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Impact of Clustering on the BER Performance of Ad Hoc Wireless Networks

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Abstract: Ad hoc wireless networks are characterized by multi-hop radio communications. The spatial distribution of the nodes is seldom perfectly regular. In particular, in a realistic ad hoc wireless network communication scenario, the nodes are likely to be clustered, *i.e.*, to configure themselves in subgroups such that the nodes inside each subgroup are relatively close to each other with respect to the distance between different subgroups. In this paper, we consider a very simple clustering scenario, defined as "uniformly clustered," which allows to derive a parameterized analytical description. The proposed clustering model, although simple and idealistic, allows to gain insights valid also in a more general case with non-regular clustering. In particular, the obtained results highlight the fact that a single long hop can significantly degrade the network communication performance and quantify this performance degradation. Topology-dependent power control is then proposed, and its advantages are evaluated.

1. INTRODUCTION

In recent years, a lot of attention has been attracted by ad hoc wireless networks, because of multiple potential applications, both civilian and military. Various approaches have appeared in the literature for the study of this type of networks. Most of these studies focus on *routing* [1, 2], but an *information-theoretic* analysis has also been proposed [3]. In [4], a novel *communication-theoretic* approach to the analysis of ad hoc wireless networks has been introduced, in which the relationship between physical and medium access control (MAC) layers is evaluated.

In [4], a regular node spatial distribution, where the nodes are at the vertices of a square grid, is first considered. Such a distribution, although very useful to understand the dynamics of multi-hop radio communication and the impact of physical layer characteristics on the upper layers, is unrealistic. Considering, as an example, the case of a *smart dust*-type sensor network [5], where nodes may be literally thrown over the terrain, it is very likely that the final distribution of the nodes will be irregular. This irregularity significantly affects the connectivity of the network [6].

The study of the performance of an ad hoc wireless network with random node distribution, and thus random clustering, requires a statistical analysis and usually entails the use of computer simulations [4, 7]. Moreover, the identification of disjoint clusters could be problematic as well. In order to gain insights regarding the impact of clustering on the performance of multi-hop ad hoc wireless networks, in this paper we impose some regularity in the cluster distribution. The obtained network topology, referred to as *uniformly clustered*, will be completely characterized by only two parameters. The considered topology constraints, although idealistic, lead to a simple parameterized analytical model which compactly allows to evaluate the network performance. In particular, the bit error rate (BER) at the end of a multi-hop communication route is analyzed. Moreover, a meaningful comparison between the performance in a uniformly clustered network communication scenario and that in a regular (square grid) network communication scenario is proposed. Our results show that a single "long" inter-cluster hop can significantly degrade the performance. A simple power control strategy is proposed to combat the negative effects of clustering. The proposed approach can describe many realistic situations, especially for sensor networks. In fact, it is very likely that these networks will be clustered and that regularity inside each cluster may be deliberately introduced (*e.g.*, a seismic sensor network, where sensors concentrate in specific regions of a wide area).

The remainder of this paper is structured as follows. In Section 2, preliminary assumptions regarding the considered ad hoc wireless network communication scenario are presented. In Section 3, basic characteristics of the packetized circuit-switched ad hoc wireless network communication model proposed in [4] are recalled. In Section 4, uniformly clustered ad hoc wireless networks are proposed: a parameterized model is introduced and an expression for the BER at the end of an average multi-hop route is derived. Numerical results are presented in Section 5, and concluding remarks are finally given in Section 6.

2. PRELIMINARY ASSUMPTIONS

In the following, we enumerate basic assumptions for the considered ad hoc wireless network communication model.

- Peer-to-peer communications are considered.
- The packetized circuit-switched ad hoc wireless network communication model introduced in [4] is considered. In particular, a source node, in need of communicating with

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a destination node, after reserving a multi-hop route to its destination, *reserves* the intermediate relay nodes for the entire transmission. The intermediate nodes are released once the entire message has been transmitted.

- Different multi-hop routes are *disjoint*. In other words, a node can not serve as a relay in more than one route.
- The *route discovery phase*, based on broadcast percolation [6, 8], is not explicitly considered, since it goes beyond the scope of the paper. In this paper, a simple routing strategy will be considered, which can be concisely described as follows. Two types of hop are possible: (i) *intracluster* hop, between two neighbors in the same cluster; (ii) *inter-cluster* hop, between two nodes in adjacent clusters. In particular, we will assume that two adjacent clusters communicate through the nodes placed at their centers. This does not correspond to the most effective routing strategy. For instance, the most desirable inter-cluster communication should be between the two closest nodes in the two clusters, and it is obvious that these two nodes will not be in the centers of the clusters.
- The nodes are fixed. This is meaningful for the case of a wireless sensor network where the sensors are static (*e.g.*, fixed sensors monitoring environmental parameters).
- A node can start transmitting only after reserving a multi-hop to the desired destination. In other words, there is *no buffer* at a node. This implies that the considered network communication scenarios are not affected by instability phenomena. Such a communication paradigm could describe a situation where a sensor node, after reserving a multi-hop route to its destination (which could correspond to a "sink-node" collecting the information generated by the various nodes in the cluster), measures physical quantities of interest (*e.g.*, temperature, density, friction, pressure, etc.) and transmits this data in real-time.
- We assume that there is no inter-node interference (INI). This allows to isolate the effect of clustering and would correspond to a scenario where only a single source/ destination pair is active at a time. Extensions of the approach proposed in this paper to a more realistic case with INI and use of specific MAC protocols, can be dealt with by using the techniques introduced in [4].

3. AD HOC WIRELESS NETWORKS WITH REGULAR NODE SPATIAL DISTRIBUTION

We assume that N nodes are placed at the vertices of a square grid inside a circular area A. Defining by $\rho_s \triangleq N/A$ the node spatial density, it is possible to show that the minimum inter-node distance can be written as $r_{\text{link}} \approx 1/\sqrt{\rho_s}$ [4]. Indicating by BER_{link} the BER at the end of a single link, assuming that (i) there is regeneration (*i.e.*, detection and possibly error correction) at each intermediate node, and that (ii) the uncorrected errors made in successive links accumulate, it is possible to show that the BER at the end of the n-th link of a multi-hop route, indicated by $\text{BER}^{(n)}$, can be expressed as

$$\text{BER}^{(n)} \approx 1-(-\text{BER}_{\text{link}})^n \tag{1}$$

An average BER expression can be obtained by evaluating (1) in correspondence to an average number of hops $\bar{n}_h$. Assuming that the number of hops n_h can be described as a discrete random variable uniformly distributed between one and the maximum number over a diameter of the circular network area, it is possible to show that $\bar{n}_h \triangleq \lfloor \sqrt{N/\pi} \rceil$ [4], where the notation $\lfloor * \rceil$ indicates the integer value closest to $*$. Note that the link BER depends on the SNR at the ending node of the link, indicated as SNR_{link}, and on the characteristics of the transmission channel.

In the rest of this paper, we will assume that the signal is transmitted over an additive white Gaussian noise (AWGN) channel and is affected by free-space loss. Hence, according to Friis free space formula [9], the received signal power at distance from the transmitter, indicated by $P_r^{(d)}$, can be expressed as follows:

$$P_r^{(d)} = \frac{\alpha P_t}{d^2} \approx \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_{\text{loss}} f_c^2 d^2} \tag{2}$$

where: P_t is the transmit power from each node; G_t and G_r are the transmitter and receiver antenna gains, respectively; f_c is the carrier frequency; c is the speed of light, and $f_{\text{loss}} \geq 1$ is a loss factor. As stated in Section 2, we consider an ideal communication scenario where there is no INI. In this case, the only noise at the receiver is represented by *thermal noise*, and the corresponding noise power can be written as $P_{\text{thermal}} = FkT_0B$, where F is the noise figure [9], $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann's constant, T_0 is the room temperature ($T_0 \approx 300$ K), and B is the transmission bandwidth. In this case, the link SNR can be written as follows:

$$\text{SNR}_{\text{link}} = \frac{P_c^{(r_{\text{link}})}}{P_{\text{thermal}}} \approx \frac{\alpha P_t \rho_s}{FkT_0 B}. \tag{3}$$

In the remainder of the paper, uncoded binary phase shift keying (BPSK) [10] will be the considered modulation format, and the link BER thus is

$$\text{BER}_{\text{link}} = Q\left(\sqrt{2\text{SNR}_{\text{link}}}\right) = Q\left(\sqrt{\frac{2\alpha P_t \rho_s}{FkT_0 R_b}}\right) \tag{4}$$

where the fact that the 3-dB bandwidth B is equal to the transmission data-rate R_b has been used. Moreover, we will assume that $G_t = G_r = 1$ (omnidirectional antennas), $f_{\text{loss}} = 1$ (no losses not related to propagation), $f_c = 2.4$ GHz, and $F = 6$ dB.

4. UNIFORMLY CLUSTERED AD HOC WIRELESS NETWORKS

Considering a global circular network area A, in a realistic network communication scenario, nodes could organize themselves in randomly shaped clusters, indicated by the shaded regions in Fig. (**1a**). An analysis of such a randomly clustered network communication scenario requires a statistical model of the node distribution and involves computer simulations [4, 7]. Moreover, it is

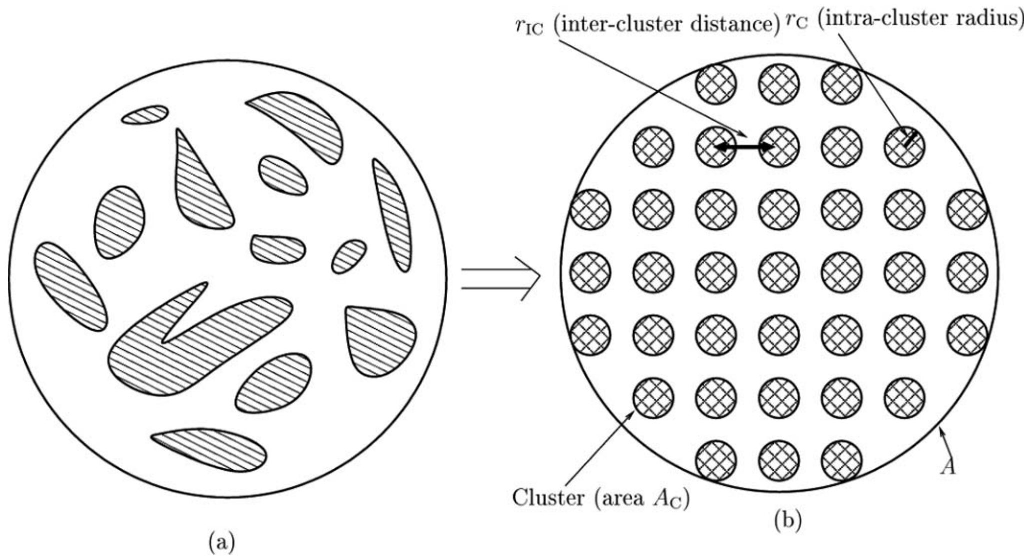

Fig. (1). Clustered ad hoc wireless networks: (**a**) random and (**b**) *uniformly clustered*.

extremely difficult to model analytically the shapes of non-regular clusters. In order to derive a simple analytical model, we impose a geometric regularity in the cluster structure. In particular, we assume that: (i) all the clusters are circular and have the same dimension; (ii) the centers of the clusters are at the vertices of a square grid. This topology is depicted in Fig. (**1b**) and will be referred to as *uniformly clustered*. We further assume that inside each cluster the nodes are distributed over a regular grid—in other words, each cluster is a small-scale version of a uniform ad hoc wireless network.

4.1. Uniformly Clustered Network Topology Parameters

A uniformly clustered node topology can be simply characterized by the following distances (also indicated in Fig. (**1b**)).

- The *inter-cluster* distance, indicated as r_{IC} and corresponding to the distance between the centers of two neighboring clusters. This distance is formally defined as

$$r_{IC} \triangleq \frac{r_A}{\Gamma_{IC}} \tag{5}$$

where $r_A \triangleq \sqrt{A/\pi}$ is the radius of the overall circular area A and $\Gamma_{IC} \geq 1$ is a parameter which quantifies how many clusters lie over a radius of the global area.

- The *radius* of a cluster, indicated as r_C, and defined (considering the inter-cluster distance r_{IC} as a reference) as

$$r_C \triangleq \frac{r_{IC}}{\Gamma_C} = \frac{r_A}{\Gamma_{IC}\Gamma_C} \tag{6}$$

where the parameter $\Gamma_{IC} \geq 2$ quantifies how small is a cluster compared to the inter-cluster distance.

In order to get a better idea about the meaning of the parameters Γ_{IC} and Γ_C, in Fig. (**2**) the clustered structures corresponding to a few combinations of Γ_{IC} and Γ_C are shown. Note that, by simply changing the values of Γ_{IC} and Γ_C, we can characterize many significant clustering situations.

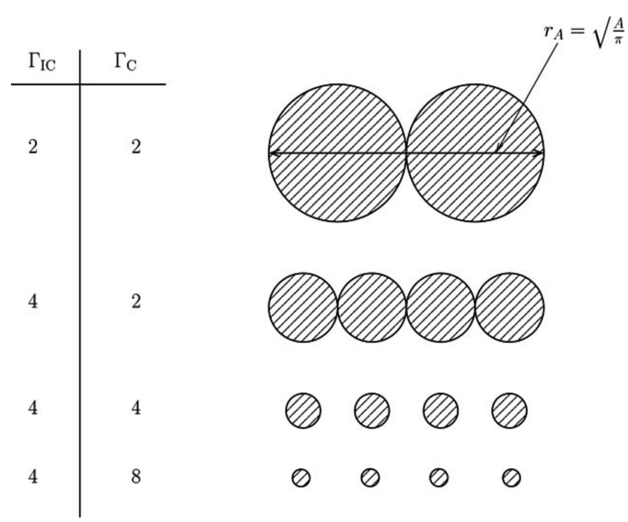

Fig. (2). Realizations of uniformly clustered topologies for particular values of the parameters Γ_C and Γ_{IC}.

Upon the introduction of the parameters Γ_{IC} and Γ_C in (5) and (6), relevant quantities for performance analysis can be computed. In particular, the cluster area A_C can be written as

$$A_C = \pi r_C^2 = \pi \left(\frac{r_{IC}}{\Gamma_C}\right)^2. \tag{7}$$

By associating to each cluster a square "tile" of side r_{1C} and neglecting border effects, the ensemble of the cluster tiles should approximately cover the entire area A. Hence, it is possible to write the total number of clusters as

$$T_{\mathrm{C}} \approx \left\lfloor \frac{A}{r_{\mathrm{IC}}^2} \right\rceil - \left\lfloor \frac{\pi r_A^2}{r_{\mathrm{IC}}^2} \right\rceil = \lfloor \pi \Gamma_{\mathrm{IC}}^2 \rceil. \tag{8}$$

Assuming that the nodes are equally distributed among the various clusters, the average number of nodes per cluster T_{C} is

$$N_{\mathrm{C}} = \frac{N}{T_{\mathrm{C}}} = \left\lfloor \frac{N}{\pi \Gamma_{\mathrm{1C}}^2} \right\rceil \tag{9}$$

and the cluster node spatial density, indicated by $\rho_{\mathrm{s}}^{\mathrm{C}}$, can be written as

$$\rho_{\mathrm{s}}^{\mathrm{C}} = \frac{N_{\mathrm{C}}}{T_{\mathrm{C}}} = \frac{N}{\pi \Gamma_{\mathrm{IC}}^2} \frac{1}{\frac{\pi r_{\mathrm{IC}}^2}{\Gamma_{\mathrm{C}}^2}} = \frac{1}{\pi} \rho_{\mathrm{s}} \Gamma_{\mathrm{C}}^2 \tag{10}$$

where ρ_{s} is the overall node spatial density. Imposing that the number of nodes N and the overall area A are the same in both the cases of uniform and uniformly clustered node distributions, equation (10) allows to directly relate the two topologies. We point out that this is a possible way of relating uniform and uniformly clustered topologies. In fact, the assumption that N and A are the same in both topologies implies that the cluster node spatial density in a uniformly clustered network is higher than that in a network with uniform topology. Another (equivalent) perspective to compare uniform and uniformly clustered topologies could be that of fixing $\rho_{\mathrm{s}} = \rho_{\mathrm{s}}^{\mathrm{C}}$: in this case, it is obvious that the performance of a uniformly clustered network can not be better than that of a uniform network.

In the remainder of this paper, the two possible network topologies introduced above (*i.e.*, uniform and uniformly clustered) will be compared according to the latter proposed perspective. Due to the cluster uniformity, the relation between ρ_{s} and $\rho_{\mathrm{s}}^{\mathrm{C}}$ depends only on the parameter Γ_{C}-should clustering be more general, it would be very difficult to concisely describe this relation with a single parameter. Recalling that inside each cluster the node distribution is perfectly uniform, it is possible to conclude that the cluster radius can be written as

$$r_{\mathrm{link}}^{\mathrm{C}} \approx \frac{1}{\sqrt{\rho_{\mathrm{s}}^{\mathrm{C}}}} = \frac{\sqrt{\pi}}{\Gamma_{\mathrm{C}} \sqrt{\rho_{\mathrm{s}}}}. \tag{11}$$

Since $A_{\mathrm{C}} = \pi r_{\mathrm{C}}^2$ and $r_{\mathrm{link}}^{\mathrm{C}} \approx \sqrt{A_{\mathrm{C}} / N_{\mathrm{C}}}$, the relationship between the inter-cluster distance and the cluster radius is the following:

$$r_{\mathrm{link}}^{\mathrm{C}} \approx \frac{\pi \Gamma_{\mathrm{IC}}}{\sqrt{N}} r_{\mathrm{C}}. \tag{12}$$

4.2. BER at the End of a Multi-Hop Path in a Uniformly Clustered Network

In general, a node in a cluster might want to communicate with a node in another cluster. Depending on the routing strategy, a multi-hop path could cross a few clusters. Recall from Section 2 that (i) each "short" hop (inside a cluster) is between neighboring nodes and that (ii) each "long" hop is between the central nodes of neighboring clusters. The intra-cluster and inter-cluster link BERs are indicated by $\mathrm{BER}_{\mathrm{C}}$ and $\mathrm{BER}_{\mathrm{IC}}$, respectively. According to the assumption of signal regeneration at intermediate nodes used to derive (1), since in a uniformly clustered topology there can be two types of hops (long or short), the final BER can be written as

$$\mathrm{BER}_{\mathrm{CI}} = 1 - (1 - \mathrm{BER}_{\mathrm{IC}})^{n_{\mathrm{h}}^{\mathrm{IC}}} (1 - \mathrm{BER}_{\mathrm{C}})^{n_{\mathrm{h}}^{\mathrm{C}}} \tag{13}$$

where $n_{\mathrm{h}}^{\mathrm{IC}}$ and $n_{\mathrm{h}}^{\mathrm{C}}$ indicate the number of intra-cluster (short) and inter-cluster (long) hops. In order to make a direct comparison with the case of perfectly uniform node distribution, we assume that the average number of hops $\bar{n}_{\mathrm{h}}$ remains the same in both cases,[1] *i.e.*, $\bar{n}_{\mathrm{h}} = \lfloor \sqrt{N/\pi} \rceil$. In other words, (13) can rewritten as follows:

$$\mathrm{BER}_{\mathrm{C1}}^{(n_{\mathrm{h}}^{\mathrm{IC}})} = 1 - (1 - \mathrm{BER}_{\mathrm{IC}})^{n_{\mathrm{h}}^{\mathrm{IC}}} (1 - \mathrm{BER}_{\mathrm{C}})^{\bar{n}_{\mathrm{h}} - n_{\mathrm{h}}^{\mathrm{IC}}} \tag{14}$$

where we have explicitly indicated $n_{\mathrm{h}}^{\mathrm{IC}}$ as a parameter.

The received powers at the end of an intra-cluster link $(P_{\mathrm{r}}^{\mathrm{C}})$ and at the end of an inter-cluster link $(P_{\mathrm{r}}^{\mathrm{IC}})$ can be written, respectively, as

$$P_r^{\mathrm{C}} = \frac{\alpha P_{\mathrm{t}}^{\mathrm{C}}}{(r_{\mathrm{link}}^{\mathrm{C}})^2} \approx \frac{\alpha P_{\mathrm{t}}^{\mathrm{C}}}{\frac{\pi}{\Gamma_{\mathrm{C}}^2 \rho_{\mathrm{s}}}} = \frac{\Gamma_{\mathrm{C}}^2}{\pi} \alpha \rho_{\mathrm{s}} P_{\mathrm{t}}^{\mathrm{C}} \tag{15}$$

$$P_r^{\mathrm{IC}} = \frac{\alpha P_{\mathrm{t}}^{\mathrm{IC}}}{r_{\mathrm{IC}}^2} = \frac{\alpha P_{\mathrm{t}}^{\mathrm{IC}}}{\Gamma_{\mathrm{C}}^2 r_{\mathrm{C}}^2} \approx \frac{\alpha P_{\mathrm{t}}^{\mathrm{IC}}}{\Gamma_{\mathrm{C}}^2 \frac{N (r_{\mathrm{link}}^{\mathrm{C}})^2}{\pi^2 \Gamma_{\mathrm{IC}}^2}} = \frac{\pi \Gamma_{\mathrm{IC}}^2}{N} \alpha \rho_{\mathrm{s}} P_{\mathrm{t}}^{\mathrm{IC}} \tag{16}$$

where $P_{\mathrm{t}}^{\mathrm{C}}$ and $P_{\mathrm{t}}^{\mathrm{IC}}$ represent the intra-cluster and inter-cluster transmit powers, respectively. As it will be shown in Section 5, the use of different transmit powers for intra-cluster and inter-cluster communications can improve the BER performance of uniformly clustered ad hoc wireless networks. In the case of uncoded BPSK signaling, the intra-cluster and inter-cluster link BER expressions become:

$$\mathrm{BER}_{\mathrm{C}} = Q\left(\sqrt{\frac{2\alpha \Gamma_{\mathrm{C}}^2 \rho_{\mathrm{s}} P_{\mathrm{t}}^{\mathrm{C}}}{\pi F k T_0 R_{\mathrm{b}}}} \right) \tag{17}$$

$$\mathrm{BER}_{\mathrm{IC}} = Q\left(\sqrt{\frac{2\alpha \pi \Gamma_{\mathrm{IC}}^2 \rho_{\mathrm{s}} P_{\mathrm{t}}^{\mathrm{IC}}}{N F k T_0 R_{\mathrm{b}}}} \right). \tag{18}$$

In order to compare the performance in the case of a uniformly clustered distribution with that obtained in the case of a uniform distribution, it is expedient to rewrite (17)-

[1]Note that in the case of a clustered node distribution, the average number of hops is strongly dependent on the topology information available at each node.

(18) as functions of the cluster node spatial density ρ_s^C. Recalling that in a uniformly clustered ad hoc wireless network $\rho_s = \pi\rho_s^C / \Gamma_C^2$, one obtains:

$$\mathrm{BER}_C = Q\left(\sqrt{\frac{2\alpha\rho_s^C P_t^C}{FkT_0R_b}}\right) \quad (19)$$

$$\mathrm{BER}_{IC} = Q\left(\sqrt{\frac{2\alpha\pi^2\Gamma_{IC}^2\rho_s^C P_t^{IC}}{FkT_0N\Gamma_C^2R_b}}\right). \quad (20)$$

4.3. Average BER Performance

Since an ad hoc wireless network does not have a hierarchical structure, but rather possesses a flat architecture, it is of interest to derive an average expression for the BER, determining an average number of inter-cluster and intra-cluster hops. Since a single cluster can be considered as a small-scale version of a perfectly uniform ad hoc wireless network, the average number of hops *inside a cluster* can be written as

$$\bar{n}_h^C \triangleq \left\lceil\sqrt{\frac{N_C}{\pi}}\right\rceil = \left\lceil\sqrt{\frac{N}{\pi\Gamma_{IC}}}\right\rceil. \quad (21)$$

The destination node may often be in a cluster different from the cluster containing the source node. Several *inter-cluster* hops between neighboring clusters are thus necessary for a packet to reach its final destination. Considering an average number of hops $\bar{n}_h^C$ inside each cluster (including the source cluster and the destination cluster), we can derive the average number of inter-cluster hops, indicated as $\bar{n}_h^{IC}$. In particular, it must hold that

$$\bar{n}_h^{IC} + (\bar{n}_h^C + 1)\bar{n}_h^C = \bar{n}_h = \left\lceil\sqrt{\frac{N}{\pi}}\right\rceil \quad (22)$$

from which it is possible to derive[2]

$$\bar{n}_h^{IC} = \left\lceil\frac{\sqrt{\pi}\Gamma_{IC} - 1}{1 + \frac{\pi\Gamma_{IC}}{\sqrt{N}}}\right\rceil. \quad (23)$$

We observe that, for large N, (23) can be simplified as

$$\bar{n}_h^{IC} \approx \left\lceil\sqrt{\pi}\Gamma_{IC} - 1\right\rceil. \quad (24)$$

From the derived formulas, it is immediate to recognize that the "geometry" of an average communication path depends only on the parameter Γ_{IC}, whereas the parameter Γ_C affects the node spatial density inside each cluster. In Table **1**, we show a few numerical examples, relative to the clustered structure and the characteristics of an average communication route, corresponding to various values of the parameter Γ_{IC}. Finally, the average BER in a uniformly clustered network, indicated as $\overline{\mathrm{BER}}_{C1}$, can be written as follows:

$$\overline{\mathrm{BER}}_{C1} \approx 1 - (1 - \mathrm{BER}_{IC})^{\bar{n}_h^{IC}}(1 - \mathrm{BER}_C)^{(\bar{n}_h^{IC}+1)n_h^{IC}}. \quad (25)$$

[2]The integer part operation is considered only on the final result, but it is not considered during the intermediate calculations.

Table 1. Examples of Uniformly Clustered Distribution for Various Values of *N* and Γ_{IC}

N	Γ_{IC}	T_C	N_C	$\bar{n}_h^C$	$\bar{n}_h^{IC}$
10^3	2	12	79	5	2
10^3	4	50	18	2	5
10^3	8	201	5	1	8
10^6	2	12	7957	50	2
10^6	4	50	1989	25	5
10^6	8	201	497	12	12
10^6	16	804	124	6	24

We observe that a routing strategy such that $\bar{n}_h^C$ hops are made inside each intermediate cluster between the source cluster and the destination cluster is likely not to be optimized. However, specific path selection (depending on the source/destination pair) would make a unified parameterized network model extremely difficult. It is reasonable to assume that if a single node, in each cluster, is in charge of relaying messages to adjacent clusters, then this node should be the central node of the cluster.

5. NUMERICAL RESULTS

The performance of a uniformly clustered ad hoc wire less network is then evaluated in several situations, in terms of BER versus node spatial density. The extension of the current analysis in other directions (*e.g.*, to evaluate the *average sustainable number of hops*, relative to the connectivity level, in clustered node distributions) can be done according to the approach proposed in [4]. We only point out that in all the figures considered in the following, the node spatial density in the horizontal axis corresponds to either the *cluster* node spatial density (ρ_s^C), for the curves relative to the clustered distribution, or the *overall* node spatial density (ρ_s), for the (reference) curves relative to the case of uniform topology. In all cases, the transmit power in the case of a perfectly uniform node distribution is set equal to the value of the intra-cluster transmit power (P_t^C). The considered transmit power values are typical of a smart-dust type of network [5]. Extensions of the obtained results to the case of wireless local area networks (WLANs) [11, 12] and non-smart dust sensor networks [13] are straightforward, by suitably increasing the transmit power.

In Fig. (**3**), the performance in a network scenario with N = 10^3 nodes and n_h^{IC} = 1 inter-cluster hop, is shown. As expected from the link BER expressions in (19)-(20), the

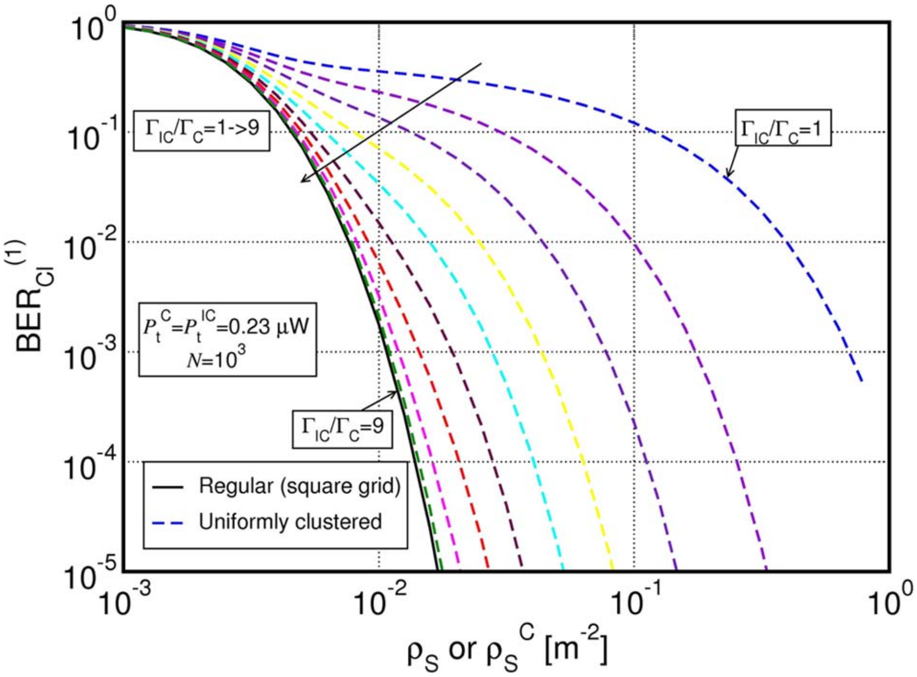

Fig. (3). BER performance of uniformly clustered net works (dashed lines) for $N = 10^3$ nodes and $n_h^{IC} = 1$ inter-cluster hop. Various uniformly clustered geometries (in terms of Γ_C and Γ_{IC}) are considered. For comparison, the BER performance of a perfectly uniform network (solid line) is also shown.

performance strongly depends on the ratio Γ_{IC}/Γ_C. In particular, the following comments can be made considering the cases of large and small values for the ratio Γ_{IC}/Γ_C, respectively.

- *Large ratio* Γ_{IC}/Γ_C. This means that Γ_{IC} is large and/or Γ_C is small-recall that $\Gamma_C \geq 2$. Considering Fig. (**2**), the fact that Γ_{IC} is large means that there are many clusters, while the fact that Γ_C is small means that the clusters are close to each other. In this case, the uniformly clustered distribution approaches a globally uniform distribution. Moreover, an inter-cluster hop is not significantly longer than an intra-cluster hop.
- *Small ratio* Γ_{IC}/Γ_C. This implies that Γ_{IC} is small and/or Γ_C is large. The fact that Γ_{IC} is small means that there are relatively few clusters, and the fact that Γ_C is large means that the clusters are relatively small compared to the global area *A*, *i.e.*, they are far apart from each other. In this case, an inter-cluster hop is significantly longer than an intra-cluster hop, and the performance is thus significantly degraded.

In order to understand the impact of the number of nodes *N* on the performance of an ad hoc wireless network, we also consider the case with $N = 10^4$ nodes, and evaluate the BER in the case of a multi-hop communication route with $n_h^{IC} = 1$ inter-cluster hop. The results are shown in Fig. (**4**). Given the expression (20) of the inter-cluster link BER, we expect that an increase of the number of nodes *N* significantly degrades the performance. As one can immediately see comparing Fig. (**3**) with Fig. (**4**), for a given ratio Γ_{IC}/Γ_C the performance, for a fixed cluster node spatial density, becomes much worse (with respect to an ad hoc wireless network with uniform topology) when the number of nodes increases. This phenomenon can be explained as follows. Let us assume that the cluster node spatial density ρ_s^C and the clustering geometry, *i.e.*, the ratio Γ_{IC}/Γ_C, are fixed. Since

$$\rho_s^C = \frac{1}{\pi}\rho_s\Gamma_C^2 = \frac{1}{\pi}\frac{N}{A}\Gamma_C^2 \tag{26}$$

two cases can be distinguished for increasing values of *N* (note that the two described situations can overlap).

- Γ_C remains constant and *A* increases. Since for fixed Γ_C the number of clusters remains unchanged, as indicated in Fig. (**2**), the clusters widen, and thus the inter-cluster distance increases. This increases the inter-cluster link BER, with deleterious effects on the overall BER.
- *A* remains constant and Γ_C decreases. Since the ratio Γ_{IC}/Γ_C is fixed, Γ_{IC} has to reduce proportionally to Γ_C. Hence, while a reduction of Γ_C does not affect the distance between the centers of two neighboring clusters, a reduction of significantly affects the distance between the centers of two clusters. Hence, larger the ratio Γ_{IC}/Γ_C larger is the performance degradation with respect to the case with a lower number of nodes.

In order to further understand the effect of inter-cluster hops, we evaluate the performance in scenarios with more than a single inter-cluster hop. We fix the clustered structure by setting $\Gamma_{IC}/\Gamma_C = 6$, and we evaluate the final BER as a function of the cluster node spatial density, for increasing values of the number of inter-cluster hops n_h^{IC} The obtained

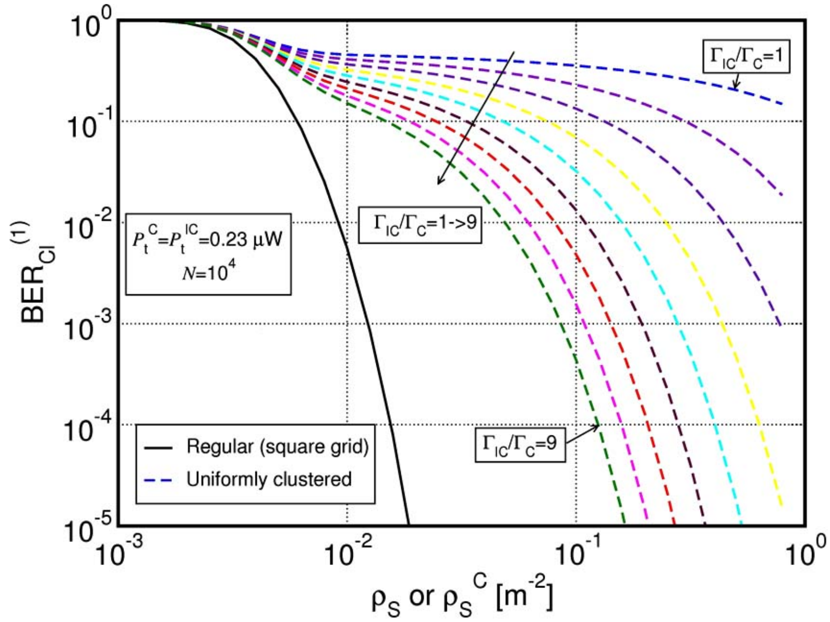

Fig. (4). BER performance of uniformly clustered networks (dashed lines) for N = 104 nodes and n_h^{IC} = 1 1inter-cluster hop. Various uniformly clustered geometries (in terms of Γ_C and Γ_{IC}) are considered. For comparison, the BER performance of a perfectly uniform network (solid line) is also shown.

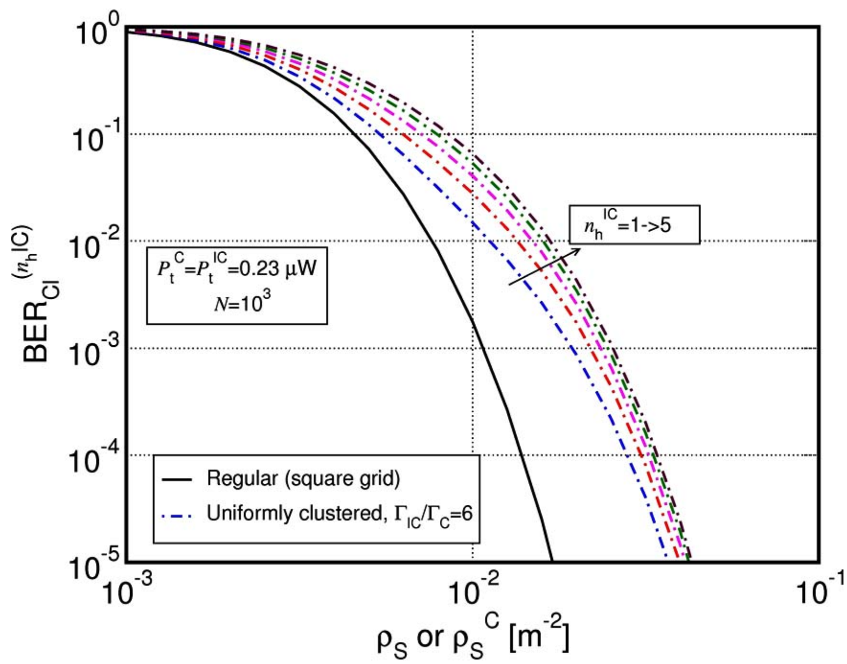

Fig. (5). BER performance of uniformly clustered networks (dashed lines) for $N = 10^4$ nodes and $\Gamma_{IC}/\Gamma_C = 6$. Various values of the number of intercluster hops n_h^{IC} are considered. For comparison, the BER performance of a perfectly uniform network (solid line) is also shown.

results are shown in Fig. (**5**). It is immediate to conclude that the first inter-cluster hop has the strongest impact, while successive inter-cluster hops do not further degrade the performance in a significant manner.

Numerical results relative to the average BER performance, according to (25), are shown in Fig. (**6**). Various values of the ratio Γ_{IC}/Γ_C are considered, and in each case the parameter Γ_{IC} takes four possible values. From Fig. (**6**), it immediate to recognize that the performance depends basically on the ratio Γ_{IC}/Γ_C, while, for a given value of this ratio, it marginally depends on the parameter Γ_{IC}.

Note that in the previous figures, the intra-cluster transmit power is equal to the inter-cluster transmit power. A possible counter-measure against the effect of clustering (in particular long hops) could consist in increasing the transmit power in correspondence to a long hop. This also suggests that routing protocols should be directly related to physical

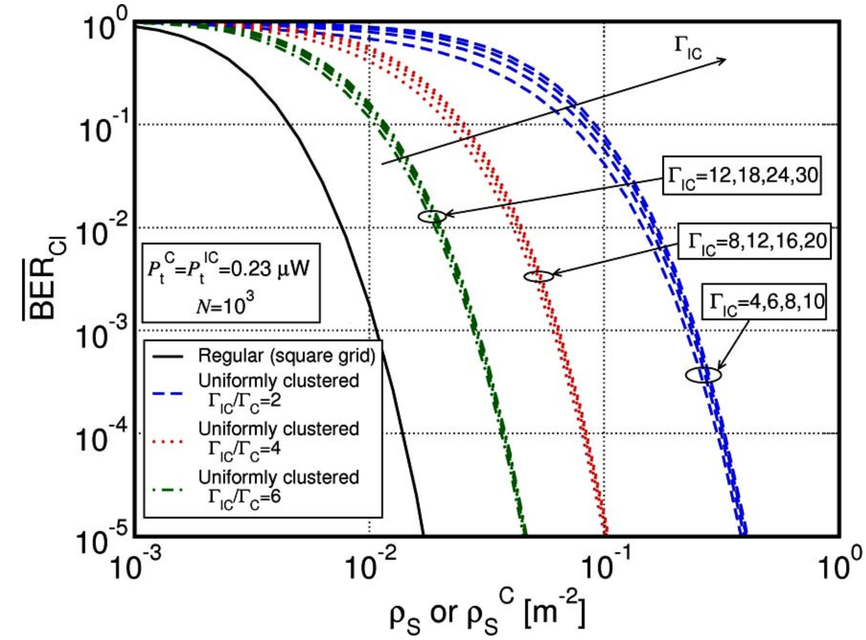

Fig. (6). Average BER performance of uniformly clustered networks (dashed lines) for $P_t^{IC} = P_t^C$. Various values of the ratio Γ_{IC}/Γ_C are considered. For comparison, the BER performance of a perfectly uniform network (solid line) is also shown.

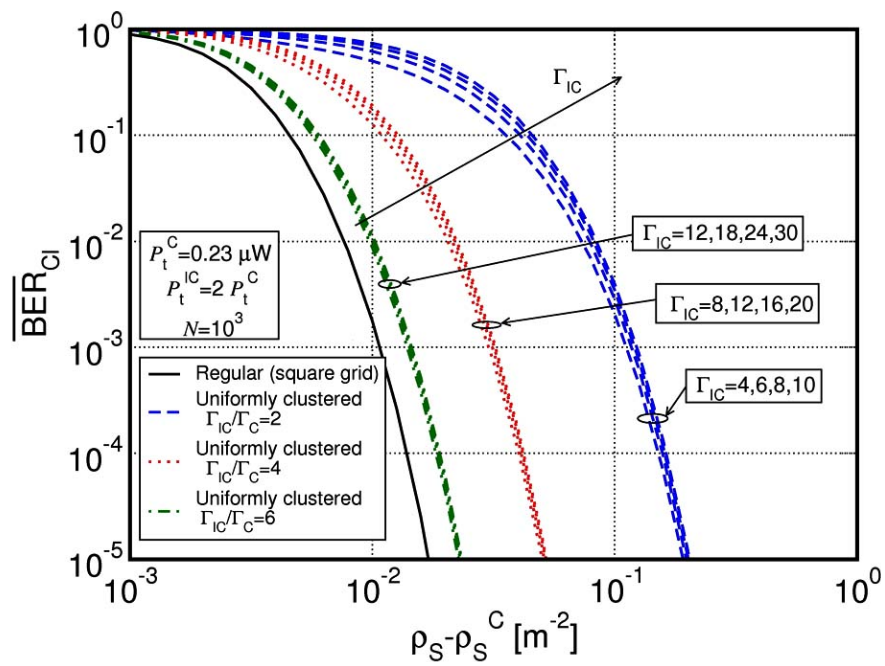

Fig. (7). Average BER performance of uniformly clustered networks (dashed lines) for $P_t^{IC} = 2 \times P_t^C$. Various values of the ratio Γ_{IC}/Γ_C are considered. For comparison, the BER performance of a perfectly uniform network (solid line) is also shown.

layer parameters. As an instance, in multi-hop communications over a clustered ad hoc wireless network the power transmitted from a node should be adaptively adjusted, depending on the next hop characteristics (long or short). For example, provided that the nodes have partial knowledge of the topology, in each cluster there could be a "cluster head" in charge of transmitting to the cluster heads of the neighboring clusters. A simple and efficient strategy could consist in increasing the transmit power for inter-cluster communication. In Fig. (**7**), the BER performance over an average communication route is evaluated by maintaining the same value of the intra-cluster transmit power as in Fig. (**6**), but doubling the inter-cluster transmit power. Comparing the BER curves in Fig. (**7**) with those in Fig. (**6**), it is immediate to notice the beneficial effects of an efficient power control.

6. CONCLUDING REMARKS

Altough in realistic ad hoc wireless networks, the nodes are likely to form clusters, rather than being regularly distributed according to some grid model, it is of interest to derive a simple parameterized analytical model, which can capture some of the key issues involved with clustering. To reach this goal, we have introduced the concept of uniformly clustered ad hoc wireless networks, and derived a simple analytical model which, through the use of a few parameters, can provide significant insights. In particular, the following conclusions can be drawn.

- A single inter-cluster (long) hop damages the BER performance over a multi-hop route. Successive inter-cluster hops have a further limited impact.
- The BER performance depends significantly on the ratio between the inter-cluster distance and the cluster radius.
- A simple power control strategy to improve the performance consists in increasing the transmit power in long hops (with respect to the transmit power considered for short hops). However, in order to regulate the transmit power, each node should be aware of the network topology. The availability of this information in a fully decentralized network communication scenario, under the constraint of minimum power consumption, is an open research problem. Moreover, in a more realistic scenario with INI, increasing the transmit power for a long hop could have deleterious effects for the neighboring nodes. A simple countermeasure could consist in using separate channels for short and long hops, respectively.

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