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Pulse-Width Modulated Amplifier for DC Servo System and Its Matlab Simulation

Ma Xiaocheng[1,*], Zhang Haotian[1,2], Cheng Yiqing[1,2], Zhu Lina[1,2] and Wu Dan[1,2]

[1]*School of Electrical and Computer Engineering, Nanjing University of Posts and Telecommunications, Jiangsu, China*

[2]*School of Engineering and Computation, New York Institute of Technology, New York, NY, USA*

Abstract: This paper introduces a mathematical model for Pulse-Width Modulated Amplifier for DC Servo Motor. The relationship between pulse-width modulated (PWM) signal and reference rotation speed is specified, and a general model of motor represented by transfer function is also put forward. When the input signal changes, the rotation speed of the servo motor will change accordingly. By changing zeros and poles, transient performance of this system is discussed in detail, and optimal ranges of the parameters is recommended at the end of discussion.

Keywords: Pulse-Width Modulated Amplifier, DC Servo Motor, Duty Cycle, Transient Performance.

1. INTRODUCTION

DC servo system plays an important role in the daily life. For example, they can be applied to spark machines, manipulators and other accurate machines (Tal, J. *et al*, 1999). Besides, reduction box can be added to DC servo system for the purpose of high accuracy and torque (Dai *et al*, 2014). Since DC servo system has the advantage of fast dynamic responses and strong anti-interference ability, it has been applied broadly in many fields. Meanwhile, pulse-width modulation's principle is that the average value of voltage (and current) fed to the load is controlled by turning the switch between supply and load on and off at a fast rate. The longer the switch is on compared to the off periods, the higher the total power supplied to the load. The main advantage of pulse width modulation is that power loss in the switching devices is very low. When a switch is off there is practically no current, and when it is on and power is being transferred to the load, there is almost no voltage drop across the switch. Power loss, being the product of voltage and current, is thus in both cases close to zero. PWM also works well with digital controls, which, because of their on/off nature, can easily set the needed duty cycle (M. Barr, 2003).

In normal motor speed (RPM) control system, resistors are used to control the speed (RPM). However, it may generate much heat, which people don't like to see. Instead, we can use pulse width modulation to achieve the goal of controlling the motor speed (RPM) [1]. Specifically, the motor speed (RPM) can be controlled by short pulses, and these pulses vary in duration (duty cycle) to change the speed (RPM) of the motor. The longer the pulses, the faster the motor turns, and *vice versa* (Milosavljevic *et al*, 2013).

*Address correspondence to these authors at the School of Overseas Education, Nanjing University of Posts and Telecommunications, Jiangsu, China; Tel: +86-18362975967; Fax:+ 86-025-85866256; E-mail: 979564225@qq.com

2. CONVERSION FROM MOTOR SPEED TO PWM SIGNAL

The following block diagram Fig. (**1**) depicts how the input (desired input speed) relates to the output (speed of the DC servo motor):

The reference rotation speed is proportional to the effective value of a rectangular pulse, so we can use a series of rectangular voltage pulse with certain duty cycle to model it. The relationship is:

$$\omega_r = KV_e \tag{1}$$

$$V_e = V_{max} \times d \tag{2}$$

where ω_r stands for the rotation speed of the reference motor, V_e stands for the effective voltage of the impulse rectangular train, and d stands for the duty cycle. In this context, we set the coefficient $K = 1$ for convenience.

Once we receive the rectangular voltage pulse, its effective voltage can be used as the input of the DC servo motor. Considering the duty cycle is not constant [2-4] (because the rotation speed of the reference motor can change from time to time), we can model the turning points using combination of unit step inputs, which is shown in Fig. (**2**).

The magenta line is the effective input voltage of the DC servo motor, and we hope the output of the DC servo motor can follow this line (because the proportionality between the rotation speed and the voltage is 1, as declared in the previous part (Table **1**).

When only considering the turning point at 5 second, as shown in Fig. (**2**), the function can be written as:

$$f_1(t) = \begin{cases} 1 & 0 < t < 5 \\ 0.5 & t > 5 \end{cases} \tag{3}$$

And it can be written in unit step function $u(t-5)$:

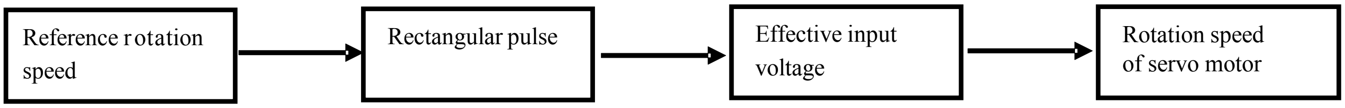


Fig. (1). Block diagram of input and output relationship.

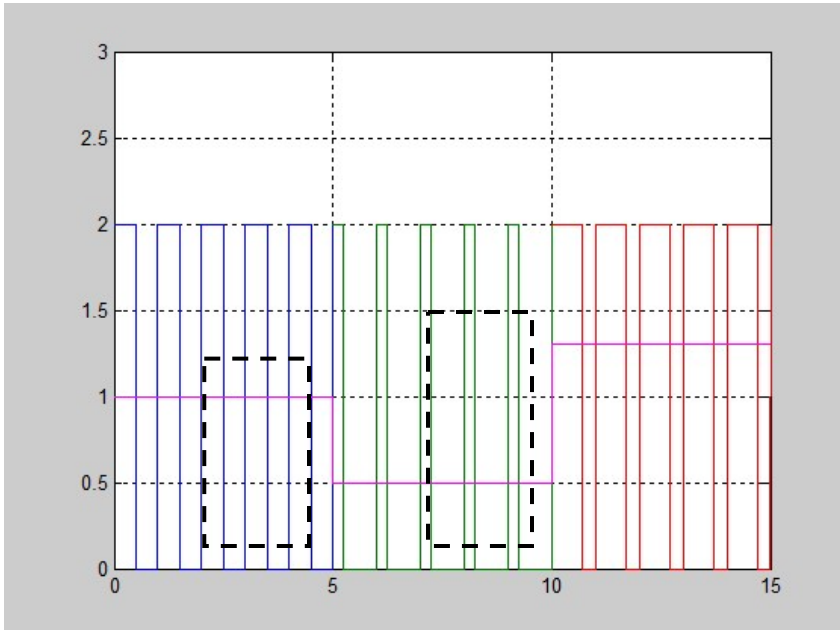


Fig. (2). PWM signal and its turning points.

$$f_1(t) = 1 - 0.5u(t-5) \tag{4}$$

Similarly, when only considering the turning point at 10 second, the function can be written as:

$$f_2(t) = \begin{cases} 0.5 & 5 < t < 10 \\ 1.4 & t > 10 \end{cases} \tag{5}$$

Table 1. Definition of symbols.

Symbols	Definitions
..	The rotor velocity of DC servo motor
t	Time
τ_j	electromechanical time constant of DC servo motor
τ_d	Electromagnetism time constant of DC servo motor
K_m	The static state of amplification coefficient of DC servo motor
u_a	The armature controlling voltage of motor
R_a	The armature resistor
L_a	The armature inductance
C_g and C_m	The electromotive constant and torque constant, which has something with motor structure
Φ	Air-gap flux
T_L	load torque

And it can be written by using combinations of $u(t-10)$:

$$f_2(t) = 0.5 + 0.9u(t-10) \tag{6}$$

In summary, the model can be established as:

$$f_x(t) = \begin{cases} a & t < t_1 \\ b & t > t_1 \end{cases} \tag{7}$$

$$f_x = (b-a)u(t-t_1) + a \tag{8}$$

3. THE TRANSFER FUNCTION

For a typical DC servo motor model, the dynamic characteristic equation for closed-loop servo mechanism is equation (9):

$$\tau_j \tau_d \frac{d^2\omega}{dt^2} + \tau_j \frac{d\omega}{dt} + \omega = K_m u_a - \frac{R_a}{C_e C_m \Phi^2} T_L - \frac{L_a}{C_e C_m \Phi^2} \frac{dT_L}{dt} \tag{9}$$

According to Eq. (9), the rotation speed ω is controlled by u_a, and is affected by T_L.

Using Laplace transform, the transfer functions of the system can be written as:

$$G_m(s) = \frac{\Omega(s)}{U_a(s)} = \frac{K_m \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{10}$$

In this equation, ω_n is un-damped frequency of motor, ξ is damping ratio of motor. The block diagram of the transfer function is as shown in Fig. (**3**).

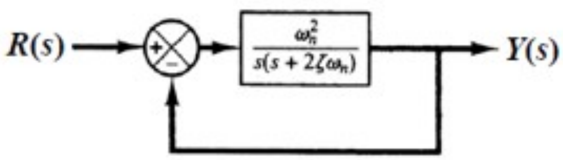


Fig. (3). Block diagram of the transfer function.

We can expand the order of the transfer function by adding poles and zeros. When adding zeros, we need to multiply $(k_z s + 1)$ to the transfer function, and the zero is $s = -1/k_z$. When adding poles, we need to multiply $1/(k_p s + 1)$ to the transfer function, and the pole is $s = -1/k_p$. By adding more and more poles and zeros, we can obtain higher order systems.

4. MODEL EXPRESSION

Modeling of the Input

As what we have discussed in the previous paragraphs, the input can be modeled as:

$$V_e = \omega_r / K \tag{11}$$

$$V_{max} = V_e / d \tag{12}$$

where ω_r is the rotation speed of the reference motor, V_e is the effective voltage of the impulse rectangular train, V_{max} is the maximum voltage of the impulse rectangular train, d is the duty cycle. Also,

$$\mathrm{r}(t) = \begin{cases} V_{e1} & t < t_1 \\ V_{e2} & t > t_1 \end{cases} \tag{13}$$

$$\mathrm{r(t)} = (V_{e2} - V_{e1})u(t - t_1) + V_{e1} \tag{14}$$

$$\mathrm{R(s)} = L[r(t)] \tag{15}$$

$R(s), r(t)$ are inputs of the system.

Modeling of the Motor

When considering add poles or zeros to Eq. (10):

$$G_m(s) = \frac{\Omega(s)}{U_a(s)} = \frac{K_m \omega_n^2 (k_{z1}s+1)(k_{z2}s+1)...}{s^2 + 2\xi\omega_n s + \omega_n^2 (k_{p1}s+1)(k_{p2}s+1)...} \tag{16}$$

with Eq. (16), the relationship between the input and the output can be written as:

$$Y(s) = R(s)G_m(s) \tag{17}$$

When performing analyses on the transient performance measures:

$$R(t) = u(t) \tag{18}$$

$$R(s) = L[R(t)] = 1/s \tag{19}$$

5. SIMULATION AND TRANSIENT PERFORMANCE ANALYSIS

Parameter Settings

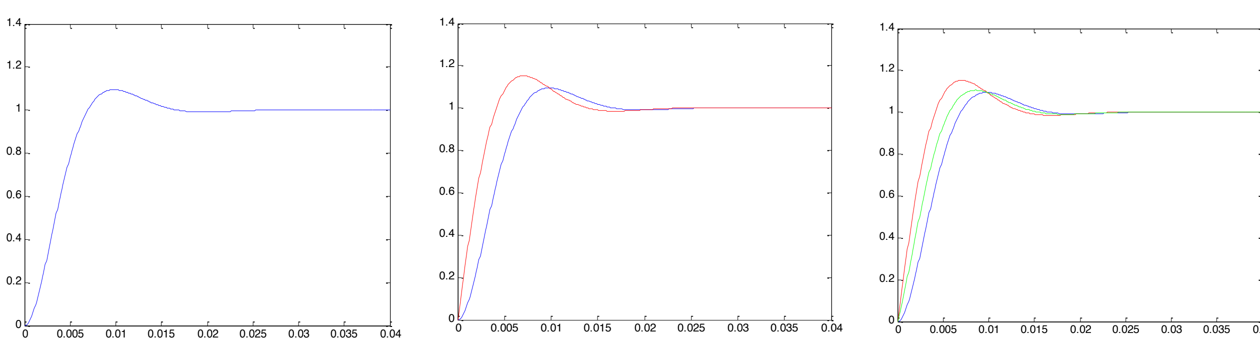
In this paper, we do transient performance analysis based on the following premises:

1. $\omega_n = 400$.
2. $K = 1$, as declared in the previous part.
3. ξ = 0.6, underdamped condition.
4. The input is the unit step function.
5. The settling time is defined as the time required for the system to settle within 2% of the amplitude.

Transient Performance Analysis

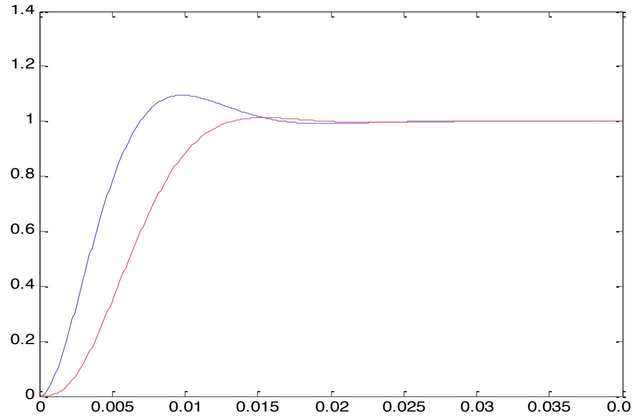
There are three criteria when considering estimate and design a control system. Firstly, the system must be stable. Secondly, the control should be accurate. Thirdly, the response should be quick-acting. What's more, there are four criteria in transient performance measures which are rise time, peak time, settling time and percent overshoot. In the below Fig. (**4**) – Fig. (**7**), x-axis represents time while y-axis represents RPM.

Fig (**4**) shows the system response under different conditions. Fig (**4a**) depicts the condition where no zero points and pole points exist; Fig (**4b**) also shows the condition where there is a zero point at -500 with the newly added red line; Fig (**4c**) illustrates the condition where there is a zero point at -1000 with green line. Table **2** contains more results.

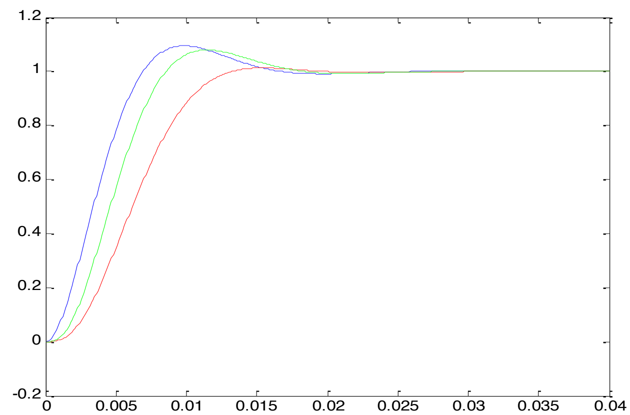


(**a**) (**b**) (**c**)

Fig. (4). System response with different zeros.



(a)



(b)

Fig. (5). System response with different poles.

Table 2. Results of different zeros.

Zeros	Rise Time	Peak Time	Settling Time	Percentage Overshoot
0	0.007	0.0098	0.0149	9.4778
-100	0.0008	0.0036	0.019	146.5061
-200	0.0017	0.0045	0.0184	54.4925
-300	0.0026	0.0055	0.0176	29.0746
-400	0.0035	0.0064	0.0123	19.4456
-500	0.0042	0.007	0.0127	15.2163
-600	0.0047	0.0075	0.0131	13.1195
-700	0.0051	0.0079	0.0133	11.9687
-800	0.0053	0.0082	0.0135	11.2790
-900	0.0056	0.0084	0.0137	10.8371
-1000	0.0057	0.0086	0.0138	10.5379

From the table above, we find that: the second-order system response with zero differs a lot from the systems without zero. In the system without zero point, [5-7] the rising time t_s only depends on system damping ξ and oscillating angular frequency ϖ_n. However, in the system response with zero points, the rising time is also related to the real component. Which can be shown in figure that the more the zero point is closed to the imaginary axis, the less the rising time would be. Also, we can infer from $r=\frac{\xi\varpi_n}{z}$ that the more r is, the more oscillatory the system would be. The percentage overshoot also relates to the position of zero point. The less the zero is, the more the φ would be, which would make t_m decrease.

Another variable that affects the output signal is the poles. Now we set the zeros back to 0, and set poles to -300, the figure will be:

Fig (**5a**) shows the condition in red lines when we set the pole to -300; In Fig (**5b**), the green line shows the condition when we set the pole to -800. More results are shown in Table **3**.

From our analysis, we infer that the output signal will be most stable when the zero is set to -828～-869 and pole is set to about -300. This conclusion is calculated according to the following steps:

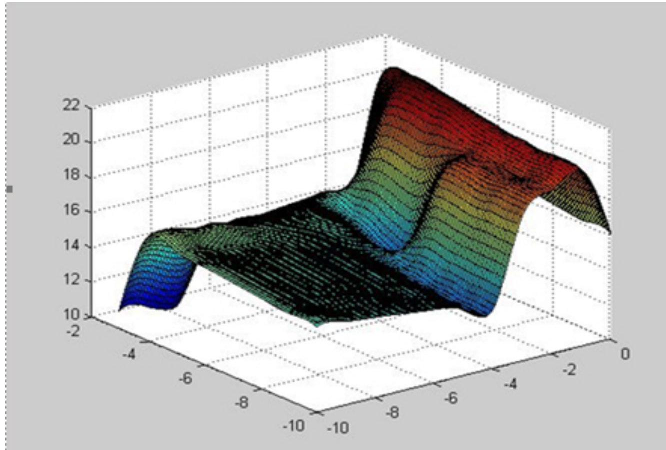


Fig. (6). Simulation of settling time, poles and zeros (three dimensional diagram).

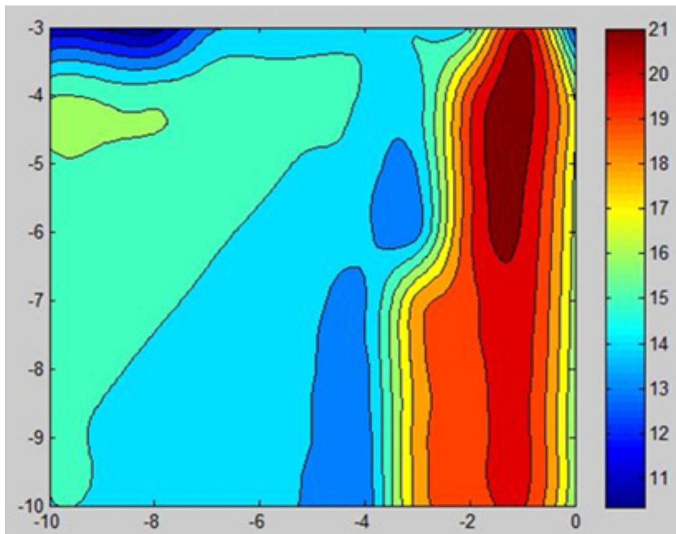


Fig. (7). Simulation of settling time, poles and zeros (Contour Plots).

Table 3. Results of different poles.

Pole	Rise Time	Peak Time	Settling Time	Percentage Overshoot
-1000	0.0082	0.0111	0.016	8.52296
-900	0.0084	0.0112	0.0161	8.287589
-800	0.0086	0.0114	0.0163	7.959381
-700	0.0089	0.0117	0.0165	7.49115
-600	0.0093	0.0121	0.0167	6.797521
-500	0.0099	0.0127	0.0169	5.733301
-400	0.0109	0.0136	0.0169	4.040086
-300	0.0133	0.0154	0.0123	1.288179

Table 4. Raw results for the shortest settling time.

Position of Zero	Position of Pole	Settling Time
-8.282828	-3.0000	10.3644

According to above data, we can get zero point, pole point, the values of settling time and percentage overshoot. If the settling time and percentage overshoot are smaller, the control system is more stable. Therefore, based on simulation, we need to find a best point for getting the most stable system.

First, according to the data of the settling time, poles and zeros, we can obtain the relationship of the settling time, poles and zeros. Using the method of interpolation, we can draw the three dimensional diagram and Contour Plots and Color Mapping. The figure is shown in Fig. (**6**).

For convenience, we narrow the coordinates of zeros and poles 100 times smaller, and enlarge the coordinates of the settling time 1000 times.

According to the diagram, we can draw a conclusion that the position of poles is set to the -3 to -4 if the settling time is the shortest [8, 9].

Secondly, according to the data of the percentage overshoot, poles and zeros, we can get the relationship between percentage overshoot, poles and zeros, which is three-dimensional diagram and Contour Plots and Color Mapping Figs. (**7-9**).

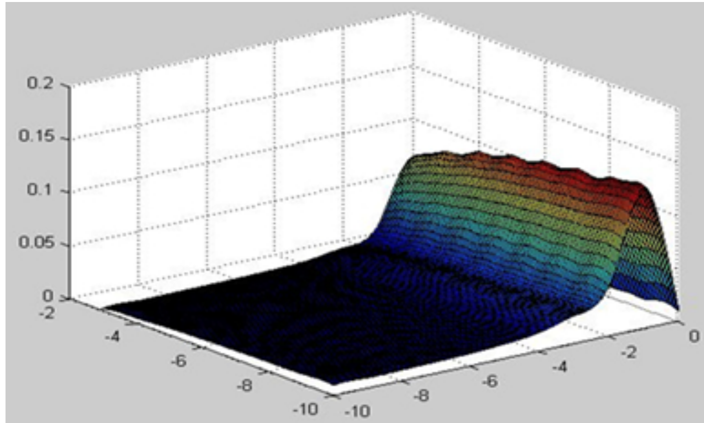


Fig. (8). Simulation of percentage overshoot, poles and zeros (three dimensional diagram).

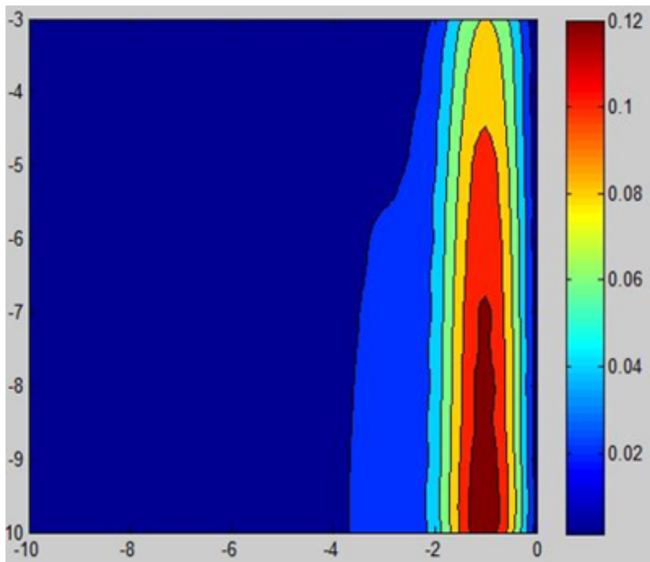


Fig. (9). Simulation of percentage overshoot, poles and zeros (Contour Plots).

Table 5. Raw results for the smallest percentage overshoot.

Position of Zero	Position of Pole	Percentage Overshoot
-8.686868	-3.0000	1.52593

In sum, we can get the range of the pole and the zero. The range of pole is from -3 to -4, and the range of zero is from -10 to -8. The simulation's results are shown below:

Because those data were processed by dividing the values of zeros and poles by 100, we will restore those data. So, if making the settling time is the shortest, then the position of zero is -828 and that of pole is -300; if making the percentage overshoot is the smallest, then the position of zero is -869 and that of pole is -300.

In sum, the optimal position is that the zero is set to -828 ～-869, and the pole is set to about -300, which makes the system most stable.

CONCLUSION

In this paper, a mathematical model for Pulse Width Modulated Amplifier for DC Servo Motor is devised. Poles and zeros can be added to the system in order to enhance the transient performance. Based on the parameters declared in the context, we find the best position for zero is -828~-869, and the best position for pole is about -300.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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Declared none.

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