



# The Open Electrical & Electronic Engineering Journal

Content list available at: [www.benthamopen.com/TOEEJ/](http://www.benthamopen.com/TOEEJ/)

DOI: 10.2174/1874129001711010177



## RESEARCH ARTICLE

# Optimal Power Flow Using an Improved Hybrid Differential Evolution Algorithm

Gonggui Chen<sup>1,2,\*</sup>, Zhengmei Lu<sup>1,2</sup>, Zhizhong Zhang<sup>3</sup> and Zhi Sun<sup>4</sup><sup>1</sup>Key Laboratory of Network control & Intelligent Instrument (Chongqing University of Posts and Telecommunications), Ministry of Education, Chongqing, China<sup>2</sup>Research Center on Complex Power System Analysis and Control, Chongqing University of Posts and Telecommunications, Chongqing 400065, China<sup>3</sup>Key Laboratory of Communication Network and Testing Technology, Chongqing University of Posts and Telecommunications, Chongqing 400065, China<sup>4</sup>Guodian Enshi Hydropower Development, Enshi, 445000, China

Received: August 09, 2017

Revised: August 18, 2017

Accepted: August 20, 2017

### Abstract:

### Objective:

In this paper, an improved hybrid differential evolution (IHDE) algorithm based on differential evolution (DE) algorithm and particle swarm optimization (PSO) has been proposed to solve the optimal power flow (OPF) problem of power system which is a multi-constrained, large-scale and nonlinear optimization problem.

### Method:

In IHDE algorithm, the DE is employed as the main optimizer; and the three factors of PSO, which are inertia, cognition, and society, are used to improve the mutation of DE. Then the learning mechanism and the adaptive control of the parameters are added to the crossover, and the greedy selection considering the value of penalty function is proposed. Furthermore, the replacement mechanism is added to the IHDE for reducing the probability of falling into the local optimum. The performance of this method is tested on the IEEE30-bus and IEEE57-bus systems, and the generator quadratic cost and the transmission real power losses are considered as objective functions.

### Results:

The simulation results demonstrate that IHDE algorithm can solve the OPF problem successfully and obtain the better solution compared with other methods reported in the recent literatures.

**Keywords:** Power system, Optimal power flow, Improved hybrid differential evolution algorithm, Particle swarm optimization, Learning mechanism, Replacement mechanism.

## 1. INTRODUCTION

Due to the development and extension of the traditional economic scheduling theory, the optimal power flow (OPF) problem considering the economy and security of the power system has drawn a lot of attention, which is a multi-constrained, non-linear and non-convex optimization problem containing continuous and discrete control variables. Since the OPF formulation by Carpentier in 1962, its usefulness is progressively being recognized, and nowadays it becomes

\* Address correspondence to this author at the Research Center on Complex Power System Analysis and Control, Chongqing University of Posts and Telecommunications, Chongqing 400065, P.R. China; Tel: +86-15696106539; Fax: +86-23-62461535; E-mails: [chenggpw@163.com](mailto:chenggpw@163.com), [chenggpw@126.com](mailto:chenggpw@126.com)

the most important tool used in operation and planning of power system [1]. The OPF problem is aimed to give the optimal settings of determined control variables for optimizing a specific objective function, such as the generation cost function, transmission real power losses and voltage deviation, while satisfying some equality and inequality constraints. It can respect the system static security constraints and schedule active and reactive power [2].

Nowadays, many optimization methods are employed successfully to solve the OPF problem and can be divided into two categories: the traditional mathematical methods and the intelligent optimization methods. The traditional mathematical methods include linear programming method [3], nonlinear programming method [4], and simplified gradient method [5] and so on. Usually, these methods have fast calculation speed and good convergence characteristics, but some restrictions on the variables and the objective functions. Compared with the traditional mathematical methods, the intelligent optimization methods based on the combination of computer technology and biological simulation can be a better way when solving the large-scale, multi-constrained, non-linear and non-convex OPF problem which contains the discrete and continuous variables. Some of the proposed intelligent methods, such as Particle Swarm Optimization with an Aging Leader and Challengers (ALC-PSO) algorithm [6], Artificial Bee Colony (ABC) [7], Lévy Mutation Teaching–Learning-Based Optimization (LTLBO) algorithm [8], Krill Herd (KH) algorithm [9], Adaptive Real Coded Biogeography-based Optimization (ARCBBO) [10], Moth Swarm algorithm (MSA) [11] and Differential Search algorithm (DSA) [12], have been proved to be successful in solving the OPF problem. Therefore, there is an increasing number of studies about the improvement and combination of algorithms or mechanisms.

As a relatively new member of evolutionary algorithms (EAs), DE is also a stochastic model which simulates the biological evolution and has been widely applied to the optimization problems in the power system. The DE consists of four operators: initialization, mutation, crossover and selection, and its performance mainly affected by the strategy of the mutation and control parameters, such as the mutant scale factor  $F$  and the crossover factor  $CR$ . In the evolutionary process, the difference among individuals will gradually decrease due to the influence of the greedy selection, which affects the diversity caused by the mutation and leads the algorithm to fall into the local optimum. So far, many modified DE algorithms have been proposed to overcome the shortcomings. The authors in [13] proposed Gaussian random variable instead of scaling factor and the proposed approach was able to provide better solution compared with other evolutionary method for economic dispatch. In [14], the proposed MBDE method which used memory mechanism to modify mutation and crossover operators provided better results. Obviously, parameter modification, the strategies of mutation and crossover affect the performance of the algorithm to a certain extent. Considering all of the above factors to aptly modify DE algorithm will be a new direction in the further.

Numerous studies indicate that a variety of intelligent algorithms show different strength and weakness in solving some problems and the combination of two or more algorithms can solve the problem more efficiently [15 - 17]. Hence, a combination of advantages of some algorithms is considered in this paper. Particle swarm optimization (PSO) with inherent parallel search mechanism is especially suitable for complex optimization fields where traditional methods are difficult to work [15]. And it can be used to improve the search ability of DE. Thus, the novel hybrid differential evolution (IHDE) algorithm based on DE and PSO is proposed in this paper. Furthermore, what is greatly significant is not only to improve the current methods but also to handle constraints on state variables well simultaneously [18]. Briefly, the contributions of this paper can be summarized as follows:

(1) An improved hybrid differential evolution algorithm considering the mutation and crossover strategies and parameter values is proposed, namely IHDE. The mutation and crossover strategies of DE are modified by the memory mechanism of PSO and learning mechanism, respectively. Moreover, the values of the mutant scale factor  $F$ , the inertia weight  $\omega$  and crossover factor  $CR_i$  are varied according to evolutionary process. Finally, the replacement mechanism is designed for reducing the probability of falling into the local optimum.

(2) A novel constraint handling method including greedy selection strategy and penalty function is proposed to keep the variable within its limits, especially in larger systems.

(3) The OPF problem is implemented on standard IEEE30-bus and IEEE57-bus systems successfully using IHDE algorithm

In the proposed algorithm, the DE is employed as the main optimizer and the three factors of PSO which are inertia, cognition, and society are used to improve the mutation for speeding up the convergence rate. Then the learning mechanism and the adaptive control of the parameters are added to the crossover to increase the diversity of population in some direction. To ensure the optimal solution satisfies the security constraints, the greedy selection mechanism

considering the value of penalty function is proposed. Furthermore, the replacement mechanism is added to the IHDE for reducing the probability of falling into the local optimum. Finally, the IHDE, DE and PSO are tested on the IEEE30-bus and IEEE57-bus systems, and the generator quadratic cost and the transmission real power losses are considered as objective functions. The simulation results demonstrate that IHDE algorithm can solve the OPF problem successfully and obtain better solutions compared with DE, PSO and other methods reported in the recent literatures.

The rest of the paper is organized as follows: Section 2 describes the mathematical formulation of the OPF problem. Section 3 introduces the DE and PSO briefly. Section 4 proposes an improved hybrid differential evolution (IHDE) algorithm. Section 5 presents the simulation results and analysis. Finally, the conclusion is drawn in section 6.

## 2. PROBLEM FORMULATION

As previously mentioned, the objective of the OPF problem is to optimize a specific objective function by finding the optimal settings of determined control variables while satisfying a range of equality and inequality constraints. The model of OPF problem contains five parts: control variable, state variable, objective function, equality constraint, and inequality constraint.

The OPF problem can be mathematically formulated as follows:

$$\text{Minimize } F(\mathbf{x}, \mathbf{u}) \tag{1}$$

Subject to

$$\begin{cases} g(\mathbf{x}, \mathbf{u}) = 0 \\ h(\mathbf{x}, \mathbf{u}) \leq 0 \end{cases} \tag{2}$$

Where  $\mathbf{x}$  and  $\mathbf{u}$  are the vector of state variables and the vector of control variables, respectively.  $F(\mathbf{x}, \mathbf{u})$  is the objective function to be minimized.  $g(\mathbf{x}, \mathbf{u})$  is the set of equality constraints.  $h(\mathbf{x}, \mathbf{u})$  is the set of inequality constraints.

### 2.1. Control Variable

The control variables of this model are active power generation  $P_G$  at PV buses except the slack bus, voltage magnitude  $V_G$  at PV buses, transformer taps settings  $T$  and shunt VAR compensation  $Q_C$ . The control variables can be expressed as:

$$\mathbf{u}^T = [P_{G2}, \dots, P_{GNG}, V_{G1}, \dots, V_{GNG}, T_1, \dots, T_{NT}, Q_{C1}, \dots, Q_{CNC}] \tag{3}$$

Where NG, NT and NC represent the number of generator buses, the number of regulating transformers and the number of shunt compensators, respectively.

### 2.2. State Variable

The state variables of this model are active power generation  $P_{G1}$  at the slack bus, voltage magnitude  $V_L$  at PQ buses, reactive power output  $Q_G$  at PV buses and transmission line loadings (or line flow)  $S_l$ . The state variables can be described as follows:

$$\mathbf{x}^T = [P_{G1}, V_{L1}, \dots, V_{LNL}, Q_{G1}, \dots, Q_{GNG}, S_{l1}, \dots, S_{lNTL}] \tag{4}$$

Where NL is the number of load buses and NTL is the number of transmission lines;  $P_{G1}$  as the state variable is the active power generation at the slack bus.

### 2.3. Equality Constraint

The power balance constraints are considered as basic equality constraints and reflect the physics of the power system which can be represented as follows:

$$\left. \begin{aligned} Q_{Gi} - Q_{Li} - V_i \sum_{j=1}^{N_i} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) &= 0 \quad (i = 1, 2, \dots, N_{PQ}) \\ P_{Gi} - P_{Li} - V_i \sum_{j=1}^{N_i} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) &= 0 \quad (i = 1, 2, \dots, N_S) \end{aligned} \right\} \tag{5}$$

Where  $\delta_{ij} = \delta_i - \delta_j$ ,  $\delta_i$  and  $\delta_j$  are voltage angles at bus  $i$  and  $j$ , respectively.  $N_i$  is the number of buses which are adjacent to bus  $i$ , including bus  $i$ .  $N_{PQ}$  is the number of PQ buses and  $N_s$  is the number of system buses excluding slack bus.  $Q_{Gi}$  and  $P_{Gi}$  represent the reactive power output and the active power generation at bus  $i$  which belongs PV buses, respectively.  $Q_{Li}$  and  $P_{Li}$  represent the reactive load demand and active load demand at bus  $i$ , respectively.  $G_{ij}$  and  $B_{ij}$  are the conductance and susceptance between bus  $i$  and bus  $j$ , respectively.  $V_i$  is the voltage magnitude at bus  $i$ .

**2.4. Inequality Constraint**

The operating limits of power system are considered as inequality constraints which guarantee the system security.

(a) Generator constraints

$$V_{Gi\min} \leq V_{Gi} \leq V_{Gi\max} \quad (i = 1, \dots, NG) \tag{6}$$

$$P_{Gi\min} \leq P_{Gi} \leq P_{Gi\max} \quad (i = 1, \dots, NG) \tag{7}$$

$$Q_{Gi\min} \leq Q_{Gi} \leq Q_{Gi\max} \quad (i = 1, \dots, NG) \tag{8}$$

(b) Transformer constraints

$$T_{i\min} \leq T_i \leq T_{i\max} \quad (i = 1, \dots, NT) \tag{9}$$

(c) Shunt VAR compensator constraints

$$Q_{Ci\min} \leq Q_{Ci} \leq Q_{Ci\max} \quad (i = 1, \dots, NC) \tag{10}$$

(d) Security constraints

$$V_{Li\min} \leq V_{Li} \leq V_{Li\max} \quad (i = 1, \dots, NL) \tag{11}$$

$$S_{li} \leq S_{li\max} \quad (i = 1, \dots, NTL) \tag{12}$$

**2.5. Objective Function**

In this paper, two objective functions are considered to evaluate the effectiveness of the proposed algorithm.

**2.5.1. Quadratic Cost Function**

The objective of the total fuel cost is widely used and it can be formulated by a quadratic curve as follows:

$$f_{Cost} = \sum_{i=1}^{NG} f_i (\$/h) \tag{13}$$

$$f_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{14}$$

Where  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of the  $i$ th generator.

**2.5.2. Transmission Real Power Losses**

The function determined by the bus voltage magnitude and angle is the total active power loss of all transmission lines and can be calculated as:

$$f_{Loss} = \sum_{k=1}^{NTL} G_k [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] (MW) \tag{15}$$

Where  $G_k$  is the conductance between bus  $i$  and bus  $j$ .

### 3. BRIEF ON DE&PSO

#### 3.1. Difference Evolution (DE) Algorithm

In DE, the forward direction is based on the differences among all the individuals in the population and the diversity is increased by crossover. Then the greedy selection is used to realize the population evolution. The process of DE includes four steps: initialization, mutation, crossover and selection.

##### 3.1.1. Initialization

The initial population of NP is randomly selected by uniform probability in the search space of the optimization problem. Random values are assigned to each D-dimensional individual according to:

$$x_{j,i|g=0} = x_{j,\min} + \text{rand}(0,1) \times (x_{j,\max} - x_{j,\min}) \quad j=1,\dots,D \quad i=1,\dots,\text{NP} \quad (16)$$

Where  $x_{j,\min}$  and  $x_{j,\max}$  represent the lower and upper bounds of the  $j$ th decision variable;  $g$  and NP represent the number of the iteration and population size, respectively.

##### 3.1.2. Mutation

The operator is utilized to generate the mutant vector  $x'_{i,g+1}$  by changing the value of the individual through the difference of other individuals. Recently, there are many mutation schemes of DE in the literature [19] and the strategy commonly used is expressed as follows:

$$x'_{i,g+1} = x_{r1,g} + F(x_{r3,g} - x_{r2,g}) \quad i=1,\dots,\text{NP} \quad (17)$$

Where  $x_{r1,g}$ ,  $x_{r2,g}$  and  $x_{r3,g}$  ( $r1 \neq r2 \neq r3 \neq i$ ) are randomly chosen from the current population;  $F \in [0, 2]$  is the mutant scale factor.

##### 3.1.3. Crossover

The operator is utilized to generate a new trial vector  $x''_{i,g+1}$  and there are two crossover strategies which include the binomial crossover and the exponential crossover [20]. In this paper, the binomial crossover is selected for the research and defined as follows:

$$x''_{ij,g+1} = \begin{cases} x'_{ij,g+1} & \text{if } \text{rand}(0,1) \leq \text{CR or } j = m \\ x_{ij} & \text{otherwise} \end{cases} \quad j=1,\dots,D \quad i=1,\dots,\text{NP} \quad (18)$$

Where  $\text{CR} \in [0,1]$  is the crossover factor;  $m \in [1,D]$  is a random integer which ensures  $x''_{i,g+1}$  can get at least one parameter from  $x'_{i,g+1}$ .

##### 3.1.4. Selection

The operator is utilized to choose a better vector between the trial vector  $x''_{i,g+1}$  and the target vector  $x_i$  based on fitness values.  $f(x)$  is the fitness function value of  $x$ . Therefore the selection criterion is expressed as follows:

$$x_{i,g+1} = \begin{cases} x''_{i,g+1} & \text{if } f(x''_{i,g+1}) \leq f(x_{i,g}) \\ x_{i,g} & \text{otherwise} \end{cases} \quad (19)$$

#### 3.2. Particle Swarm Optimization (PSO)

In PSO, each particle represents a potential solution for the specific issue and varies with its position and velocity in the feasible space. At iteration  $g$ , the D-dimensional position vector and velocity vector of the particle  $i$  can be represented as  $x_{i,g} = (x_{i1,g}, \dots, x_{iD,g})$  and  $v_{i,g} = (v_{i1,g}, \dots, v_{iD,g})$ , respectively. The individual best position of the particle  $i$  achieved and the global best position of the whole swarm based on fitness values up to the current iteration can be represented as  $p_{\text{best}i} = (p_{\text{best}i1}, \dots, p_{\text{best}iD})$  and  $g_{\text{best}} = (g_{\text{best}1}, \dots, g_{\text{best}D})$ , respectively. There exist three components that have an impact on the velocity of a particle, namely inertia, cognition, and society [18]. The particle tries to modify its position according to the current velocity and it can be formulated as:

$$v_{i,g+1} = \omega v_{i,g} + c_1 r_1 (p_{besti} - x_{i,g}) + c_2 r_2 (g_{best} - x_{i,g}) \quad i = 1, \dots, NP \tag{20}$$

$$x_{i,g+1} = x_{i,g} + v_{i,g+1} \quad i = 1, \dots, NP \tag{21}$$

Where  $\omega$  is the inertia weight;  $c_1$  and  $c_2$  are positive constants and considered as acceleration coefficients to make the particle accelerate toward the best position;  $r_1$  and  $r_2$  are uniformly distributed random variables between 0 and 1.

#### 4. PROPOSED EFFICIENT APPROACH

In this section, an efficient approach is proposed to enhance the search ability of the DE and PSO, which named improved hybrid differential evolution algorithm (IHDE).

##### 4.1. Mutation

The mutation strategy of IHDE algorithm contains the effect of other individual difference and the impact on the velocity of a particle, namely, inertial, cognitive, and social, which can be described as follows:

$$x'_{i,g+1} = x_{i,g} + v_{i,g+1} + F \times (x_{r1,g} - x_{r2,g}) \quad i = 1, \dots, NP \tag{22}$$

$$v_{i,g+1} = \omega v_{i,g} + c_1 r_1 (x_{pbest} - x_{i,g}) + c_2 r_2 (x_{Gbest} - x_{i,g}) \tag{23}$$

Where  $x_{pbest}$  is randomly selected from the excellent individuals rather than the individual best position of the particle  $i$  and the excellent individuals are the best ( $p \times NP$ ) individuals in the population based on the fitness value;  $x_{Gbest}$  represents the global best solution. The larger the values of  $F$  and  $\omega$  are, the stronger the exploration capability will be, which may lead to slower convergence. On the contrary, the exploitation capability will be stronger, which may lead to premature convergence. Therefore, the improvement of the parameters should be implemented, which directly affect the search precision and convergence speed. In IHDE algorithm, both the mutant scale factor  $F$  and the inertia weight  $\omega$  are varied in its range aimed to perform a global search firstly and then perform a local search, which can be expressed as:

$$F_g = F_{max} - \frac{F_{max} - F_{min}}{g_{max}} \times g \tag{24}$$

$$\omega_g = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{g_{max}} \times g \tag{25}$$

##### 4.2. Crossover

The learning mechanism is added to the crossover which can be formulated as follows:

$$x''_{ij,g+1} = \begin{cases} x'_{ij,g+1} & \text{if } rand(0,1) \leq CR_i \text{ or } j = m \\ x_{learnij} & \text{otherwise} \end{cases} \quad (i = 1, \dots, NP, j = 1, \dots, D) \tag{26}$$

Where  $x_{learnij}$  is generated by the target vector  $x_{i,g}$  and the mutant vector  $x'_{i,g+1}$  through the learning mechanism and expressed as:

$$x_{learnij} = x_i \begin{cases} x'_{ij,g+1} + r_3 \times (x_{ij} - x'_{ij,g+1}) & \text{if } f(x_i) \leq f(x'_{i,g+1}) \\ x_{ij} + r_4 \times (x'_{ij,g+1} - x_{ij}) & \text{otherwise} \end{cases} \tag{27}$$

Where  $r_3$  and  $r_4$  are uniformly distributed random variables between 0 and 1. The crossover factor  $CR_i$  is generated according to a normal distribution of mean  $\mu_i$  and standard deviation 0.1 which can be defined as follows:

$$CR_i = randN(\mu_i, 0.1) \tag{28}$$

$$\mu_i = (1-cr) \times \mu_i + cr \times mean_c(C_R) \tag{29}$$

Where  $cr$  is a positive constant between 0 and 1;  $\text{mean}_c()$  is the arithmetic mean of  $C_R$ ;  $\mu_i$  is initialized to be 0.5 and  $C_R$  represents the set of all crossover factors which successfully obtained a better solution at generation  $g$ .

**4.3. Selection**

In order to find the solution in the feasible space, the algorithm proposes a greedy selection strategy considering the constraint condition. If only one of the two solutions' penalty functions is zero, the section is performed as follows:

$$x_{i,g+1} = \begin{cases} x_{i,g+1}^* & \text{Penalty}(x_{i,g+1}^*) = 0, \text{Penalty}(x_{i,g}) \neq 0 \\ x_{i,g} & \text{Penalty}(x_{i,g}) = 0, \text{Penalty}(x_{i,g+1}^*) \neq 0 \end{cases} \tag{30}$$

Otherwise, the selection criterion is defined as DE as follows:

$$x_{i,g+1} = \begin{cases} x_{i,g+1}^* & \text{if } f(x_{ij,g+1}^*) \leq f(x_{ij,g}) \\ x_{i,g} & \text{otherwise} \end{cases} \tag{31}$$

The penalty function will be explained in detail in section 4.5.

**4.4. Replacement Mechanism**

The mechanism replaces the individual which is not improved by a predetermined number of trials with a new one, and its steps are summarized as follows:

Step 1: Set the un-updated number,  $u=0$ .

Step 2: Check whether the individual has been updated.

Step 3: If the individual is not updated, the un-updated number adds one,  $u=u+1$ . On the contrary, the number is changed to zero.

Step 4: Check whether the individual has not updated for predefined limit trails. If exists, replace it with a new one by Eq.(16) and change the un-updated number to zero.

**4.5. Constraint Handling Method**

In this method, the inequality constraints of state variables which contain real power generation at slack bus, reactive power generation at PV buses, load bus voltage magnitude and line loading are incorporated into the objective function using a penalizing strategy to keep the variables within its limits. The strategy introduces a penalty function which can be formulated as:

$$\begin{aligned} \text{Penalty} = & K_V \sum_{i=1}^{NL} (V_{Li} - V_{Li}^{\text{lim}})^2 + K_Q \sum_{i=1}^{NG} (Q_{Gi} - Q_{Gi}^{\text{lim}})^2 \\ & + K_P (P_{G1} - P_{G1}^{\text{lim}})^2 + K_S \sum_{i=1}^{NL} (S_{li} - S_{li}^{\text{lim}})^2 \end{aligned} \tag{32}$$

Where  $K_V, K_Q, K_P$  and  $K_S$  are the penalty factors of the state variable, respectively.  $X^{\text{lim}}$  depended by the corresponding state variable can be expressed as follows:

$$X_i^{\text{lim}} = \begin{cases} X_{i\text{min}} & X_i < X_{i\text{min}} \\ X_i & X_{i\text{min}} \leq X_i \leq X_{i\text{max}} \\ X_{i\text{max}} & X_i > X_{i\text{max}} \end{cases} \tag{33}$$

Where  $X_{i\text{min}}$  and  $X_{i\text{max}}$  are the lower and upper bounds of the state variable, respectively. Furthermore, the values of the penalty factors are dynamic in the optimization process and can be calculated as:

$$K_{Fg} = K_{F\text{min}} + \frac{K_{F\text{max}} - K_{F\text{min}}}{g_{\text{max}}} \times g \tag{34}$$

Where  $K_{Fg}$  represents the penalty factor at the generation  $g$ ;  $K_{Fmin}$  and  $K_{Fmax}$  are the lower and upper bounds of the penalty factor, respectively. At the beginning of the optimization, the penalty is smaller aimed to pick out the solution with better function value and then it increases gradually as the iteration to obtain the solution which satisfies security constraints. The penalty function and the original objective function form a new fitness function to avoid the state variable violating the limits, which can be formulated as:

$$F(x) = f(x) + \text{Penalty} \quad (35)$$

On the other hand, the control variable is self-constrained. When the independent variable violates limits, the position of the individual will be adjusted as follows:

$$x_j = \begin{cases} r_1 \times x_{j,max} + (1-r_1) \times x_{j,best} & \text{if } x_j > x_{j,max} \\ r_2 \times x_{j,min} + (1-r_2) \times x_{j,best} & \text{if } x_j < x_{j,min} \end{cases} \quad j = 1, \dots, D \quad (36)$$

Where  $x_{j,best}$  is the corresponding variable of the best individual,  $r_1$  and  $r_2$  are uniformly distributed random variables between 0 and 1.

#### 4.6. Implementation of IHDE Algorithm

The proposed IHDE algorithm for solving OPF problem is summarized as follows:

Step 1: Establish the OPF problem model and enter power system data and algorithm parameters.

Step 2: Initialize the population by Eq.(16) and set the iteration number,  $g=1$ .

Step 3: Evaluate the penalty function value and the fitness value of the population and select the excellent individuals and the global best solution.

Step 4: Update the mutant scale factor  $F$ , the inertia weight  $\omega$  and the crossover factor  $CR_i$ .

Step 5: Apply the mutation and the crossover of IHDE algorithm to generate a new trial vector  $x''_{i,g+1}$ .

Step 6: Evaluate the penalty function value and the fitness value of  $x''_{i,g+1}$ .

Step 7: Apply the selection of IHDE algorithm between  $x''_{i,g+1}$  and  $x_{i,g}$ .

Step 8: Update the population, the excellent individuals and the global best solution.

Step 9: Apply replacement mechanism.

Step 10: Stop and memorise the global best solution if the iteration number  $g$  reaches the maximum, else set  $g=g+1$  and go back to Step 4.

### 5. SIMULATION RESULTS and DISCUSSION

In this section, model systems and simulation analysis are represented. The IHDE algorithm has been applied to the IEEE 30-bus and IEEE 57-bus for solving the OPF problem and compared with DE, PSO and the simulation results of other methods presented in the recent literatures. In addition, 3 different cases are studied which summarized in Table 1. The simulations were performed in matlab2014 on a personal computer with 3.30GHz processor and 8.00GB for RAM.

**Table 1. Summary of the studied cases.**

Test system	Control variables	Name	Objective function	Constraints
IEEE 30	24	Case 1	Quadratic cost function	Equality and non-equality
IEEE 30	24	Case 2	Transmission real power losses	Equality and non-equality
IEEE 57	33	Case 3	Quadratic cost function	Equality and non-equality

As mentioned before, there are many control parameters of DE, PSO and IHDE need to be set. Here, the population sizes ( $N_p$ ), maximum iteration number ( $g_{max}$ ) and so on are given in the Table 2. For each case 30 runs are conducted to get the solution quality. The lower limit of the penalty parameters are all chosen as 10, and the upper limit of the penalty parameters are chosen as 100 in IEEE 30-bus system while 1000 in IEEE 57-bus.



Table 2. Parameters of the three algorithms.

Algorithm	$N_p$	$g_{max}$	$F$	$C_R$	$\omega$	$c_1$	$c_2$
DE	30	500	0.3/0.8	0.8	--	--	--
PSO	30	500	--	--	0.4/0.9	2	2
IHDE	30	500	0.3/0.8	--	0.4/0.9	2	2

5.1. IEEE 30-Bus System

The system has 6 generators, 41 branches, 9 shunt VAR compensations and 4 transformers, which also has 2.834p.u. for the active power demand and 1.262p.u. for the reactive power demand on base of 100MVA. In addition, the detailed line data, the bus data and the cost coefficients are given in [21] and [2]. The minimum and maximum limits of transformer taps are 0.9p.u. and 1.1p.u., respectively. The lower and upper limits of bus shunt capacitors are 0.0p.u. and 0.05p.u., respectively. Furthermore, the voltage magnitudes of generator buses are assumed to vary in the range [0.95, 1.1] p.u. and the lower and upper limits of load buses are considered to be 0.95p.u. and 1.05p.u., respectively.

5.1.1. Case 1: Minimization of Quadratic Cost Function

The objective function of minimization of quadratic cost is defined as Eq.(13) and Eq.(14). The optimal control variables of IHDE, DE, and PSO for case 1 are shown in Table 3. In order to verify the efficiency of the proposed algorithm, the results are compared with other methods reported in the literatures and their details are shown in Table 4. As shown in the tables, the minimization of quadratic cost obtained by IHDE, DE, and PSO are 800.4152 \$/h, 800.5409 \$/h and 800.6488 \$/h, respectively. And the result of IHDE is better than DE, PSO, ARCBBO [10], MGBICA [22], MSA [11] and ABC [7]. Although the results obtained from GSA [23] and BBO [24] get less cost but the optimal solutions violate system security constraints mentioned before. Moreover, Fig. (1) shows the optimal convergence curves and Fig. (2) shows the results in 30 independent simulations of IHDE, DE, and PSO for case 1. It can be seen in Fig. (2) that IHDE has a stronger robustness.

Table 3. Optimal solutions for case 1 and case 2 on IEEE 30 system.

Control variables	Case 1			Case 2		
	IHDE	DE	PSO	IHDE	DE	PSO
$P_1$ (MW)	177.2248	177.1034	177.3276	51.48791	51.55068	51.54542
$P_2$ (MW)	48.74866	48.68600	48.67364	79.99838	79.99396	79.99778
$P_3$ (MW)	21.39375	21.48266	21.44029	49.99992	49.99714	49.99992
$P_8$ (MW)	21.07999	21.20626	21.19052	34.99997	34.99712	34.99697
$P_{11}$ (MW)	11.96386	11.94555	11.84440	30.00000	29.99629	30.00000
$P_{13}$ (MW)	12.00000	12.00841	12.00239	39.99947	39.99320	39.99238
$V_1$ (p.u.)	1.083017	1.079011	1.082761	1.061236	1.061394	1.060080
$V_2$ (p.u.)	1.063646	1.059780	1.062633	1.057166	1.057288	1.056278
$V_5$ (p.u.)	1.032327	1.028427	1.030119	1.037699	1.037502	1.036530
$V_8$ (p.u.)	1.036608	1.033294	1.032159	1.043903	1.044415	1.043915
$V_{11}$ (p.u.)	1.093351	1.063881	1.068400	1.085657	1.095213	1.075596
$V_{13}$ (p.u.)	1.048811	1.071938	1.056255	1.055954	1.059464	1.063790
$T_{11}$ (p.u.)	1.040000	1.020000	1.000000	1.070000	1.040000	1.050000
$T_{12}$ (p.u.)	0.930000	0.920000	0.980000	0.900000	0.950000	0.930000
$T_{15}$ (p.u.)	0.970000	1.010000	1.000000	1.000000	1.000000	1.010000
$T_{36}$ (p.u.)	0.970000	0.960000	0.950000	0.970000	0.980000	0.970000
$Q_{C10}$ (p.u.)	0.000000	0.005000	0.004000	0.001000	0.004000	0.050000
$Q_{C12}$ (p.u.)	0.001000	0.005000	0.029000	0.000000	0.012000	0.016000
$Q_{C15}$ (p.u.)	0.039000	0.041000	0.038000	0.040000	0.048000	0.044000
$Q_{C17}$ (p.u.)	0.050000	0.050000	0.049000	0.050000	0.048000	0.050000
$Q_{C20}$ (p.u.)	0.038000	0.038000	0.041000	0.040000	0.036000	0.038000
$Q_{C21}$ (p.u.)	0.050000	0.050000	0.050000	0.050000	0.050000	0.050000
$Q_{C23}$ (p.u.)	0.029000	0.025000	0.021000	0.030000	0.028000	0.022000
$Q_{C24}$ (p.u.)	0.050000	0.050000	0.035000	0.050000	0.050000	0.050000

(Table 3) contd....

Control variables	Case 1			Case 2		
	IHDE	DE	PSO	IHDE	DE	PSO
$Q_{C29}$ (p.u.)	0.023000	0.009000	0.000000	0.019000	0.026000	0.022000
Fuel cost (\$/h)	<b>800.4152</b>	800.5409	800.6488	967.6228	967.6734	967.7104
Power loss (MW)	9.011081	9.032239	9.078871	<b>3.085644</b>	3.128384	3.132461

Table 4. Comparison of the simulation results for Case 1 on IEEE 30 system (\$/h).

Algorithms	Min
IHDE	800.4152
DE	800.5409
PSO	800.6468
ARCBBO [10]	800.5159
MGBICA [22]	801.1409
MSA [11]	800.5099
ABC [7]	800.6600
GSA [23]	798.6751 <sup>a</sup>
BBO [24]	799.1116 <sup>a</sup>

<sup>a</sup> Infeasible solution.

5.1.2. Case 2: Minimization of Transmission Real Power Losses

The objective function of minimization of transmission real power losses is defined as Eq.(15). The optimal control variables of IHDE, DE, and PSO for case 2 also are shown in Table 3 and the comparison of the results obtained from other methods reported in the literatures are shown in Table 5. As shown in the tables, the minimization of transmission real power losses obtained by IHDE, DE, and PSO are 3.085644 MW, 3.128384 MW and 800.3.132461 MW, respectively. And the result of IHDE is better than DE, PSO, ABC [7], DSA [12], MSA [11] and MGBICA [22]. On the other hand, the result of HS [25] is less than IHDE’s and defined as infeasible solution because it doesn’t satisfy the security constraints. Furthermore, Fig. (3) shows the optimal convergence curves and Fig. (4) shows the results in 30 independent simulations of IHDE, DE, and PSO for case 2. Fig. (3) demonstrates that IHDE has better convergence.

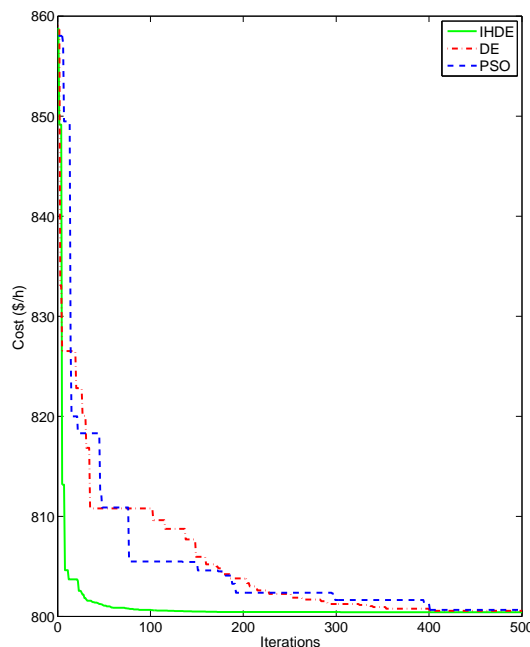


Fig. (1). The optimal convergence curves for Case 1 of IEEE 30-bus system.

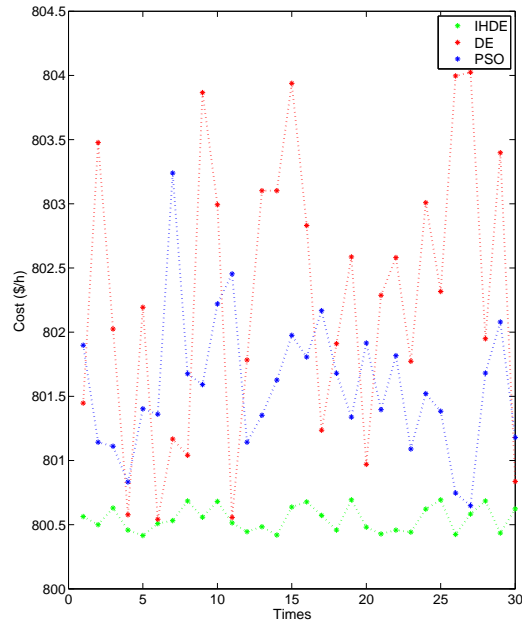


Fig. (2). The results' distribution for Case1 of IEEE 30-bus system.

Table 5. Comparison of the simulation results for Case 2 on IEEE 30 system (MW).

Algorithms	Min
IHDE	3.085644
DE	3.128384
PSO	3.132461
ABC [7]	3.1078
DSA [12]	3.0945
MSA [11]	3.1005
MGBICA [22]	4.937
HS [25]	2.9678 <sup>a</sup>

<sup>a</sup> Infeasible solution.

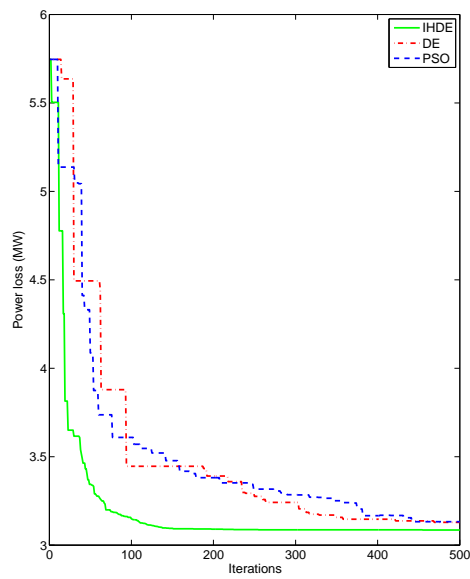


Fig. (3). The optimal convergence curves for Case 2 of IEEE 30-bus system.

### 5.2. IEEE 57-Bus System

The system has 7 generators, 3 shunt VAR compensations and 15 transformers, which also has 12.508 p.u. for the active power demand and 3.364 p.u. for the reactive power demand on base of 100MVA. All detailed line data, the bus data and the cost coefficients are given in [26]. The minimum and maximum limits of transformer taps are 0.9p.u. and 1.1p.u., respectively. The lower and upper limits of bus shunt capacitors are 0.0p.u. and 0.3p.u., respectively. Furthermore, the voltage magnitudes of generator buses are assumed to vary in the range [0.9, 1.1] p.u. and the lower and upper limits of load buses are considered to be 0.94p.u. and 1.06p.u., respectively. In this system, the maximum iteration number  $g_{max}$  is set as 1000.

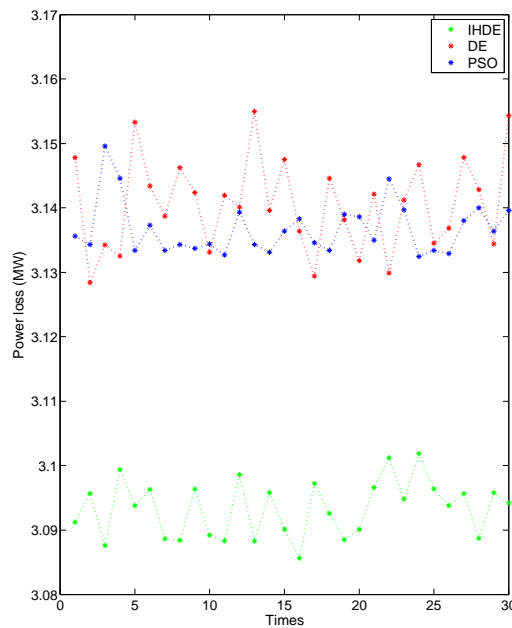


Fig. (4). The results' distribution for Case2 of IEEE 30-bus system.

#### 5.2.1. Case 3: Minimization of Quadratic Cost Function

The objective function is defined as case 1. The optimal control variables of IHDE, DE, and PSO for case 3 are shown in Table 6 and the comparison of the results obtained from other methods reported in the literatures are shown in Table 7. As shown in the tables, the minimization of quadratic cost obtained by IHDE, DE, and PSO for case 3 are 41667.99 \$/h, 41699.16 \$/h and 41702.78 \$/h, respectively, and the result of IHDE is better than DE, PSO, ABC [7], ICBO [1], DSA [12] and MSA [11]. The Table 6 demonstrates IHDE algorithm can get less cost than other intelligent algorithms in the feasible space. Moreover, Fig. (5) shows the optimal convergence curves and Fig. (6) shows the results in 30 independent simulations of IHDE, DE, and PSO for case 3.

Table 6. Optimal solutions for case 3 on IEEE 57 system.

Control variables	Case 3		
	IHDE	DE	PSO
$P_1$ (MW)	142.6683	143.2926	148.6137
$P_2$ (MW)	88.85812	88.70183	91.71634
$P_3$ (MW)	45.05665	45.06715	40.88899
$P_6$ (MW)	72.40753	68.82424	63.90829
$P_8$ (MW)	460.4850	459.7235	468.2037
$P_9$ (MW)	96.94107	98.92860	100.0000
$P_{12}$ (MW)	359.3236	361.7904	353.3791
$V_1$ (p.u.)	1.064728	1.024065	1.046639
$V_2$ (p.u.)	1.062703	1.021103	1.044469
$V_3$ (p.u.)	1.056291	1.012882	1.040994

(Table 6) contd....

Control variables	Case 3		
	IHDE	DE	PSO
$V_6$ (p.u.)	1.061397	1.026209	1.060628
$V_8$ (p.u.)	1.072778	1.041820	1.072790
$V_9$ (p.u.)	1.047832	1.011943	1.038355
$V_{12}$ (p.u.)	1.050065	1.008183	1.031182
$T_{4-18}$ (p.u.)	1.035900	1.035200	0.956400
$T_{4-18}$ (p.u.)	0.920300	0.910300	1.046200
$T_{21-20}$ (p.u.)	1.016100	1.021000	1.051500
$T_{24-25}$ (p.u.)	0.979300	1.099900	1.099300
$T_{24-25}$ (p.u.)	1.054800	0.982600	0.957100
$T_{24-26}$ (p.u.)	1.026000	1.033100	1.044100
$T_{7-29}$ (p.u.)	0.996500	0.965000	1.023800
$T_{34-32}$ (p.u.)	0.972800	0.972400	0.980600
$T_{11-41}$ (p.u.)	0.904800	0.905200	0.952500
$T_{15-45}$ (p.u.)	0.981100	0.941700	0.975100
$T_{14-46}$ (p.u.)	0.965900	0.923900	0.970800
$T_{10-51}$ (p.u.)	0.975700	0.938600	0.978900
$T_{13-49}$ (p.u.)	0.929600	0.900200	0.935900
$T_{11-43}$ (p.u.)	0.973100	0.930800	0.972200
$T_{40-56}$ (p.u.)	1.018300	1.020200	1.016700
$T_{39-57}$ (p.u.)	0.956700	0.971500	0.990800
$T_{9-55}$ (p.u.)	0.993100	0.972600	0.993300
$Q_{C18}$ (p.u.)	0.002000	0.129400	0.093600
$Q_{C25}$ (p.u.)	0.139900	0.163200	0.092400
$Q_{C53}$ (p.u.)	0.125600	0.140200	0.132600
Fuel cost (\$/h)	41667.99	41699.16	41702.78

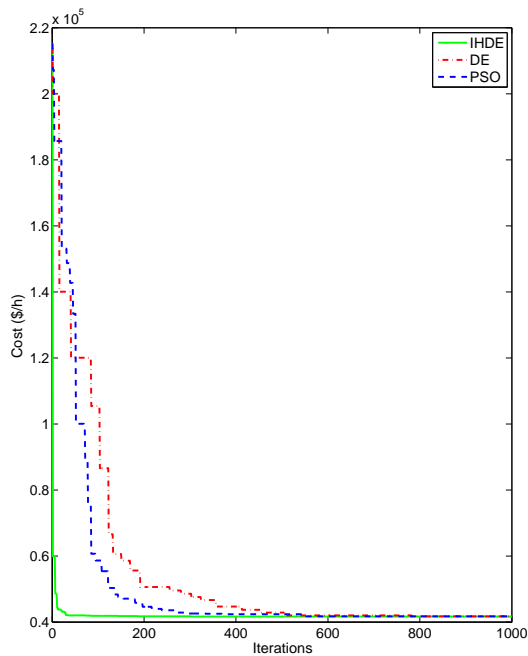
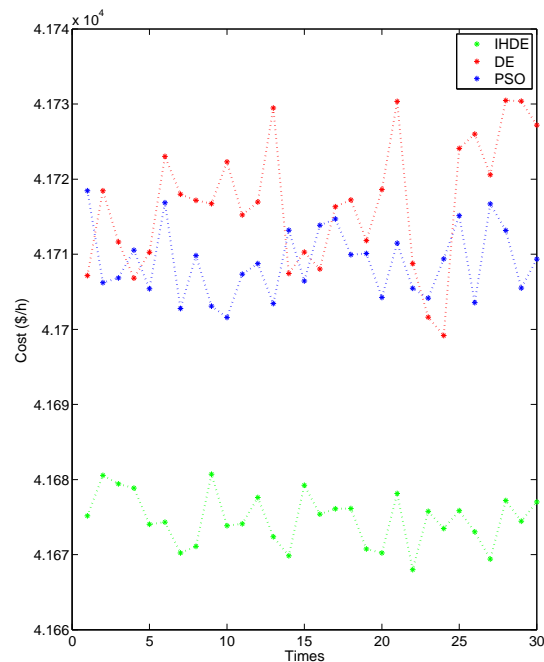


Fig. (5). The optimal convergence curves for Case 3 of IEEE 57-bus system.

**Table 7. Comparison of the simulation results for Case 3 on IEEE 57 system (\$/h).**

Algorithms	Min
IHDE	41667.9900
DE	41699.1600
PSO	41702.7800
ABC [7]	41693.9589
ICBO [1]	41697.3324
DSA [12]	41686.8200
MSA [11]	41673.7231

**Fig. (6).** The results' distribution for Case3 of IEEE 57-bus system.

## CONCLUSION

An improved hybrid differential evolution algorithm (IHDE) based on DE and PSO has been proposed in this paper to solve the optimal power flow problem which is a large-scale, multi-constrained and nonlinear optimization problem. In order to show the practicability of the proposed algorithm, the generator quadratic cost and the transmission real power losses objective functions are considered for the OPF problem. Then the IHDE, DE and PSO are tested on the two systems: IEEE30-bus system and IEEE57-bus system. Furthermore, the results obtained from the IHDE algorithm are compared with DE, PSO and other methods reported in the recent literatures. As the simulation results indicated, the IHDE algorithm can solve the OPF problem successfully and has better robustness and convergence characteristics while getting the solutions with high quality.

## CONSENT FOR PUBLICATION

Not applicable.

## CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

## ACKNOWLEDGEMENTS

The authors would like to thank the editors and the reviewers for their constructive comments. This work was supported by the National Natural Science Foundation of China (Nos. 51507024 and 61263030) and Science and Technology Research Project of Chongqing Municipal Education Commission (No. KJ1500401).

## REFERENCES

- [1] H.R. Boucekara, A.E. Chaib, M.A. Abido, and R.A. El-Sehiemy, "Optimal power flow using an Improved Colliding Bodies Optimizationalgorithm", *Appl. Soft Comput.*, vol. 42, pp. 119-131, 2016.  
[http://dx.doi.org/10.1016/j.asoc.2016.01.041]
- [2] S.S. Reddy, and P.R. Bijwe, "Efficiency improvements in meta-heuristic algorithms to solve the optimal power flow problem", *Elect. Power Energy Sys.*, vol. 82, pp. 288-302, 2016.  
[http://dx.doi.org/10.1016/j.ijepes.2016.03.028]
- [3] B. Mota-Palomino, and V.H. Quintana, "Sparse Reactive Power Scheduling by a Penalty Function - Linear Programming Technique", *Ieee T Power Syst.*, vol. 1, pp. 31-39, 1986.  
[http://dx.doi.org/10.1109/TPWRS.1986.4334951]
- [4] R. Shoults, and D. Sun, "Optimal Power Flow Based Upon P-Q Decomposition", *IEEE Trans. Power Apparatus Syst.*, vol. PAS-101, pp. 397-405, 1982.  
[http://dx.doi.org/10.1109/TPAS.1982.317120]
- [5] J. Carpentier, "Contribution a l'Etude du Dispatching Economique", *Bulletin de la Societe Francaise des Electriciens*, vol. 3, pp. 431-474, 1962.
- [6] R.P. Singh, V. Mukherjee, and S.P. Ghoshal, "Particle swarm optimization with an aging leader and challengers algorithm for the solution of optimal power flow problem", *Appl. Soft Comput.*, vol. 40, pp. 161-177, 2016.  
[http://dx.doi.org/10.1016/j.asoc.2015.11.027]
- [7] M.R. Adaryani, and A. Karami, "Artificial bee colony algorithm for solving multi-objective optimal power flow problem", *Elect. Power Energy Sys.*, vol. 53, pp. 219-230, 2013.  
[http://dx.doi.org/10.1016/j.ijepes.2013.04.021]
- [8] M. Ghasemi, S. Ghavidel, M. Gitizadeh, and E. Akbari, "An improved teaching-learning-based optimization algorithm using Lévy mutation strategy for non-smooth optimal power flow", *Elect. Power Energy Sys.*, vol. 65, pp. 375-384, 2015.  
[http://dx.doi.org/10.1016/j.ijepes.2014.10.027]
- [9] P. Roy, and C. Paul, "Optimal power flow using krill herd algorithm", *Inte. Transac. Elect. Energy Sys.*, vol. 25, pp. 1397-1419, 2014.  
[http://dx.doi.org/10.1002/etep.1888]
- [10] A.R. Kumar, and L. Premalatha, "Optimal power flow for a deregulated power system using adaptive real coded biogeography-based optimization", *Elect. Power Energy Sys.*, vol. 73, pp. 393-399, 2015.  
[http://dx.doi.org/10.1016/j.ijepes.2015.05.011]
- [11] A.A. Mohamed, Y.S. Mohamed, and A.A. El-Gaafary, "Optimal power flow using moth swarm algorithm", *Electr. Power Syst. Res.*, vol. 142, pp. 190-206, 2017.  
[http://dx.doi.org/10.1016/j.epsr.2016.09.025]
- [12] K. Abaci, and V. Yamacli, "Differential search algorithm for solving multi-objective optimal power flow problem", *Elect. Power Energy Sys.*, vol. 79, pp. 1-10, 2016.  
[http://dx.doi.org/10.1016/j.ijepes.2015.12.021]
- [13] M. Basu, "Improved differential evolution for economic dispatch", *Elect. Power Energy Sys.*, vol. 63, pp. 855-861, 2014.  
[http://dx.doi.org/10.1016/j.ijepes.2014.07.003]
- [14] R.P. Parouha, and K.N. Das, "A memory based differential evolution algorithm for unconstrained optimization", *Appl. Soft Comput.*, vol. 38, pp. 501-517, 2016.  
[http://dx.doi.org/10.1016/j.asoc.2015.10.022]
- [15] G. Chen, S. Huang, and Z. Sun, "A Chaotic Quantum Behaved Particle Swarm Optimization Algorithm for Short-term Hydrothermal Scheduling", *Open Elect. Electro. Engi. J.*, vol. 11, pp. 23-37, 2017.  
[http://dx.doi.org/10.2174/1874129001711010023]
- [16] S. Xie, R. Zhai, X. Liu, B. Li, K. Long, and Q. Ai, "Self-adaptive Genetic Algorithm and Fuzzy Decision Based Multiobjective Optimization in Microgrid with DGs", *Open Elect. Electro. Engi. J.*, vol. 10, pp. 46-57, 2016.  
[http://dx.doi.org/10.2174/1874129001610010046]
- [17] X. Wang, C. Wang, and Q. Li, "Short-term Wind Power Prediction Using GA-ELM", *Open Elect. Electro. Engi. J.*, vol. 11, pp. 48-56, 2017.  
[http://dx.doi.org/10.2174/1874129001711010048]
- [18] G. Chen, L. Liu, Z. Zhang, and S. Huang, "Optimal reactive power dispatch by improved GSA-based algorithm with the novel strategies to handle constraints", *Appl. Soft Comput.*, vol. 50, pp. 58-70, 2017.  
[http://dx.doi.org/10.1016/j.asoc.2016.11.008]
- [19] G. Li, Q. Lin, L. Cui, Z. Du, Z. Liang, and J. Chen, "A novel hybrid differential evolution algorithm with modified CoDE and JADE", *Appl. Soft Comput.*, vol. 47, pp. 577-599, 2016.  
[http://dx.doi.org/10.1016/j.asoc.2016.06.011]
- [20] Z. Zhao, J. Yang, Z. Hu, and H. Che, "A differential evolution algorithm with self-adaptive strategy and control parameters based on symmetric Latin hypercube design for unconstrained optimization problems", *Eur. J. Oper. Res.*, vol. 250, pp. 30-45, 2016.  
[http://dx.doi.org/10.1016/j.ejor.2015.10.043]

- [21] O. Alsac, and B. Stott, "Optimal Load Flow with Steady-State Security", *IEEE Trans. Power Apparatus Syst*, vol. 93, pp. 745-751, 1974. [<http://dx.doi.org/10.1109/TPAS.1974.293972>]
- [22] M. Ghasemi, S. Ghavidel, M.M. Ghanbarian, and M. Gitizadeh, "Multi-objective optimal electric power planning in the power system using Gaussian bare-bones imperialist competitive algorithm", *Inf. Sci.*, vol. 294, pp. 286-304, 2015. [<http://dx.doi.org/10.1016/j.ins.2014.09.051>]
- [23] S. Duman, U.U. Güvenç, Y. Sönmez, and N. Yörükeren, "Optimal power flow using gravitational search algorithm", *Energy Convers. Manage.*, vol. 59, pp. 86-95, 2012. [<http://dx.doi.org/10.1016/j.enconman.2012.02.024>]
- [24] A. Bhattacharya, and P.K. Chattopadhyay, "Application of biogeography-based optimisation to solve different optimal power flow problems", *IET Gener. Transm. Distrib.*, vol. 5, pp. 70-80, 2011. [<http://dx.doi.org/10.1049/iet-gtd.2010.0237>]
- [25] S. S, and S. K.S, "Multi-objective harmony search algorithm for optimal power flow problem", *Int J Elec Power*, vol. 33, pp. 745-752, 2011. [<http://dx.doi.org/10.1016/j.ijepes.2010.12.031>]
- [26] R.D. Zimmerman, C.E. Murillo-Sánchez, and R.J. Thomas, Matpower, Available: <http://www.pserc.cornell.edu/matpower>

---

© 2017 Chen *et al.*

This is an open access article distributed under the terms of the Creative Commons Attribution 4.0 International Public License (CC-BY 4.0), a copy of which is available at: <https://creativecommons.org/licenses/by/4.0/legalcode>. This license permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.