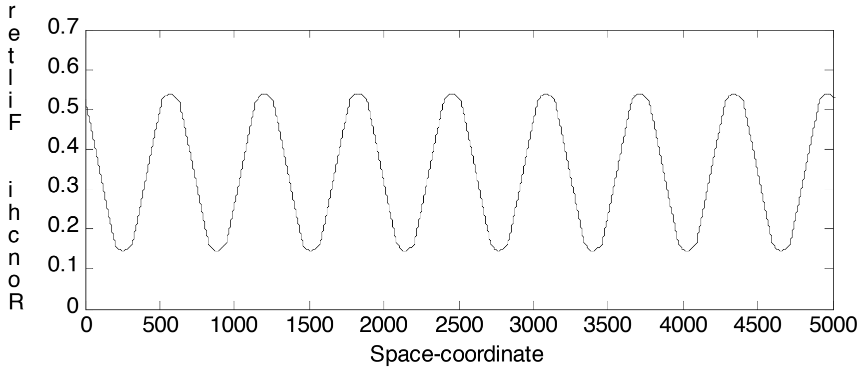

Fig. (10). Ronchi pattern after filtering.

Harmonic has a great influence on the light intensity of projection. We define the intensity of light and shade contrast as: $K = \frac{Y(t)_{\max} - Y(t)_{\min}}{Y(t)_{\max} + Y(t)_{\min}}$. According to the filtering operation in formula 3 and 4, for the suboptimal sine grating patterns in Fig. (**2**), we use broadband gaussian filtering to recover sine wave as shown in Fig. (**6**), the light and shade contrast K = 0.8541. For grating pattern corresponding to the Ronchi grating spectrum in Fig. (**9**), we must develop a narrowband gauss filter on it, the recovery sine wave as shown in Fig. (**10**), and its light and shade contrast K=0.55. Therefore, the Suboptimal PWM sine grating in this article, its fringe pattern on the surface of the object to be tested by the lens has a better light and shade contrast than Ronchi grating.

5. CONCLUSION

This paper has designed a sinusoidal grating based on Suboptimal PWM. Applying the Pulse Width Modulation in time dimension to the space dimension, according to nonuniform spacing interval after modulation to rule grating and after selecting the optimal parameter values, the grating's low pass filter conditions are very low, and the pattern contrast of this sinusoidal grating through the lens defocusing is higher than the conventional Ronchi grating's. Therefore, the design of the PWM sinusoidal grating can improve the precision of three-dimensional contour measurement, it can be widely used in the field of 3 d profile measurement.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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