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Estimating Value-at-Risk in Electricity Market Based on Grey Extreme Value Theory

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Abstract: How to effectively evaluate price of volatility risk is the basis of risk management in electricity market. Electricity price connotes a grey system, due to uncertainty and incomplete information for partial external or inner parameters. A two-stage model for estimating value-at-risk based on grey system and extreme value theory is proposed. Firstly, in order to capture the dependencies, seasonalities and volatility-clustering, a GM(1,2) model is used to filter electricity price series. In this way, an approximately independently and identically distributed residual series with better statistical properties is acquired. Then extreme value theory is adopted to explicitly model the tails of the residuals of GM(1,2) model, and accurate estimates of electricity market value-at-risk can be produced. The empirical analysis based on the historical data of the PJM electricity market shows that the proposed model can be rapidly reflect the most recent and relevant changes of electricity prices and can produce accurate forecasts of value-at-risk at all confidence levels, and the computational cost is far less than the existing two-stage value-at-risk estimating models, further improving the ability of risk management for electricity market participants.

Keywords: Value-at-risk, Grey system theory, Extreme value theory, GM(1,2) model, Peaks over thresholds.

1. INTRODUCTION

The introduction of market competitive mechanism has provided more lucrative opportunities for electricity market participants, but also brought the price of volatility risk hitherto unknown at the same time. The distinctive characteristics of electric energy, which cannot be effectively stored through time and space and needs instantaneous balance for supply and demand, make electricity price present highly unusual volatility and occasional extreme movements of magnitudes rarely seen in traditional financial markets. Once financial risk occurs in electricity market, there have more serious negative effects on society and economy than in traditional financial market [1]. Therefore, how to effectively make an accurate assessment on the price of volatility risk in electricity market has become a very current and important issue.

Value-at-risk (VaR) is a risk management tool to quantify the level of risk exposure in advance, which overcomes the defect of ex-post evaluation for traditional risk management method, so VaR has become one of the most popular risk measurement tools in practice. By introducing capacity sufficient rate and must-run rate as exogenous explanatory variables to depict the generators' market power and the supply-demand relationship, a generalized autoregressive conditional heteroskedasticity model with Gaussian distribution innovations (NGARCH) has been used to assess the price of volatility risk in electricity markets [2]. In view of leverage effects of electricity prices, an exponential GARCH (EGARCH) model with Gaussian distribution innovations is developed to estimate the trading risk for distribution companies [3]. Considering that NGARCH based VaR calculating model cannot effectively address the leptokurtosis and heavy-tailed phenomenon in the data of profit and loss, a re-sampling method based on a bias-correction step and the bootstrap has been developed, further improving the VaR forecasting accuracy of the NGARCH model [4]. By utilizing Gram-Charlier series expansion of normal density function and student-t distribution to depict the residuals distribution of ARMAX-GARCH model, an estimating model of VaR considering the characteristics of electricity price series such as seasonalities, heteroscedasticities, skewnesses and lepkurtosises, has been proposed, showing that the model with Gram-Charlier series expansion of normal density function can rapidly reflect the recent and relevant changes of electricity prices and produce accurate forecasts of VaR at all confidence levels [5].

With GARCH-based model, the impacts of probability distribution assumption for innovations on VaR estimation accuracy are analyzed for four distributions: normal, student-t, skewed student-t and general error distribution (GED). The numerical example based on the historical data of the Pennsylvania-New Jersey-Maryland (PJM) market shows that the accuracy and stability of estimated values of VaR are heavily dependent on the selection of probability distribution for innovations and the model with GED distribution performs better in predicting VaR values [6]. Extreme value theory (EVT) provides a firm theoretical foundation to study the asymptotical distribution of extreme value for order statistics, without assuming the probability distribution for the sample data. EVT allows extrapolation beyond the sample and can accurately describe the behavior of the tails of the real data. R Rozario [7] estimated the VaR of electricity market using a technique from EVT known as peaks over thresholds (POT), showing that the estimated results perform well for moderate to very high confidence levels (95-90%), but struggle at higher levels (>99%) owing to the extreme clustering and other dependence evident in the data. Hans NE Bystrom [8] extended the classic unconditional EVT approach by first filtering the data via GARCH specification to capture some of the dependencies in return series, and thereafter applying ordinary EVT techniques. In this way the independently and identically distributed (IID) assumption behind the EVT-based tail-quantile estimator is less likely to be violated, and the better tail estimates in-sample and better predictions of future extreme price changes can be acquired. To describe the leverage effects of volatility of electric power price, an EGARCH specification [9] is used to filter the return series to obtain nearly IID residuals, showing that EGARCH-EVT model can rapidly reflect the most recent and relevant changes of electricity prices and produce accurate forecasts of VaR in the more volatile markets where the distribution of returns is characterized by higher levels of skewness and excess kurtosis. Up to now electric power energy cannot be stored economically and therefore the influencing factors such as loads, climates and installed capacity have an un-tempered effect on electricity prices. In particular, electricity price exhibits considerably richer structure than load curve and has the following characteristics: mean reversion, seasonalities, heteroscedasticities and extreme behavior with fast-reverting spikes. To obtain an approximately IID residual series with better statistical properties, an ARMAX-GARCH model with Gram-Charlier series expansion of normal density function and skewed-t distribution over the error items is used to pre-filter the raw data to capture the dependences of price series, further improving the effectiveness of the VaR estimates via POT model [10, 11].

Although the approximately IID residual series can be acquired by using GARCH models to pre-filter the electricity price series, the high non-linearity for the GARCH models leads to very large computational costs and hinders the wide application in practice. Considering that the properties of incomplete and uncertain information for the spot prices are in line with the characteristics of grey variables, a grey system and extreme value theory based two-stage model for estimating VaR is proposed in this paper (referred as GM(1,2)-POT-VaR). In stage one, to acquire the approximately IID residuals with better statistical properties, a grey GM(1,2) model is used to pre-filter the electricity price series. In stage two, an EVT based POT model is employed to explicitly deal with the right tail of the residuals of the GM(1,2), and accurate estimates of VaR in electricity market can be produced. There are several contributions. First, the paper proposes a model that has the potential to generate more accurate quantile estimates for electricity market. The heteroscedasticities, skewnesses, kurtosises, seasonalities and relationship to system loads of electricity prices are accommodated via a grey GM(1,2) specification. In forecasting VaR, EVT is applied to the residuals from this model. Clearly, the proposed GM(1,2)-POT-VaR model is a sophisticated approach to forecasting VaR. The second contribution is to acquire an approximately IID residual series with better statistical properties by using a GM(1,2) model. The effectiveness of the VaR estimates via POT model can be further improved. The third contribution of this paper is to compare the accuracy of VaR forecasts under the proposed model with two conventional approaches: ARMAX-GARCH-st [10] and ARMAX-GARCHSK [11]. Tail quantiles are estimated under each competing model and the frequency with which realized returns violate these estimates provides an initial measure of model success. The empirical analysis based on the historical data of the PJM electricity market indicates that the GM(1,2)-POT-VaR model can rapidly reflect the most recent and relevant changes of electricity prices and can produce accurate forecasts of VaR at all significance levels. Moreover, the computational costs is far less than the proposed models in [10, 11], further improving the risk management ability of electricity market participants. These results suggest that the proposed approach is robust and therefore useful.

2. GM(1,2) MODEL

The grey system theory was proposed by J L Deng in 1982, which is a multidisciplinary theory dealing with those systems with lack information. The grey model is a modelling method based on the concept of grey generating function and differential fitting, having the advantages that the predicted results can be tested and less original data are needed. Assuming that the observed series of electricity prices and loads are $X_1^{(0)} = \{x_1^{(k)}(1)\}$ and $X_2^{(0)} = \{x_2^{(k)}(1)\}$ respectively, the first-order accumulated generating operation (1-AGO) series of $X_1^{(0)}$ be $X_1^{(1)} = \{x_1^{(1)}(k) \mid x_1^{(1)}(k) = \sum_{j=1}^{k} x_1^{(1)}(j)\}$, and the 1-AGO series of $X_2^{(0)}$ be $X_2^{(1)} = \{x_2^{(1)}(k) \mid x_2^{(1)}(k) = \sum_{j=1}^{k} x_2^{(1)}(j)\}$, among them, $k = 1, 2, \cdots, n$. Then, the dynamic process of $x_1^{(1)}(k)$ can be described by the grey GM(1,2) model:

$$x_1^{(0)}(k) + a z_1^{(1)}(k) = b x_2^{(1)}(k). \tag{1}$$

The corresponding whitening differential equation is

$$\frac{dx^{(1)}(t)}{dt} + a x^{(1)}(t) = b x_2^{(1)}(k), \tag{2}$$

where, $dx^{(1)}(t)/dt$ is the grey derivative of $X^{(1)}$, a and b are the model parameters to be estimated, $z_1^{(1)}(k) = \lambda x_1^{(1)}(k) + (1-\lambda) x_1^{(1)}(k-1)$ $(0 \le \lambda \le 1)$ is the background value. In traditional GM(1,2) model the λ is usually taken to be a fixed value 0.5. Let $\hat{a} = [a, b]^T$, then the estimated values by least squares method is

$$\hat{a} = (B^T B)^{-1} B^T Y_N, \tag{3}$$

in which,

$$Y_N = [x_1^{(0)}(2), x_1^{(0)}(3), \cdots, x_1^{(0)}(n)]^T,$$

$$B = \begin{bmatrix} -\lambda x_1^{(1)}(2) - (1-\lambda)x_1^{(1)}(1) & x_2^{(1)}(2) \\ -\lambda x_1^{(1)}(3) - (1-\lambda)x_1^{(1)}(2) & x_2^{(1)}(3) \\ \vdots & \\ -\lambda x_1^{(1)}(n) - (1-\lambda)x_1^{(1)}(n-1) & x_2^{(1)}(n) \end{bmatrix}.$$

After calibrated $\hat{a}$, the solution to equation (1) with initial condition $x_1^{(1)}(1) = x_1^{(0)}(1)$ is

$$\hat{x}_1^{(1)}(k+1) = \left[x_1^{(0)}(1) - \frac{b}{a}x_2^{(1)}(k+1)\right]e^{-ak} + \frac{b}{a}x_2^{(1)}(k+1). \tag{4}$$

From equation (4), and by the first-order inverse accumulated generating operation of $\hat{x}_1^{(1)}(k+1)$, the modelling value $\hat{x}_1^{(0)}(k+1)$ can be derived to be

$$\begin{aligned} \hat{x}_1^{(0)}(k+1) &= \hat{x}_1^{(1)}(k+1) - \hat{x}_1^{(1)}(k) \\ &= \left[x_1^{(0)}(1) - \frac{b}{a}x_2^{(1)}(k+1)\right]e^{-ak} + \frac{b}{a}x_2^{(1)}(k+1). \end{aligned} \tag{5}$$

With the operation of electricity market, the new data of electricity price continue to emerge. In order to utilize the rich information contained in the new observed values, the new-information grey model is used in this paper. That is, each new obtained value will be added to the tail of the series, at the same time, the first observed value will be removed from the series. The research has shown that new-information grey model have some advantages such as small data sets required, less computational complexity, objective and reliable forecasted results [12].

3. EVT MODEL

There exists strong temporal dependence in the electricity price series due to the specific features of electric power. It violates the underlying assumption that the data series to which EVT is applied should be a sequence of IID random variables. In this paper, a two-stage approach, provided by McNeil and Frey [13], is used to this problem. Firstly, the heteroscedasticities, skewnesses, lepkurtosises and seasonalities of electricity price series are filtered by the GM(1,2) model in section 2 to obtain a nearly IID normalized residual series. In stage two, the EVT framework is applied to the standardized residuals to better capture the heavy-tails and improve the accuracy of VaR estimation.

POT is to model the excess distribution for the IID sample data that exceed a high threshold. Given the distribution function $F_z(z)$ of a random variable Z,the distribution function of values of z above a certain threshold u, $F_u(y)$, is called the conditional excess distribution function and is defined as

$$F_u(y) = \mathrm{Prob}(Z - u \le y \mid Z > u), \forall 0 \le y \le z_F - u, \tag{6}$$

where Z is a random variable, u is a given threshold, y=z-u are the excesses and $z_F \le \times$ is the right endpoint of $F_z(z)$. We verify $F_u(y)$ that can be written in terms of $F_z(z)$, i.e.

$$F_u(y) = \frac{F_z(u+y) - F_z(u)}{1 - F_z(u)} = \frac{F_z(z) - F_z(u)}{1 - F_z(u)}. \tag{7}$$

The theorem of Balkema-De Haan-Pickands states that for large u, the conditional excess distribution function $F_u(y)$ is well approximated by the generalized Pareto distribution (GPD) $G_{\xi,\sigma}(y)$, which is defined as

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi}{\sigma}y\right)^{-1/\xi} & \forall \xi\ ?\ 0 \\ 1 - e^{-y/\sigma} & \forall \xi = 0 \end{cases} \tag{8}$$

for $y \in [0, \infty)$ if $\xi \ge 0$ and $y \in [0, -\sigma/\xi]$ if $\xi < 0$. ξ is the shape parameter or tail index and $\sigma > 0$ is the scaling parameter. In general, we cannot fix an upper bound for financial losses, so only distributions with shape parameter $\xi > 0$ are suited to model fat-tailed distributions.

If T is the total number of observations and T_u the number of observations above the threshold u, the value of $F_z(u)$ can be well approximated by the estimate $(T-T_u)/T$ for sufficiently high u. Replacing $F_u(y)$ by the GPD and $F_z(u)$ by $(T-T_u)/T$, we obtain the estimate of $F_z(z)$ from Equation (7)

$$\hat{F}_z(z) = \begin{cases} 1 - \dfrac{T_u}{T}\left(1 + \dfrac{\xi}{\sigma}(z-u)\right)^{-1/\xi} & \xi\ ?\ 0 \\ 1 - \dfrac{T_u}{T}e^{-(z-u)/\sigma} & \xi = 0 \end{cases} \tag{9}$$

for $z>u$.

A reasonable threshold u must be chosen to effectively estimate the values of parameters ξ and σ. So far, no automatic algorithm with satisfactory performance for choice of threshold u is available. A popular graphical tool for visually selecting u is the sample mean excess plot defined by the points $(u, e_n(u))$. Let $z_{(1)} > z_{(2)} > \ldots > z_{(T)}$ represent the IID order random variables, $e_n(u)$ can be calculated by

$$e_n(u) = \sum_{i=k}^{n}(z_{(i)} - u) \Big/ (n-k+1), \tag{10}$$

where $k = min\{i \mid z_{(i)} > u\}$, n-k+1 is the number of observations exceeding threshold u [14]. If the GPD provides a good description of the data $e_n(u)$ should be approximately linear in u. So we can select the value that locates at the beginning of the sample mean excess plot which is roughly linear as the suitable threshold. If the sample mean excess plot is upward sloping when $z\ ?\ u$, the distribution of the observations exceeding threshold u is the GPD with positive ξ; If the sample mean excess plot is downward sloping when $z\ ?\ u$, the observations exceeding threshold u follows a distribution with short tails; If the sample mean excess plot is horizontal when $z\ ?\ u$, the distribution of the observations exceeding threshold u is the exponential distribution.

Having determined a threshold, the estimates of ξ and σ of the GPD can be obtained by applying maximum likelihood estimation for the excesses of a threshold u. Replacing the values of parameters by their estimates and inverting equation (9) for a given probability c, the estimates of the c-th tail quantile for the sample distribution can be gotten,

$$\hat{F}_z^{-1}(c) = \begin{cases} u + \dfrac{\hat{\sigma}}{\hat{\xi}}\left(\left(Tc/T_u\right)^{-\hat{\xi}} - 1\right) & \hat{\xi} \neq 0 \\ u - \hat{\sigma}\ln\left(Tc/T_u\right) & \hat{\xi} = 0 \end{cases}, \tag{11}$$

which is valid for positive excesses, that is $z > u$.

4. ESTIMATION AND EVALUATION OF VAR

Some characteristics of electricity spot price data naturally lend itself to EVT analysis. For instance, electricity itself is non-storable. As such the equilibrium between supply and demand must be maintained to guarantee a continuous stream of electricity. This leads to an extremely turbulent market where spot prices can rise from average levels to many times this within a very brief period. Large spot price movements expose market participants to significant market risk over short periods of time. In this situation risk managers will be interested in a risk measure like VaR. The strong temporal dependence in the sequence of electricity prices, due to the specific characteristics of electric power, violates the underlying assumption that the data sequence to which EVT models are applied should be a sequence of IID random variables. In this paper, a two-stage approach, provided by A J McNeil and R Frey [13], is used to this problem. Firstly, the dependences, heteroscedasticities, skewnesses, lepkurtosises and seasonalities of electricity price series are filtered by a grey GM(1,2) model to obtain a nearly IID residual series $\{\varepsilon_t\}$. In stage two, the EVT framework is applied to the tails of the nearly IID residuals to better capture the heavy-tails and improve the accuracy of VaR estimation.

4.1. GM(1,2)-POT-VaR Estimating Model

VaR is one of the most intuitive and comprehensible risk measures. It is based on the standard statistical technology and has become an international popular risk measurement technology. Assuming normal market conditions and no trading in a given portfolio, VaR is defined as a threshold value such that the probability that the worst loss on the portfolio over a target horizon exceeds this value is the given level of probability. Mathematically, the VaR of the portfolio with a confidence interval c, VaR_c, is defined as

$$VaR_c = \inf\left\{x \in R \mid \mathrm{Prob}(\Delta P > x) \leq 1 - c\right\}, \tag{12}$$

where Prob(•) denotes the portfolio probability distribution and ΔP the portfolio losses over the given holding period.

For a given time horizon t, suppose that the system demand for electricity is Q_t, the retail price to ultimate customers is P_0, the spot price is $p_t = E(p_t|I_{t-1}) + \varepsilon_t$, I_{t-1} the information set available at time t-1 and ε_t the random shock such that $E(\varepsilon_t)=0$ and $E(\varepsilon_t \varepsilon_s)=0, \forall t \neq s$, where E is the conditional expectation operator. The trading losses of an electric utility over the target horizon t is

$$\Delta P_t = Q_t\left(\mathrm{E}(p_t \mid \mathrm{I}_{t-1}) + \varepsilon_t - P_0\right). \tag{13}$$

As the retail price, P_0, is a regulated price approved by electricity regulatory departments and the electric power demand, Q_t, can be accurately forecasted (generally forecasted error is below 3%), Q_t and P_0 can be regarded as constant. Let $f_\varepsilon(\varepsilon_t \mid I_{t-1})$ denote the conditional probability density function of ε_t conditional on I_{t-1}. The VaR of an electric utility in the specified period t with the pre-assigned probability level c, denoted by $VaR_{c,t}$, is

$$\begin{aligned} 1 - c &= \mathrm{Prob}(\Delta P_t \geq VaR_{c,t}) \\ &= \mathrm{Prob}\left(\varepsilon_t \geq \frac{VaR_{c,t} - Q_t(\mathrm{E}(p_t \mid \mathrm{I}_{t-1}) - P_0)}{Q_t}\right) \\ &= \int_{\frac{VaR_{c,t} - Q_t(\mathrm{E}(p_t \mid \mathrm{I}_{t-1}) - P_0)}{Q_t}}^{\infty} f_\varepsilon(x \mid \mathrm{I}_{t-1})dx \end{aligned} \tag{14}$$

Now inverting equation (14) for the given probability c, we obtain

$$VaR_{c,t} = Q_t\left(\mathrm{E}(p_t \mid \mathrm{I}_{t-1}) - P_0 + F_\varepsilon^{-1}(c \mid \mathrm{I}_{t-1})\right), \tag{15}$$

where $F_\varepsilon(\cdot)$ is the conditional cumulative distribution function of ε_t, F_ε^{-1} is the quantile function defined as the inverse of the distribution function F_ε.

The spot price presents the properties of incomplete and uncertain information. It is in line with the characteristics of grey variables, so we can estimate the expected values of the electricity spot price $\mathrm{E}(p_t|I_{t-1})$ and the c-quantile $F_\varepsilon^{-1}(c \mid \mathrm{I}_{t-1})$ of the residual series ε_t by equations (5) and (11). Then we can calculate the VaR of an electric utility in the specified period t by equation (15).

4.2. Backtesting For VaR Estimates

It is of crucial importance to assess the accuracy of VaR estimates, as they are only useful insofar as they accurately characterize risk. Backtesting or verification testing is the way that we verify whether forecasted losses are in line with actual losses. The most widely known backtesting method based on failure rates has been suggested by Kupiec. Kupiec's test measures whether the number of violation exceptions (losses larger than estimated VaR) is in line with the expected number for the chosen confidence interval. Denoting the number of times that the actual portfolio returns fall outside the estimated values of VaR as N and the total number of observations as T, we may define the number of violation exceptions as:

$$\begin{aligned} N &= \sum_{t=1}^{T} I_t, \\ I_t &= \begin{cases} 1 & if \quad p_t > VaR_{c,t} \\ 0 & if \quad p_t \leq VaR_{c,t}. \end{cases} \end{aligned} \tag{16}$$

Under the null hypothesis that the VaR estimated model is correct at a pre-assigned confidence interval, the number of violation exceptions N should follow a binomial probability distribution

$$P(N|T,\alpha) = \binom{T}{N} \alpha^N (1-\alpha)^{T-N}, \tag{17}$$

where T is the sample size and α corresponding to the significance level chosen for the VaR approach. If the sample size T is input and α is set to one minus the level of confidence, the binomial function produces the likelihood that a specific number of VaR breaks is to occur.

The observed failure rate N/T should act as an unbiased measure of the level of significance α=1-c as sample size is increased. Assuming that the proposed model is accurate, the following likelihood ratio (LR)

$$LR = -2\log\left((1-c)^N c^{T-N}\right) + 2\log\left(\left(\frac{N}{T}\right)^N \left(1-\frac{N}{T}\right)^{T-N}\right) \tag{18}$$

is asymptotically χ^2 (chi-squared) distributed with one degree of freedom. If the value of LR exceeds the critical value of the χ^2 distribution, the null hypothesis will be rejected and the model is deemed as inaccurate. On the contrary, the null hypothesis will be accepted and the model should be considered correct.

5. EMPIRICAL RESULTS

The PJM is organized as a day-ahead market. Participants submit their buying and selling bid curves for each of the next 24 hours. Then the market operator aggregates bids for each hour and determines market clearing prices and volumes for each hour of the following day. In this paper, a total of 1197 observations of average daily electricity spot prices in dollars per megawatt hour ($/MWh) and average daily loads in gigawatt (Gw) are employed to validate the performance of the VaR calculating model. The sample period begins on 1st June 2007 and ends on 9th September 2010. Table **1** presents some descriptive statistics for the average daily electricity spot price and load series. It can be seen from Table **1** that electricity prices and loads are quite volatile, highly non-normal, clearly skewed rightward, and with a median well below the mean. In fact the nulls of normality of electricity price and load series are rejected with the Jarque-Bera test. This is typical of electricity spot prices in a competitive market.

5.1. Estimates of GM(1,2) Model

Taking the significant Weekly Seasonality of the spot price series into account, in order to improve the filtering effects of GM (1,2) and to acquire the approximately IID residuals with better statistical properties, the data window length is set to 7 in this paper. The estimated results of GM(1,2) have been depicted in Fig. **1**. As seen from Fig. **1**, the estimated values of GM(1,2) model and the observed ones are in good conformity, reflecting the basic change rules of electricity prices. But the electricity prices in the peak and valley periods are very unstable due to the impacts of the bidding strategies of generation companies on the market clearing prices, resulting larger estimated errors in these periods.

Table 1. Descriptive statistics of the sample.

Statistics	Price($/MWh)	Load(GW)
Mean	53.52041	81.19221
Median	49.97068	79.89221
Maximum	189.6557	115.7839
Minimum	24.87494	58.34586
Std. Dev.	20.20158	10.50560
Skewness	1.420081	0.375318
lepkurtosis	6.594566	2.582759
Jarque-Bera	1046.748	36.78506
p-value	0.000000	0.000000

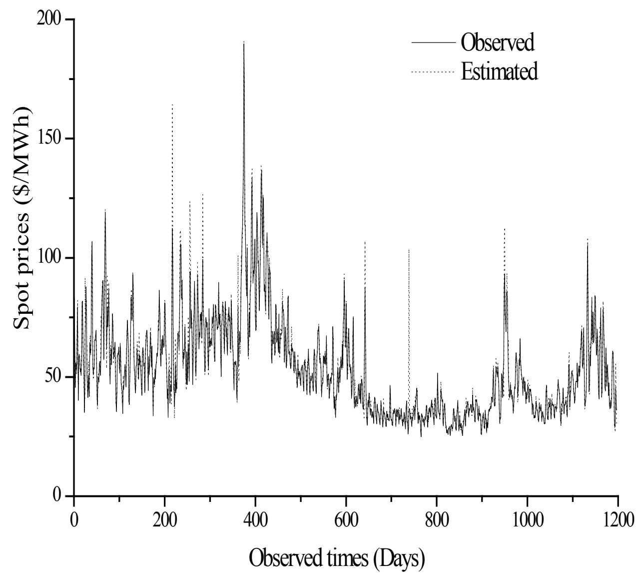

Fig. (1). Estimated results of GM(1,2) model.

Table **2** illustrates the Ljung–Box Q statistics and the corresponding probability values (p-Values) for the residuals and their square sequences. It is seen from Table **2**, the Ljung–Box Q statistics of the square series are not significant at up to 24 lags, suggesting that no potential time-varying volatility exists in the residual series; the Ljung–Box Q statistics at 7 or 24 lags for the residual series are far less than the daily average electricity spot price series, indicting that there are still weak serial dependences, so we can conclude that the residual series is a stationary series with weakly serial correlation and without volatility clustering, meeting the prerequisite of EVT modelling [15].

Table 2. Ljung-box test for residuals of GM(1,2).

Statistics	Electricity Prices ($/MWh)	Residuals ($/MWh)
Ljung-Box Q(6)	3868.28(0)	28.87(0.000)
Ljung-Box Q(24)	11348.94(0)	51.21(0.001)
Ljung-Box $Q^2(6)$	3117.13(0)	1.37(0.968)
Ljung-Box $Q^2(24)$	7892.50(0)	3.62(0.999)

5.2. Estimates of GM(1,2)-POT-VaR Model

To apply EVT, the threshold can be selected by the mean excess function or Hill plots. We use the mean excess function to calculate the threshold. Fig. **2** shows the sample mean excess function for the residuals of the grey GM(1,2) model. From a closer inspection of the plot, we find that the sample mean excess plot *e(x)* is roughly linear when the value of the threshold u is about 3.837. So we fix the threshold u to 3.837. In this case, the number of resulting excesses are 119, accounting for 9.94% of the sample, which is consistent with percentages suggested by McNeil and Frey [13].

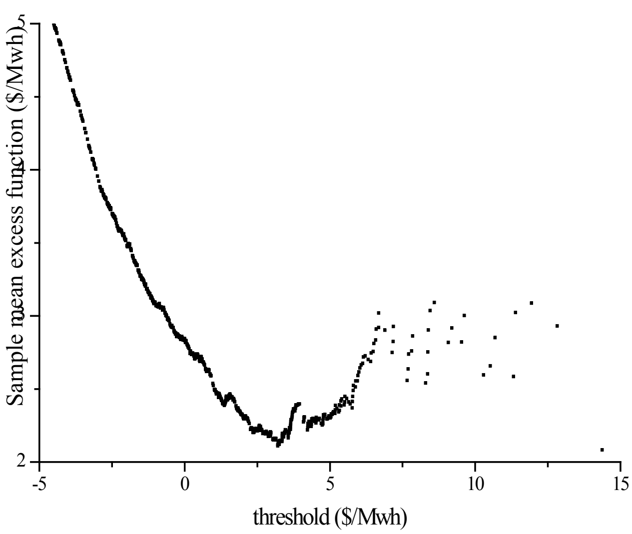

Fig. (2). Mean excess function plots of residuals.

After selecting the threshold u, the residuals above the selected threshold u, which will be used as the sample data for EVT implementation, are also determined. The estimates of the shape and scale parameters, ξ and σ, can be determined by fitting the GPD to the residuals via maximum likelihood estimator. Inserting the estimates of ξ and σ into equation (11), the tail quantiles of the standardized residual series at a given confidence level c can be calculated. Table **3** reports the estimated results for tail index, scale parameter and tail quantiles. It can be seen that the ξ estimates is positive and statistically significant, indicating that the right tail of the distribution of standardized residuals is characterized by the Fréchet distribution.

Table 3. Estimates of GPD parameters & quantiles.

Threshold	Shape	Scale	Confidence Level	Tail Quantile
3.837	0.023371	2.333667	95.0%	5.453875
			97.5%	7.111029
			99.0%	9.343275
			99.5%	11.06395

5.3. VaR Estimates and Backtesting

Without loss of generality, in this paper we assume that an electric utility has the obligation to serve 1MW of load 24 hours a day and the retail price has been frozen at a level equivalent to 0$/MWh. Substituting the calculated results at subsection 5.1and 5.2 into equation (15), the VaR at each confidence level can be estimated. Fig. **3** shows the estimated results of the dynamic VaR at the 99% confidence level. It can be seen from Fig. **3**, the estimates of GM(1,2)-POT-VaR model can reflect the changes of electricity prices sensitively, showing better dynamic characteristics.

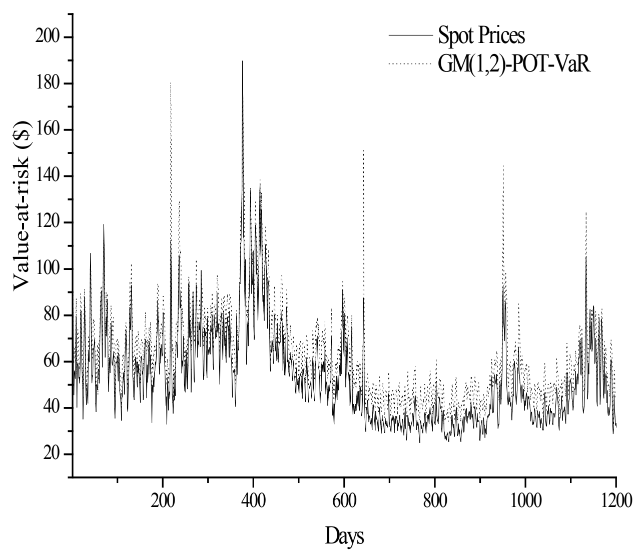

Fig. (3). Dynamic VaR for electric power company.

The Kupiec's test results for actual and forecasted losses are shown in Table **4**. It can be seen from Table 4 that the null hypotheses of ARMAX-GARCH-st-VaR [10], ARMAX-GARCHSK-VaR [11] and our proposed GM(1,2)-POT-VaR models cannot be rejected in each significance levels. Summarizing the results for the Kupiec's tests, the VaR predictions by these methods are insignificantly different from the proposed downfall probability, but because the GM(1,2)-POT-VaR model is easier to deal with and possesses the advantages of less computational costs, this further improves the risk management ability for electricity market participants to some extent.

Table 4. Backtests of estimated VaRs.

Confidence Level	Statistics	ARMAX-GARCHSK	ARMAX-GARCH-st	GM(1,2)-POT-VaR
95%	Expected	60	60	60
	Real	59	60	59
	LR	0.013	0.000	0.013
97.5%	Expected	30	30	30
	Real	28	30	28
	LR	0.130	0.000	0.130
99%	Expected	12	12	12
	Real	13	13	12
	LR	0.087	0.087	0.000
99.5%	Expected	6	6	6
	Real	7	6	7
	LR	0.164	0.000	0.164

CONCLUSION

The distinctive characteristics of electric energy make electricity price present highly volatility and occasional extreme movements of magnitudes rarely seen in markets for regular financial assets, thus volatility of price risk identification, evaluation and management in electricity market are more important than in financial markets. Considering the pertinences of electricity prices, a grey system and extreme value theory based two-stage model for estimating VaR is proposed. In stage one, to acquire the approximately IID residuals with better statistical properties, a grey GM(1,2) model is used to pre-filter the electricity price series. In stage two, an EVT based model is employed to explicitly deal with the right tail of the residuals of the GM(1,2)l, and accurate estimates of VaR in electricity market can be produced. The empirical analysis indicates that the GM(1,2)-POT-VaR model can rapidly reflect the most recent and relevant changes of electricity prices and can produce accurate forecasts of VaR at all significance levels. The computational costs is far less than the methods in [10, 11], further improving the risk management ability of market participants. These results present several potential implications for risk hedging strategies in electricity market.

CONFLICT OF INTEREST

The author confirms that this article content has no conflicts of interest.

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