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Searching for Mobile Intruders in Circular Corridors by Three 1-Searchers

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Abstract: We consider the problem of searching for a mobile intruder in a circular corridor by three mobile searchers, who hold one flashlight, a variation of the 1-searcher problem in a circular corridor. A circular corridor is a polygon with one polygonal hole such that its outer and inner boundaries are mutually weakly visible. The 1-searcher has a flashlight and can see only along the ray of the flashlight emanating from his position. In the searching process, each 1-searcher can move on the boundary or into the circular corridor, the beam of his flashlight must be irradiated on the inner boundary. The previous research of this paper suggests an algorithm which decides whether a given circular corridor is searchable by two 1-searchers or not. This paper proves that three 1-searchers always clear a given circular corridor. And a search schedule can be reported in O(m) time, where $m \le n^2$ denotes the walk instructions reported, and n denotes the total number of vertices of the outer and inner boundaries.

Keywords: 1-searcher, circular corridor search problem, computational geometry, deadlocks, visibility.

1. INTRODUCTION

Consider the following scenario. There are two roles people in a dark channel, one is extremely dangerous evader, called as the intruder, and the other is responsible for ensuring the safety of channel, called as the 1-searcher. In this paper, the channel is abstracted as a polygon with one polygonal hole such that its outer and inner boundaries are mutually weakly visible, and called as a circular corridor. Also, searching for the mobile intruders in the circular corridor is called as circular corridor search problem, in which the intruder can move arbitrarily fast, and its location is unknown for the 1-searcher who has a flashlight. In the searching process, each 1-searcher can move on the boundary or into the corridor, and the beam of the flashlight he holds must be irradiated on the inner boundary. The task of the 1-searcher is to search the entire channel to capture the intruder, and to ensure the safety of the channel. If the intruder is irradiated by the beam of the flashlight, it is considered to be captured. Clearly, one 1-searcher is impossible to clear the circular corridor completely. To clear the whole circular corridor and capture the intruder, at least two or more 1-searchers are needed. The main content of this paper consists of two aspects: i), for a given circular corridor such that its outer and inner boundaries are mutually weakly visible, we prove that it could be cleared by no more than three 1-searchers. ii), a search schedule can be reported in time linear in its size.

The polygon search problem was first introduced by Suzuki and Yamashita [1], in which they considered a single pursuer looking for a intruder inside a given simple polygon. They defined different kinds of searchers (pursuers) depending on the number of flashlights that they held, *e.g.*, a k-searcher has k flashlights($k \ge 1$), and an ∞-searcher has 360°-vision. This naturally defines a polygon search problem for each class of searchers.

Icking and Klein were the first to study the two-guard problem, they defined the "two-guard walkability problem" [2], which is a search problem for two guards whose starting and goal position are given, and who walk only on the boundary of a polygon to force the intruder out of the corridor through the goal position finally, while maintaining mutual visibility. (Note that the two-guard problem is a slightly different type of 1-searchers problem). They gave an $O(n\log n)$ time algorithm for determining whether a corridor(a simple polygon with an *entrance* and an *exit* on its boundary) can be swept by two guards, where n denotes the number of vertices of the corridor. This result has also been improved to $\Theta(n)$ by Heffernan [3].

Guibas *et al.* extended the polygon search problem, in which multiple searchers were collaborated in order to clear a given polygonal region [4]. They showed that determining the minimal number of searchers needed to clear a polygonal region with holes allowed is an NP-hard problem. It is not known whether the same problem for given simple polygons in which without any holes is also NP-hard.

Tseng *et al.* studied the two-guard problem in which the *entrance* and *exit* are not given, and gave an $O(n \log n)$ time algorithm to determine whether there is a pair of vertices for supporting a search schedule in a given polygon that allows a sweep [5]. This result has also been improved to $O(n)$ [6].

Park *et al.* gave a polynomial solution for the case of a single 2-searcher and proved that adding more flashlights to a single searcher does not increase the class of the polygons

[7]. Note that the set of polygons searchable by a single 2-searcher is a proper subset of the set of polygons searchable by two 1-searchers.

LaValle *et al.* studied the problem of searching a simple polygon (without holes) by a 1-searcher(a single mobile searcher), and gave an $O(n^2)$ time algorithm [8]. Later, they presented an $O(n^2+nm^2+m^4)$ time algorithm for searching a simple polygon without any holes by two 1-searchers, where n is the number of vertices of the polygon and m is the number of concave regions [9]. Note that m has a lower bound $\Omega(n)$ in the worst case.

Jiang and Tan first focused on searching the circular corridors by two 1-searchers and did some research work [10]. The searchability of a given circular corridor by two 1-searchers can be determined in $O(n\log n)$ time, and a search schedule can be reported in $O(m)$ time, where m denotes walk instructions.

In this paper, we study the variation of the 1-searcher problem in a circular corridor which can not be cleared by two 1-searchers, and draw a conclusion that a given circular corridor can be cleared by three 1-searchers.

2. PRELIMINARIES

2.1. Notation

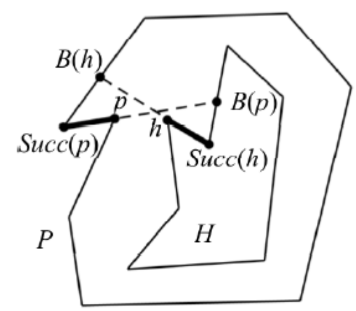
A polygon is called as a simple polygon if it contains no holes nor self-intersections. A polygon, such as inside it contains another simple polygonal hole, we call the circular corridor, denote it by *CC*. In this paper, we study the circular corridors such that its outer and inner boundaries are disjoint and mutually weakly visible. In the rest of this paper, we denote the outer boundary and the inner boundary of a given *CC* by *P* and *H* respectively. For the convenience of narration, we denote by $P[v, u]$ the clockwise closed chain of *P* from point v to point u, and $P(v, u)$ the open chain of *P* from v to u, and the points on *P* will be denoted by p, p', p_1, etc. Note that we can do the same definition for *H* but the points on *H* is denoted by h, h', h_1, *etc*.

As the definition above, for point $p \in P$ and point $h \in H$, we say that they are mutually visible if the line segment $\overline{ph}$ lies in the interior of *CC* completely, that is to say $\overline{ph}$ does not intersect with any boundary of *P* or *H*, except for two boundary endpoints p and h. For two polygonal chains $P[p, p_1]$ and $H[h, h_1]$, we say that $P[p, p_1]$ is weakly visible from $H[h, h_1]$ if each point of $P[p, p_1]$ is visible from some point of $H[h, h_1]$, and vice versa. Just for the sake of convenience, we assume that *CC* is in a general case, no three vertices are collinear and no three lines extending three edges of *CC* have a common point.

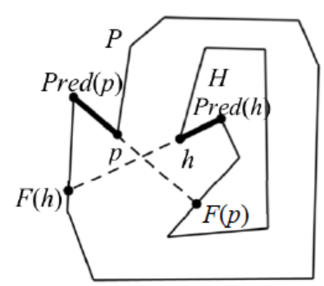
A vertex of *CC* is reflex if its interior angle is strictly larger than 180°. Otherwise, it is convex. Note that we consider the region between the boundary of *P* and the boundary of *H* as the interior of *CC*. An important definition for reflex vertices is that of ray shots. In order to definite a ray shot, we need to introduce the concepts of successive vertex and precursor vertex. For a vertex x of *CC*, the vertex of *CC* immediately preceding (succeeding) x on the boundary *P* or *H* in clockwise(the vertex order is agreed in clockwise direction) is called the precursor (successive) vertex of x, denoted by $Pred(x)$ ($Succ(x)$). Now, we define the ray shots as follows: the backward ray shot from a reflex vertex r of $P(H)$, denoted by $B(r)$, is the first point of $H(P)$, if it exists, hit by a "bullet" shot at r in the direction from $Succ(r)$ to r, and the forward ray shot $F(r)$ is the first point of $P(H)$ hit by a "bullet" shot at r in the direction from $Pred(r)$ to r. We call the vertex r as the origin of the shots $B(r)$ and $F(r)$ (See Fig. (**1**)). Note that we do not consider the ray shots which are on the same boundary as their origins, since *P* and *H* are mutually weakly visible.

In the following discussion, we assume that $B(r)$ ($F(r)$) is slightly above (below) the backward ray (forward ray) shot from r, as viewed from r, and thus $B(r)$ ($F(r)$) and $Succ(r)$ ($Pred(r)$) are mutually visible.

A backward deadlock is formed by a pair of vertices $p \in P$, $h \in H$, if both the three points p, $Succ(p)$, $B(h)$ on *P* and the three points h, $Succ(h)$, $B(p)$ on *H* are in clockwise order (see Fig.(**1**)(**a**)). Also a forward deadlock is formed by a pair of vertices $p \in P$, $h \in H$, if both the three points $F(h) \in P$, $Pred(p)$, p on *P* and the three points $F(p) \in H$, $Pred(h)$, h on *H* are in clockwise order (Fig.(**1**)(**b**)). Two vertices p and h are called the defining vertices of the deadlock. Two edges $\overline{pSucc(p)}$, $\overline{hSucc(h)}$ in Fig. (**1**)(**a**), and two edges $\overline{Pred(p)p}$, $\overline{Pred(h)h}$ in Fig. (**1**)(**b**) are called the defining edges of the deadlock. Clearly, when a backward (forward) deadlock is formed, the two backward (forward) rays are intersected.



(a) Backward deadlocks.



(b) Forward deadlocks.

Fig. (1). Illustration of ray shots and deadlocks.

2.2. Problem Definition

Let $S(t) \in CC$ (the 1-searcher can move into the inner region of *CC*) and $F(t) \in H$ denote the position of the 1-searcher *S* and his lightpoint *F* (*i.e.*, the point of the inner boundary illuminated by the flashlight) at a time $t \geq 0$. A point $x \in CC$ is said to be detected or illuminated at time t if x is on the line segment $\overline{S(t)F(t)}$. We consider three 1-

searchers and their illuminated points in this paper, denoted by S_a , S_b , S_c , F_a , F_b , F_c , respectively.

A region is considered as contaminated , if it might contain the intruder (whose position is unknown to the 1-searchers as he is capable of moving arbitrarily fast) at a time; otherwise, it is clear. If a region becomes contaminated again, it is referred to as recontaminated. A search schedule of three 1-searchers for *CC* is defined by three sequences of piecewise-continuous functions $\{S_a , S_b , S_c\}$: [0, 1]→*CC* and $\{F_a , F_b , F_c\}$:[0, 1]→*H*, such that the intruder is contained in $\overline{S_x(t)F_x(t)}$, *x* is *a*, *b* or *c*, no matter how fast the intruder moves.

Suppose that all trivial movements are removed from the considered search schedule, we can define the movements or search instructions of a 1-searcher as follows:

(i) During the movement, the 1-searcher *S* and his lightpoint *F* move along single edges of *P* and *H* in the same direction (*e.g.*, in clockwise), respectively, such that no intersections occur among all line segments $\overline{S(t)F(t)}$.

(ii) During the movement, the 1-searcher *S* and his lightpoint *F* move along single edges of *P* and *H* in the opposite direction, respectively, such that any two segments of $\overline{S(t)F(t)}$ intersect each other.

(iii) The lightpoint *F* jumps from a reflex vertex *x*∈*H* to another point *y*∈*H* or from the point *y* to the vertex *x*.

(iv) The 1-searcher *S* (e.g., S_a in Fig. (**2**)(iv)) moves in the interior of *CC*, while aiming his flashlight at a point of *H*.

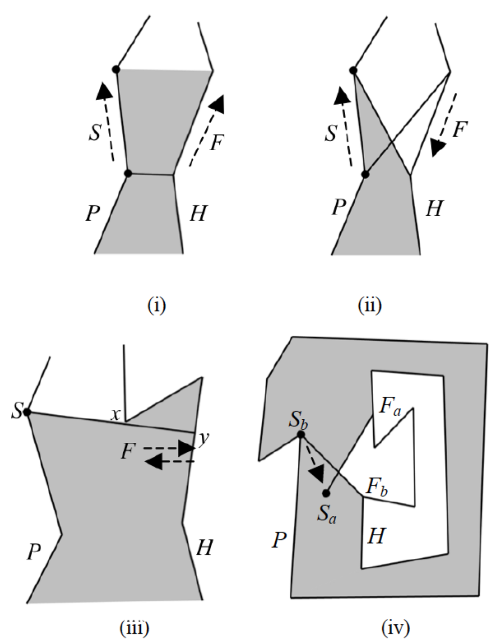


Fig. (2). Search instructions of a 1-searcher.

For the instructions of type (i) to (iv), some examples are shown in Fig. (**2**) in which the shaded region denotes the clear region and the dotted arrows give the directions of the 1-searcher or his lightpoint moves. For any instructions of type (i) or (ii), the 1-searcher *S* and his lightpoint *F* are continuous on *P* and *H*, respectively (see Fig. (**2**)(i) and (ii)). It is worth pointing out that an instruction (ii) can simply be performed by rotating the line segment connecting *S* and *F* around the intersection point of the starting and ending segments. The instruction of type (iii) gives the only possible discontinuities of the lightpoint *F* on *H*, it is usually termed as the lightpoint jumps. Finally, the instruction of type (iv) means that the 1-searcher can move into inner of *CC*. Note that instructions of type (iv) defined here is differ from the definition in [10]. In the case of two 1-searchers, the 1-searcher must be limited to the clear region to prevent recontamination occurs (Fig. (**2**)(iv)), but in the case of tree 1-searchers, one of the 1-searchers can move into the contaminated region since other 1-searchers are limited to the clear region.

In [10], Jiang and Tan defined two different time periods of the "start phase" and the "end phase", in which the two 1-searchers cooperatively clear a group of deadlocks (if it exists). More precisely, two 1-searchers start at the same position, and then cooperatively clear a group of deadlocks in the start phase. Analogously, after clearing a group of deadlocks, they finish the search at the same position in the end phase. Suppose that at least one deadlock is cleared by two 1-searchers in the start phase, and the defining vertices of the deadlock are denoted by p_0 and h_0 (See Figs. (**3**) and (**4**)). Let all points on *P* (*H*) are ordered clockwise with respect to $p_0 \in P$ ($h_0 \in H$), then the inequality $v < u$ implies that the point *v* is encountered before *u* by a clockwise searcher on *P* (*H*), starting at p_0 (h_0).

For a given *CC*, by introducing the concept of "non- separated" deadlocks, we know that a pair of non-separated deadlocks cannot be cleared in the end phase by two 1-searchers [10]. However, it can be cleared by three 1-searchers in any phases, even if the non-separated deadlocks exist. As an important concept, we need to define *BF*-deadlocks, *FB*-deadlocks, *BB*-deadlocks and *FF*- deadlocks as follows, and thus the a non-separated deadlock can be defined as well.

Definition 1. Suppose that there are two pairs of the vertices $<p_1, h_1>$ and $<p_2, h_2>$ in the contaminated region, $p_1 < p_2$ and $h_1 < h_2$ hold, thus, $<p_1, h_1>$ and $<p_2, h_2>$ form a pair of *BF*-deadlocks if $<p_1, h_1>$ causes a backward deadlock and $<p_2, h_2>$ causes a forward deadlock (Fig. (**3**) (**a**)).

Definition 2. Suppose that $<p_1, h_1>$ causes a forward deadlock, and $<p_2, h_2>$ causes a backward deadlock, $p_1 < p_2$ and $h_1 < h_2$ hold, thus, $<p_1, h_1>$ and $<p_2, h_2>$ form a pair of *FB*-deadlocks if either of the following conditions is true: The first one is that there are no points $x \in H[h_1, h_2]$ such that $H[h_1, x]$ and $H[x, h_2]$ are weakly visible from $P[p_0, p_1)$ and $P(p_2, p_0]$, respectively, and nor points $y \in P[p_1, p_2]$ such that $P[p_1, y]$ and $P[y, p_2]$ are weakly visible from $H[h_0, h_1)$ and $H(h_2, h_0]$, respectively (Fig. (**3**)(**b**)). The second one is that there exists no internal segment $\overline{ph}$, $p \in P(p_2, p_1)$ and $h \in H(h_2, h_1)$, which has two edges $\overline{p_1Pred(p_1)}$, $\overline{p_2Succr(p_2)}$ to its one side and two edges $\overline{h_1Pred(h_1)}$, $\overline{h_2Succr(h_2)}$ to its another side in the contaminated region (Fig. (**3**)(**c**)).

Definition 3. Suppose that $<p_1, h_1>$ and $<p_2, h_2>$ cause a backward deadlock, respectively, $p_1 < p_2$ and $h_1 < h_2$ hold, thus, $<p_1, h_1>$ and $<p_2, h_2>$ form a pair of *BB*-deadlocks if both the following conditions are true: The first one is that the chain $H[h_1, h_2]$ is not weakly visible from $P(p_2, p_0)$, or $P[F(v), p(h_1)]$ is not weakly visible from $H[h_1, v]$ when $F(v) < p(h_1)$ holds. The second one is that the chain $P[p_1, p_2]$ is not weakly visible from $H(h_2, h_0)$ (Fig. (**4**)).

Definition 4. Suppose that $<p_1, h_1>$ and $<p_2, h_2>$ cause a forward deadlock, respectively, $p_1 < p_2$ and $h_1 < h_2$ hold, thus, $<p_1, h_1>$ and $<p_2, h_2>$ form a pair of *FF*-deadlocks if both the following conditions are true: The first one is that the chain $H[h_1, h_2]$ is not weakly visible from $P(p_0, p_1)$, or $P[p(h_2), B(u)]$ is not weakly visible from $H[u, h_2]$ when $p(h_2) < B(u)$ holds. The second one is that the chain $P[p_1, p_2]$ is not weakly visible from $H(h_0, h_1)$.

Definition 5. If the deadlocks caused by <p1, h1> and <p2, h2> form a pair of BF-deadlocks, *FB*-deadlocks, *BB*-deadlocks, or *FF*-deadlocks in the end phase, then we call them as the non-separated deadlocks.

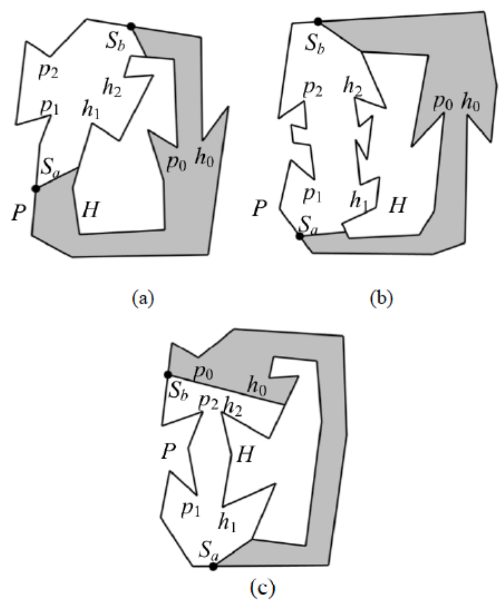


Fig. (3). (a) The pairs of *BF*-deadlocks; (b)-(c) the pairs of *FB*-deadlocks.

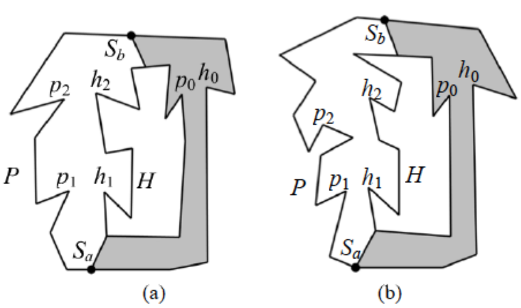


Fig. (4). The pairs of *BB*-deadlocks.

Lemma 1. (See [10]). A given *CC* is not searchable by two 1-searchers if (i) all deadlocks in *CC* cannot be split by any internal line segment $\overline{ph}$ and (ii) there are three deadlocks in *CC* such that any two of them are non-separated.

The proof of Lemma 1 is given in [10]. Here, we give some examples only which are not searchable, shown in Figs. (**3**)-(**5**).

In the rest of this paper, we usually denote by S_a, S_c the 1-searchers whose flashlight move in *CC* counterclockwise and S_b the 1-searcher whose flashlight moves in *CC* clockwise. If the given *CC* is not searchable by two 1-searchers, we need the third 1-searcher to clear the whole *CC*. Without loss of generality, assume that three 1-searchers always start(end) at the same position. At the beginning, we take S_a and S_c together as one 1-searcher with S_b to search *CC* just like two 1-searchers in [10]. When they encounter the non-separated deadlocks, let S_b stop moving, then S_a and S_c begin to separate and mutually cooperate to clear the rest of *CC*, and end at the position of S_b finally. Fig. (**5**) illustrates an example for clearing a circular corridor by three 1-searchers. The starting positions of three 1-searchers are shown in Fig. (**5**)(**a**). Before they do not encounter the non-separated deadlocks, S_a and S_c always move together, S_c does not play his role. S_a and S_b mutually cooperate to clear the deadlock formed by $<p_0, h_0>$, just like two 1-searchers (Fig. (**5**)(**b**)). When they encounter the non-separated deadlocks caused by $<p_1, h_1>$ and $<p_3, h_3>$, S_b stops moving, S_a and S_c begin to separate (Fig. (**5**)(**c**)). S_a and S_c mutually cooperate to clear the deadlock formed by $<p_1, h_1>$, then end at the same position and continue to search (Fig. (**5**)(**d**)). S_a and S_c clear the deadlock caused by $<p_2, h_2>$ (Fig. (**5**)(**e**)). S_a and S_c clear the deadlock caused by $<p_3, h_3>$. Finally, end at the position of S_b (Fig. (**5**)(**f**)). The search is completed.

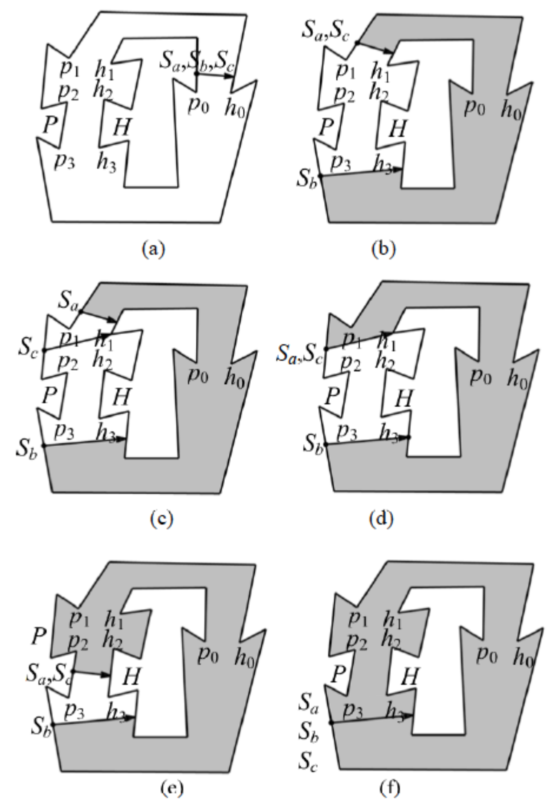


Fig. (5). Snapshots of a search schedule of three 1-searchers.

3. ALGORITHM

In this section, we prove that for a given *CC*, if the non-separated deadlocks are in start or end phase, three 1-searchers can also clear it. In other words, no matter how fast the intruder moves, the given *CC* could be searchable by no more than three 1-searchers. At the beginning, two 1-searchers move together counterclockwise and the third moves clockwise along the outer boundary. When they encounter the non-separated deadlocks, the former 1-searchers begin to separate and "mutually cooperate" to clear the deadlock, then end at the same position and continue to search. Finally, three 1-searchers meet at the same position and the search is completed. In the process of "mutually cooperating", one of two 1-searchers moves first, separate deadlocks by its flashlight and the other moves to clear the region blocked by the flashlight.

Lemma 2. (See [2, 3]). A corridor (Q, u, v) is searchable by two guards if and only if the chains L and R are mutually weakly visible and no deadlocks occur between L and R, where Q is a simple polygon, u and v are the *entrance* and *exit* on its boundary. It takes $\Theta(n)$ time to determine the walkability of a corridor, and $O(n \log n + m)$ time to output a search schedule of minimum length, where $m(\leq n^2)$ denotes the number of the instructions reported.

Without loss of generality, from lemma 3 to lemma 8, we assume that at least one deadlock caused by $<p_0, h_0>$ is cleared in the start phase, and then the three 1-searchers encounter the non-separated deadlocks. If they do not encounter the non-separated deadlocks, only two 1-searchers can clear the *CC*. From this time, S_b stops moving, S_a and S_c begin to separate and mutually cooperate to clear all the deadlocks they encountered. For ease of presentation, we also assume that all points on $P(H)$ are ordered clockwise, with respect to $p_0 \in P$ ($h_0 \in H$). So, the inequality $v<u$ implies that the point v is encountered before u by a clockwise walker on $P(H)$, starting at $p_0(h_0)$.

In this paper, there are some appoints as follows:

① Before clearing a deadlock, the 1-searcher S_a and S_c are at the same position. It is easy to be done, since they move together while without finding any deadlocks, and move to the same position after clearing a deadlock.

② The backward deadlock found by S_a and S_c is appointed that the defining vertices, denoted by p_1 and h_1, are mutually visible (see Fig.(**6**) for an example where p_1 and h_1 are mutually visible, but p' and h' are not visible). There are some backward deadlocks caused by such as $<p', h'>$, $<p', h_1>$, $<p_1, h'>$, but p' and h', p' and h_1, p_1 and h' are all not mutually visible, so we consider they are not backward deadlocks, p' and h' are not defining vertices. Since they can be easily cleared as well during S_a and S_c mutually cooperating to clear the backward deadlock caused by $<p_1, h_1>$ following lemma 7 and 8.

③ To facilitate processing, we take a concave vertex which is in the contaminated region and S_a (F_a) as a pair of defining vertices of a backward deadlock, if the concave vertex exists, and is not defining vertex of any deadlocks (see Fig. (**7**)(**a**) or Fig. (**7**)(**b**)). This case is caused by the following process. After clearing a forward deadlock, say, caused by $<p_1, h_1>$, S_a and S_c are at the same position, and they find a concave vertex which does not form a deadlock in the contaminated region (the vertex h_2 shown in Fig. (**7**)(**a**), or the vertex p_2 shown in Fig. (**7**)(**b**)). When both the three points $Pred(p_1)$, p_1, $B(h_2)$ on P and the three points $F(p_1)$, h_2, $Succ(h_2)$ on H are in clockwise order (both the three points $F(h_1)$, p_2, $Succ(p_2)$ on P and the three points $Pred(h_1)$, h_1, $B(p_2)$ on H are in clockwise order), we can consider the h_2 (p_2) and S_a (F_a) as a pair of defining vertices of a backward deadlock; Otherwise, a deadlock formed with $h_2(p_2)$ as its defining vertex.

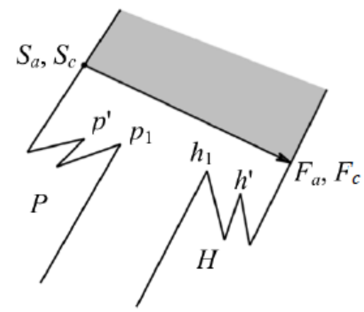


Fig. (6). p' is not visible to h'.

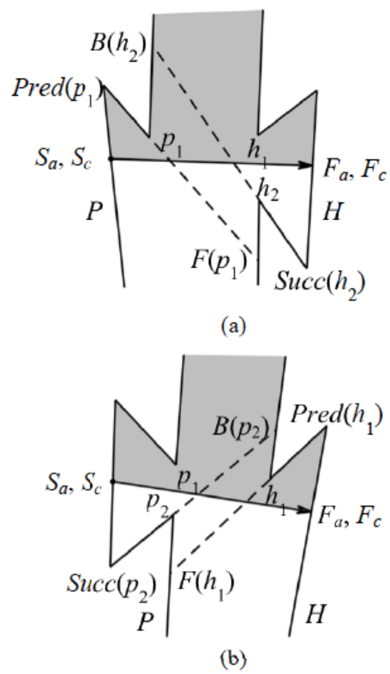


Fig. (7). Consider $h_2(p_2)$ as a defining vertex of a backward deadlock.

Now we show a search schedule for clearing a given circular corridor by three 1-searchers as follows.

Lemma 3. The first deadlock found by S_a and S_c is a backward deadlock, say, caused by $<p_1, h_1>$, $p_1 \in P$, $h_1 \in H$. If the following two conditions are satisfied, then S_a and S_c can mutually cooperate to clear this backward deadlock, and then S_a and S_c can meet at the same position.

① $H[h_1, Succ(h_1)]$ is in the cleared region, and $P[p_1, Succ(p_1)]$ is in the contaminated region.

② There are no defining edges on $P[p_1, S_a]$ except $\overline{p_1 Succ(p_1)}$.

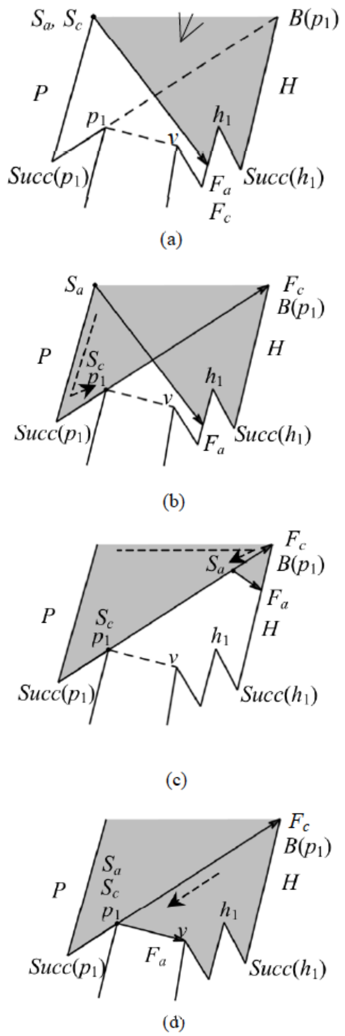


Fig. (8). Illustration for the proof of lemma 3.

Proof. Since <p_1, h_1> caused a backward deadlock, so the line segments $\overline{p_1B(p_1)}$ and $\overline{S_aF_a}$ must be intersected (Fig. (**8**)(**a**)). There are no defining edges on $P[p_1, S_a]$ except $\overline{p_1Succ(p_1)}$. Let S_a stop moving, S_c searching forward along the outer boundary arrives at p_1, and aims his flashlight at $B(p_1)$ to clear $P[p_1,S_a]$ using the instructions of type (i), (ii) and (iii) described as above (Fig. (**8**)(**b**)). Suppose that the vertex $v \in H$ is the first point visible to p_1 on the inner boundary from F_a in counterclockwise. Then S_a moves into *CC* to arrive at $B(p_1)$ (Fig. (**8**)(**c**)), and moves along $\overline{p_1B(p_1)}$ to arrive at p_1, and aims his flashlight at v to clear $H[v,B(p_1)]$ (Fig. (**8**)(**d**)). In this process, the instruction (iv) is used. Finally S_c moves to the position of S_a, since $H[v, h_1]$ is cleared. This completes the proof.

If there are defining edges on $P[p_1, S_a]$ except $\overline{p_1Succ(p_1)}$, the backward deadlock caused by <p_1, h_1> is not the first deadlock they found.

Lemma 4. The first deadlock found by S_a and S_c is a backward deadlock, say, caused by <p_1, h_1>, $p_1 \in P$, $h_1 \in H$. If the following two conditions are satisfied, then S_a and S_c can mutually cooperate to clear this backward deadlock, and then S_a and S_c can meet at the same position.

① $P[p_1, Succ(p_1)]$ is in the cleared region, and $H[h_1, Succ(h_1)]$ is in the contaminated region.

② There are no defining edges on $H[h_1, F_a]$ except $\overline{h_1Succ(h_1)}$.

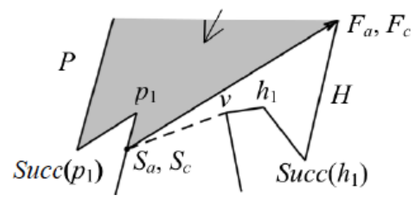


Fig. (9). Illustration for the proof of lemma 4.

Proof. The proof for Lemma 4 is similar to Lemma 3. Fig. (**9**).

If there are defining edges on $H[h_1, F_a]$ except $\overline{h_1Succ(h_1)}$, the backward deadlock caused by <p_1, h_1> is not the first deadlock they found.

Lemma 5. The first deadlock found by S_a and S_c is a forward deadlock, say, caused by <p_1, h_1>, $p_1 \in P$, $h_1 \in H$. If the following two conditions are satisfied, then S_a and S_c can mutually cooperate to clear this forward deadlock, and then S_a and S_c can meet at the same position.

① $P[Pred(p_1), p_1]$ is in the cleared region, and $H[Pred(h_1), h_1]$ is in the contaminated region.

② There are no complete deadlocks between <S_a, F_a> and <$F(h_1)$, h_1>.

Note that the complete deadlock means that one defining vertex is on $P[F(h_1), S_a]$ and the other defining vertex is on $H[h_1, F_a]$.

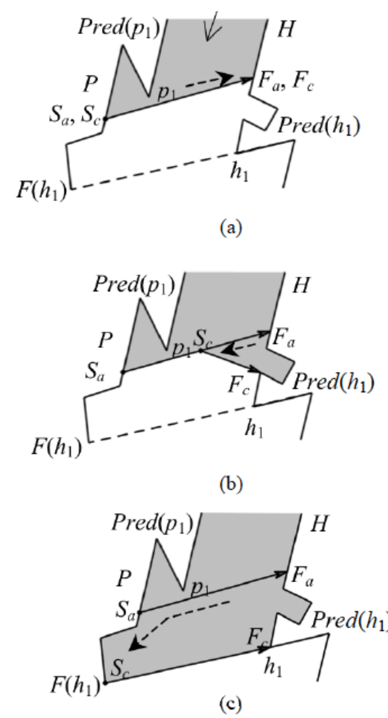


Fig. (10). Illustration for the proof of lemma 5.

Proof. Since there are no complete deadlocks between <S_a, F_a> and <$F(h_1)$, h_1>, the simple polygon P_1 composed of $P[F(h_1), S_a]$, $\overline{S_aF_a}$, $H[h_1, F_a]$ and $\overline{h_1F(h_1)}$ is a *LR*-visibility polygon with respect to F_a and $F(h_1)$, and no deadlocks formed in P_1. In this case, let S_a stop moving, S_c moves into *CC* to arrive at F_a and aims his flashlight at F_a (Fig. (**10**)(**a**)-(**b**)). Next, S_c moves along $\overline{S_aF_a}$ and the outer boundary from S_a to $F(h_1)$ and ends at $F(h_1)$ to clear P_1 following lemma 1 (Fig. (**10**)(**c**)). Finally, S_c aims his flashlight at h_1 and S_a moves to the position of S_c to clear the forward deadlock caused by <p_1, h_1>. This completes the proof.□

If there are complete deadlocks between <S_a, F_a> and <$F(h_1)$, h_1>, the forward deadlock caused by <p_1, h_1> is not the first deadlock they found.

Lemma 6. The first deadlock found by S_a and S_c is a forward deadlock, say, caused by <p_1, h_1>, $p_1 \in P$, $h_1 \in H$, and both p_1 and h_1 are in the contaminated region. S_a and S_c can mutually cooperate to clear the defining edge $\overline{Pred(p_1)p_1}$, and then S_a and S_c can meet at the same position.

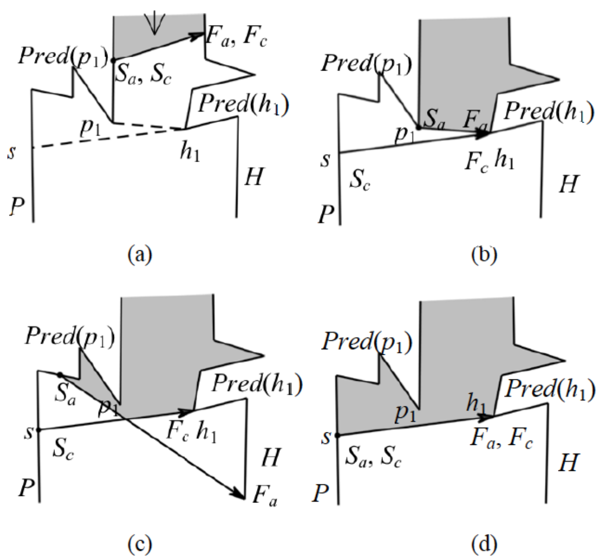


Fig. (11). Illustration for the proof of lemma 6.

Proof. Since there are no complete deadlocks between <S_a, F_a> and <p_1, h_1>, S_a and S_c can arrive at p_1 and aim their flashlights at h_1 to clear $P[p_1,S_a]$ and $H[h_1,F_a]$ following lemma 2. Suppose $s \in P$ is the first point visible to h_1 on the outer boundary from p_1 in counterclockwise, let S_a stop moving, S_c moves into *CC* to arrive at *s*, and aims his flashlight at h_1 (Fig. (**11**)(**a**)-(**b**)). At this time, $P[Pred(p_1), p_1]$ is still in the contaminated region. Next, S_a moves along the outer boundary to arrive at *s* to clear $P[s,p_1]$ (Fig. (**11**)(**c**)-(**d**)). In this process, the beam of S_a always intersects with $\overline{sh_1}$. Finally, S_a aims his flashlight at h_1. This completes the proof.□

Lemma 7. The first deadlock found by S_a and S_c is a backward deadlock, say, caused by <p_1, h_1>, $p_1 \in P$, $h_1 \in H$, and both p_1 and h_1 are in the contaminated region. If there are no defining edges on $H[h_1, F_a]$ except $\overline{h_1Succ(h_1)}$, then S_a and S_c can mutually cooperate to clear this backward deadlock and meet at the same position as shown in Fig. (**12**).

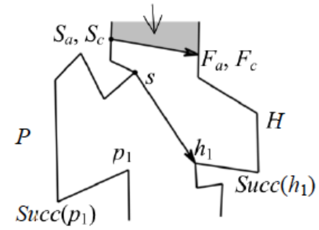


Fig. (12). Illustration for the proof of lemma 7.

Proof. Suppose $s \in P$ is the first point visible to h_1 on the outer boundary from S_a in counter clockwise, since there are no defining edges on $H[h_1,F_a]$ except $\overline{h_1Succ(h_1)}$, S_a and S_c can arrive at *s* and aim their flashlights at h_1 to clear $P[s, S_a]$ and $H[h_1, F_a]$ following lemma 2. If there are no defining edges on $P[Succ(p_1), S_a]$, then the deadlock can be cleared following lemma 3. Otherwise, they find another deadlock to be the first deadlock. These deadlocks can be cleared following lemma 3 to lemma 6. This completes the proof.

Lemma 8. The first deadlock found by S_a and S_c is a backward deadlock, say, caused by <p_1, h_1>, $p_1 \in P$, $h_1 \in H$, and both p_1 and h_1 are in the contaminated region. If there are no defining edges on $P[p_1, S_a]$ except $\overline{p_1Succ(p_1)}$, then S_a and S_c can mutually cooperate to clear this backward deadlock and meet at the same position as shown in Fig. (**13**).

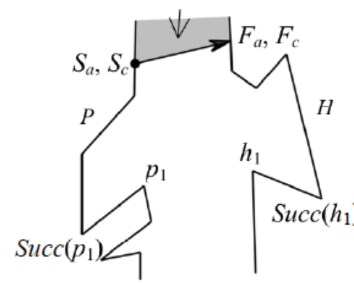


Fig. (13). Illustration for the proof of lemma 8.

Proof. The proof for Lemma 8 is similar to Lemma 7.□

For the first deadlock S_a and S_c found, they first cooperate to clear its one defining edge, and then cooperate to clear the other defining edge. The deadlock is either a forward deadlock or a backward deadlock. If it is not cleared, either its two defining edges are all in the contaminated region or one defining edge is in the contaminated region and the other is in the cleared region. So from lemma 3 to lemma 8 contain all the cases of the first deadlock S_a and S_c found. If a deadlock they found is not satisfied a lemma of lemma 3 to lemma 8, then the deadlock is not the first deadlock they found. After they clear the first deadlock, the next deadlock will be the first deadlock they found. When they encounter a non-separated deadlock, they take the nearest deadlock to them as the first deadlock to clear, then clear the other deadlock. Such as S_a and S_c encounter the non-separated deadlock caused by <p_1, h_1> and <p_2, h_2> in Fig. (**5**)(**c**), the deadlock caused by <p_1, h_1> is the first deadlock they found. They first cooperate to clear it, and then the deadlock caused by <p_2, h_2> becomes the first deadlock, they continue to clear it. The non-separated deadlock caused by <p_1, h_1> and <p_2, h_2> is cleared. So the 1-searchers S_a and S_c in counterclockwise can clear all the deadlocks one by one, and finally end at the position of S_b to clear *CC*. The search schedule can be obtained from lemma 3 to lemma 8, and the total walk instructions are $O(n^2)$ in the worst case, where *n* denotes the total number of vertices of *P* and *H*. Hence, a search schedule of a given *CC* can be reported in O(*m*) time, where $m \leq n^2$ denotes walk instructions.

CONCLUSION

We study the variation of the 1-searcher problem in a circular corridor which can not be cleared by two 1-searchers, and prove that any one circular corridor can be cleared by no more than three 1-searchers. Moreover, a search schedule can be reported in O(*m*) time, where $m \leq n^2$ denotes walk instructions, and *n* denotes the total number of vertices of *P* and *H*.

A more ambitious goal is to provide an algorithm for searching a circular corridor such that its outer and inner boundaries are not mutually weakly visible with any number of 1-searchers.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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