

# Optimization of a Multi-pendulum Wave Energy Converter

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**Abstract:** In order to improve the energy capture efficiency of a multi-pendulum wave energy converter, a mathematical model of the pendulum structure has been built. The final structure parameters of the pendulum have been obtained by using genetic algorithm based on the numerical simulation results of the pendulum structure optimization. The results show that under obtained structure parameters the proposed multi-pendulum device can obtain maximum energy conversion efficiency.

**Keywords:** Genetic algorithm, multi-pendulum, structure optimization, wave power generation.

## 1. INTRODUCTION

For the pendulum wave energy converter, the factors that influence the hydrodynamic characteristics of the pendulum include buoyancy airfoil structure, pendulum corresponding to the flow angle of attack and fluid medium, etc. When the installation site and mode are determined, the corresponding flow medium and the flow angle of attack can be determined. Thus in order to improve the water dynamic characteristics of the pendulum, the first task is to choose or design the reasonable pendulum structure. However, the pendulum structure's optimization involves the three-dimensional wave on the pendulum, such as wave force on the pendulum, and the study has the complex three-dimensional radiation, which makes the three-dimensional solution extremely difficult [1, 2]. Therefore, the present optimization for pendulum structure was conducted based on the results of the numerical simulation.

According to our previous research [3], a new multi-pendulum wave energy converter is proposed and it is shown in Fig. (1). It consists of floating body system 1, capture system 2, conversion system 3, power system 4, auto-location system 5 and anchor chain system, seen in Fig. (1). The pendulums are used to capture wave energy and achieve the maximum of capture efficiency by choosing reasonable spacing between them. The energy transfer process is shown in Fig. (2). When the wave acts on the pendulum 201, the pendulum will swing back and forth in the wave direction (assuming the direction is perpendicular to the paper face). When the direction of pendulum is the same as that of wave, the helical gear 31 will drive the ratchet wheel 33 rotating counterclockwise and the ratchet 32 is holding on in the meantime. When the direction of pendulum is opposite to

that of wave, the ratchet 32 rotates counterclockwise and the ratchet 33 is holding-on in the meanwhile. Thus the wave force of floating pendulums is collected. The acquisition of the wave energy will be input to the flywheel. Then the power system 4 generates electricity. Finally, the converted electrical energy will be output through submarine cable. The complete equipment is fixed at the bottom of the sea by the cable system.

## 2. OPTIMIZATION MODEL

### 2.1. Determination of the Objective Function

Under the action of the linear wave, ignoring the external forces generated by the wind, swell and other environmental load, the pendulum motion responses equation can be represented by rigid motion equation [4-6]:

$$\{-\omega^2([M] + [a]) - i\omega([B] + [b]) + [K]\}\{\xi\} = \{F_{0D}\} \quad (1)$$

In the formula,  $[M]$  represents the quality matrix of the pendulum,  $[a]$  represents the added mass matrix,  $[B]$  represents the damping matrix of the energy extraction system,  $[b]$  represents the damping coefficient matrix,  $[K]$  represents the restoring force matrix,  $\{\xi\}$  represents the amplitude vector of the motion of pendulum,  $\{F_{0D}\}$  represents the exciting force vector of waves.

The main optimized parameters of the pendulum with right parallelepiped structure are length  $l$ , width  $b$  and thickness  $\delta$ . The moment of inertia of the pendulum is:

$$M = \rho_0 \delta l b (\delta^2 + 4b^2) / 12 \quad (2)$$

In the formula,  $\rho_0$  represents the density of the pendulum, and it must meet the following formula:

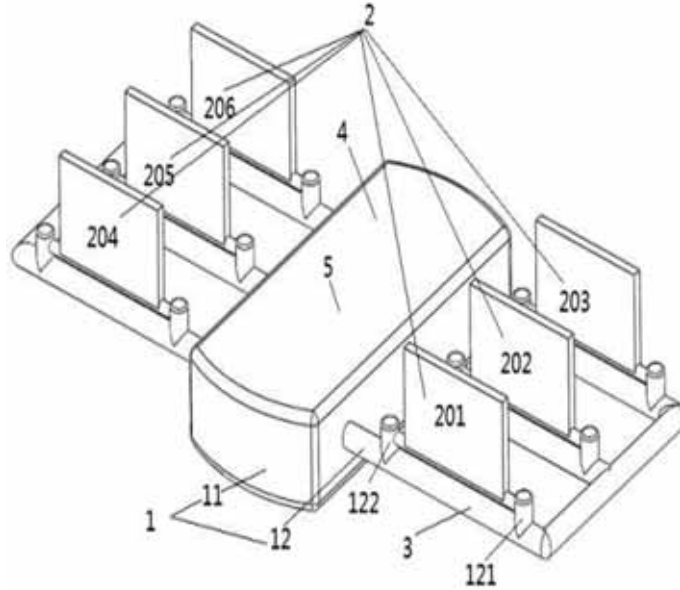


Fig. (1). The overall schematic diagram of the multi-pendulum wave energy converter.

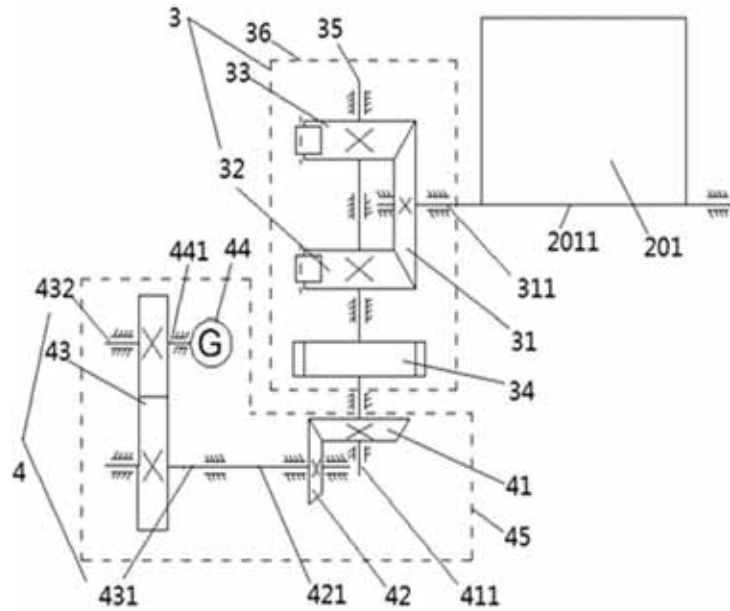


Fig. (2). The energy transfer schematic diagram of the multi-pendulum wave energy converter.

$$\rho_0 = \frac{\rho g (\delta^2 + 6b^2) - 12\omega^2 I_a / l \delta}{6gb^2 + \omega^2 b (\delta^2 + 4b^2)} \quad (3)$$

The restoring force coefficient of the pendulum is:

$$K = \rho g \delta^3 l / 12 + (\rho - \rho_0) g \delta l b^2 / 2 \quad (4)$$

The energy extraction efficiency of the pendulum can be represented as:

$$\eta = \frac{\sum_0^{nT} M_v(t) \cdot d\theta(t)}{E_\omega \cdot nT}, (n = 1, 2, 3, \dots) \quad (5)$$

In the formula,  $M_v(t)$  represents the output torque in the shaft,  $\theta$  represents the pendulum angle,  $T$  represents the period of wave,  $E_\omega$  represents the wave energy flux.  $E_\omega$  is average wave power of the wave period and it can be represented with the circular frequency  $\omega$  as:

$$E_{\omega} = \frac{1}{16} l \rho g H^2 \frac{\omega}{k_0} \left[ 1 + \frac{2k_0 h}{\sinh(2k_0 h)} \right] \quad (6)$$

In the formula,  $\rho$  represents the density of sea water,  $H$  represents the wave height,  $\lambda$  represents the wavelength,  $T$  represents the wave period,  $k_0$  represents the wave number,  $h$  represents the depth of water. In this case,  $H = 1.1m$ ,  $T = 3.6s$ ,  $\rho = 1025kg/m^3$ ,  $E_{\omega} = 19340.22w$ . By using the hydrodynamics software AQWA, we can draw the conclusion that the energy extraction efficiency is the best when the floating pitch is half a wavelength. The damping torque of shaft  $B = 7000N \cdot m$ , the force on the pendulum per unit area by wave adopts the average value of the numerical simulation results, and the torque on the per unit area can be calculated as  $T_{od} = 1266N \cdot m$ .

The purpose of the structural optimization is to maximize the energy extraction efficiency of the pendulum, and it must meet Equation (5). The objective function of the structure optimization is given as follows:

$$\xi = \frac{1266bl}{-\omega^2 \left\{ \left[ \rho_0 \delta l b (\delta^2 + 4b^2) / 12 \right] + I_a \right\} - i\omega [7000 + c_b] + \rho g \delta^3 l / 12 + (\rho - \rho_0) g \delta l b^2 / 2} \quad (7)$$

### 2.2. Constraint Condition

The thickness of the pendulum must be greater than zero and less than the length and width. The width of the pendulum should be greater than the height of the incident wave and less than the depth of the water. The length is decided according to the size of the wave force, and the Morrison method and the diffraction theory are currently methods of calculating the wave force [6].

The application range of Morison equation as well as diffraction theory depends on the values of  $D/L$  and  $H/D$  [8]. When  $D/L > 0.15$  and  $H/D < 1.0$ , the diffraction is taken into account without considering viscosity effect. So the calculation is carried out based on the linear diffraction. Since numerical simulation software is based on the three dimensional potential flow theory, the pendulum length needs to meet the conditions as:

$$l/\lambda \geq 0.15 \text{ and } H/l \leq 1.0$$

### 2.3. Mathematical Model of Structural Optimization

The objective function [7] involves the calculation of hydrodynamic coefficients, the added mass and damping coefficient. At present, the research methods are roughly divided into two kinds: analytical method and numerical calculation. The analytical solution basically solves the problem of two-dimensional, rectangle as a typical streamlined shape, and it also solves the three dimensional case whose additional

quality has not been the theoretical calculation value [7]. Therefore, the float is placed to solve the problem of the two-dimensional rectangle pendulum. Two-dimensional case, float the rotational inertia of the pendulum is:

$$M = \left[ \rho_0 b (\delta^2 + 4b^2) / 6 \right] + I_a \quad (8)$$

Restoring force coefficient is as follows:

$$K = \rho g \delta^3 / 12 + (\rho - \rho_0) g \delta b^2 / 2 \quad (9)$$

Equation 7 can be expressed as follows:

$$\xi = \frac{1266b}{-\omega^2 \left\{ \left[ \rho_0 \delta l b (\delta^2 + 4b^2) / 12 \right] + I_a \right\} - i\omega [7000 + c_b] + \rho g \delta^3 / 12 + (\rho - \rho_0) g \delta b^2 / 2} \quad (10)$$

In this formula,  $g$  is the gravitational acceleration. The floating pitches, additional mass moment and radiation damping moment are as follows:

$$I_a = \sum_{m=1}^{\infty} \frac{2\rho}{N_m k_m^5 \cos^2 k_m b} (k_m b \sin k_m b + \cos k_m b - 1)^2 \quad (11)$$

$$c_b = \frac{2\rho\omega}{N_0 k_0^5 \cosh^2 k_0 b} (k_0 b \sinh k_0 b - \cosh k_0 b + 1)^2 \quad (12)$$

In this formula:

$$N_0 = \frac{1}{\cosh^2 k_0 b} \left( \frac{b}{2} + \frac{\sinh 2k_0 b}{4k_0} \right) \quad (13)$$

$$N_m = \frac{1}{\cosh^2 k_m b} \left( \frac{b}{2} + \frac{\sinh 2k_m b}{4k_m} \right) \quad (14)$$

$$k_m \cdot \tan(k_m h) = -\omega^2 / g, m = 1, 2, 3, \quad (15)$$

Assumes that  $\rho_0 = 300kg/m^3$ . The incident wave number is determined by the dispersion relation through MATLAB numerical calculation, it also can be calculated by Equation (15).

For the complex, only real numbers can be compared. In addition to the real numbers, the nonzero imaginary part of complex cannot be compared. Therefore, the two-dimensional case of pendulum can establish mathematical model of structural optimization as follows:

$$\begin{aligned} & \max_{\xi} (b, \delta) \\ & \xi = \frac{1266b}{-\omega^2 \left[ \rho_0 b (\delta^2 + 4b^2) / 6 \right] - \omega^2 I_a + \rho g \delta^3 / 12 + (\rho - \rho_0) g \delta b^2 / 2} \\ & s.t. \\ & H < b < h, 0 < \delta < H \end{aligned} \quad (16)$$

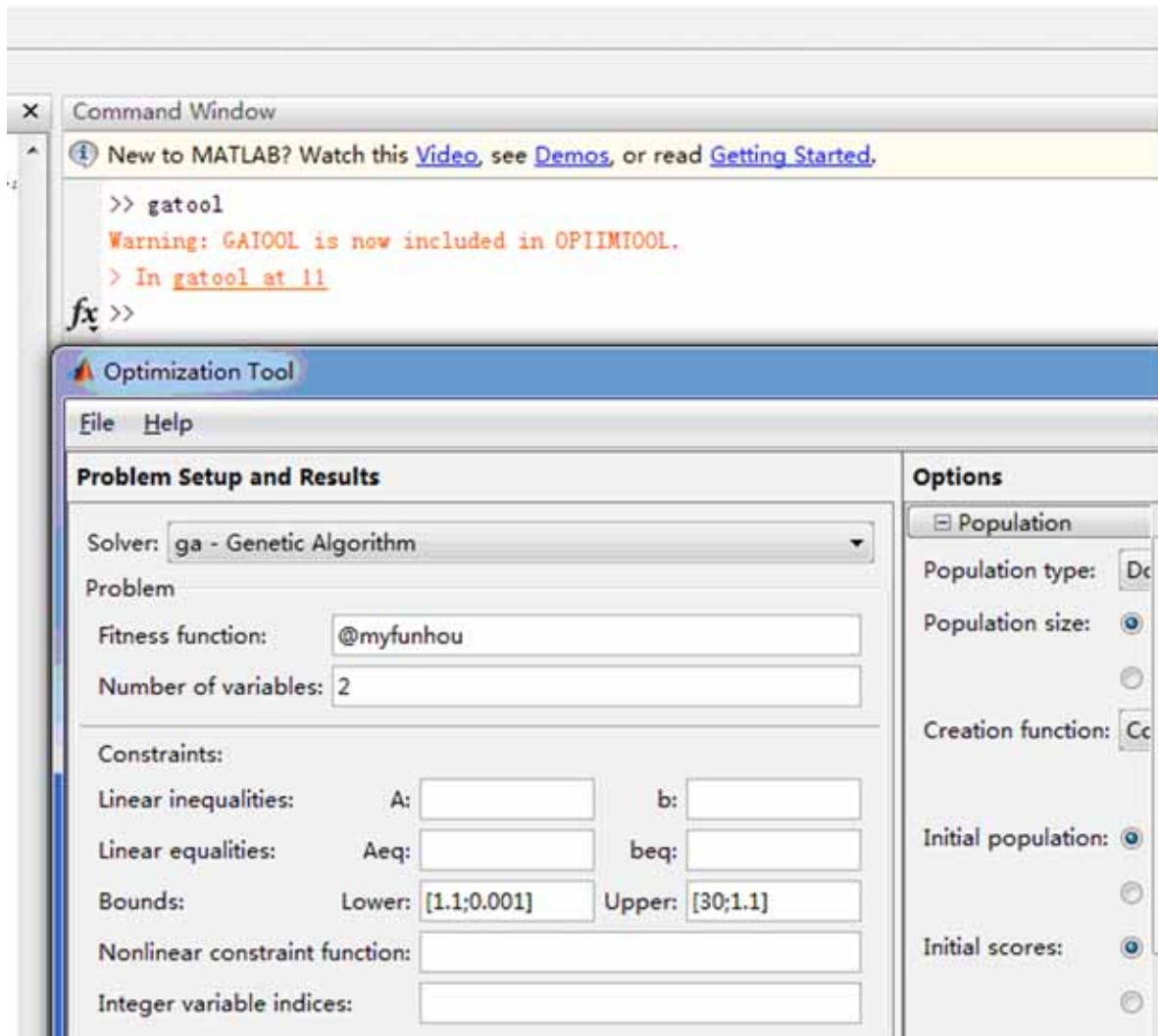


Fig. (3). Setting of objective function and constraint conditions.

### 3. OPTIMIZATION PROCESS AND RESULTS

#### 3.1. Algorithm

In this paper, the genetic algorithm is adopted to Equation 16 to seek the optimal solution. The GA toolbox of the MATLAB r2012a is used to optimize and the procedure of optimization is shown as follows:

- (1) Calculating the value  $k_m$  according to Equation (15).
- (2) Writing the objective function program myfunhou.m.

(3) In the MATLAB command window, entering the gatool invoking Genetic Algorithm Toolbox, then entering the objective function and constraint conditions, as shown in Fig. (3).

#### 3.2. Control Parameters

Genetic evolution number  $G=100$ , choosing Double vector as population type, size  $N = 20$ , crossing-over rate  $P_c = 0.8$ , choosing constraint dependent as mutation rate

$P_m$ . The update frequency of the graphics is set to 1, that is to say, the iteration algorithm updates once an image. Selecting the best individual and optimal variables as the output image, and the rest of the parameters keep the default values as in Fig. (4).

#### 3.3. Selection Strategy

Adopting the Roulette strategy proposed by Holland we can get the computational results, as shown in Figs. (5 and 6).

Using the genetic algorithm we can get the optimal individual:  $x_1 = 4.368$ ,  $x_2 = 0.241$ , and the maximum pendulum angle  $\xi = -10995.39$ , the minus sign for  $\xi$  expresses plate counterclockwise rotation after the incident wave. Because the imaginary part of the objective function is discarded, the maximum pendulum angle reaches more than ten thousand, this means no damping torque. In this case, when  $x_1 = 4.368$  and  $x_2 = 0.241$ , the rotation range of the pendulum is the

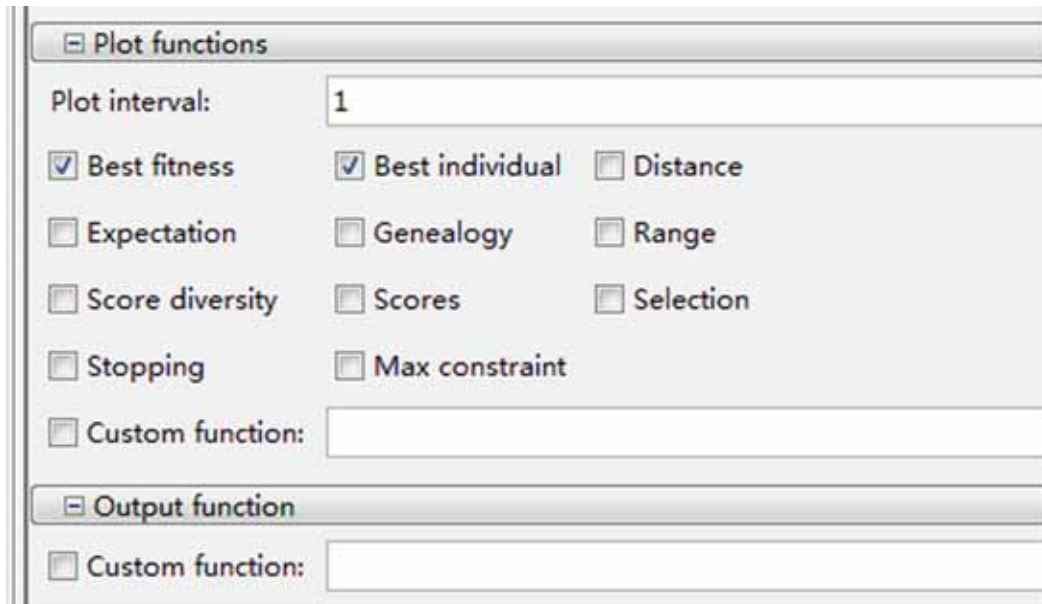


Fig. (4). setting of algorithm options.

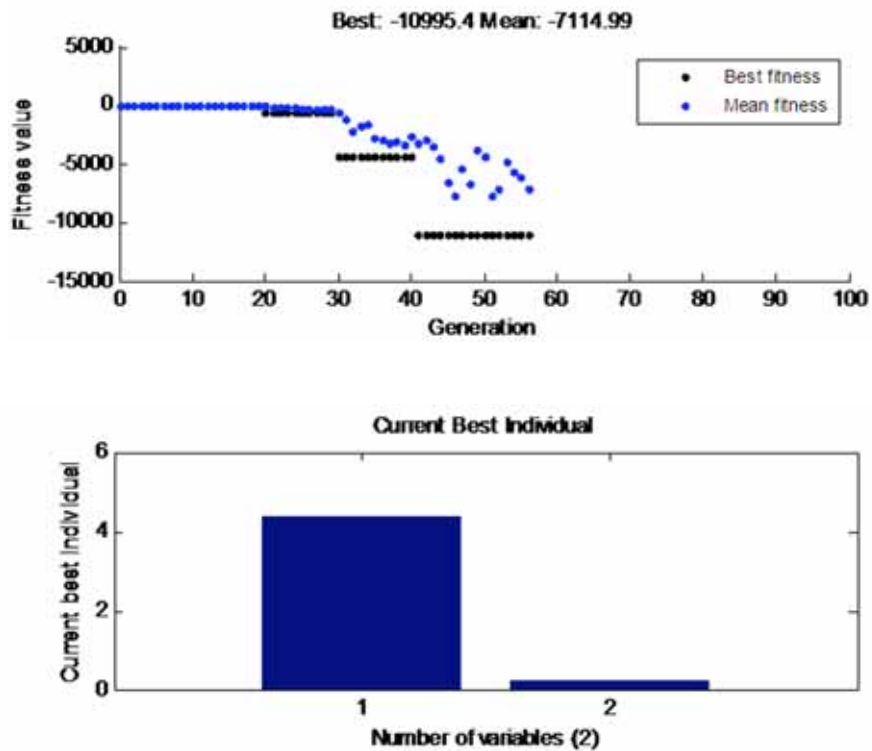
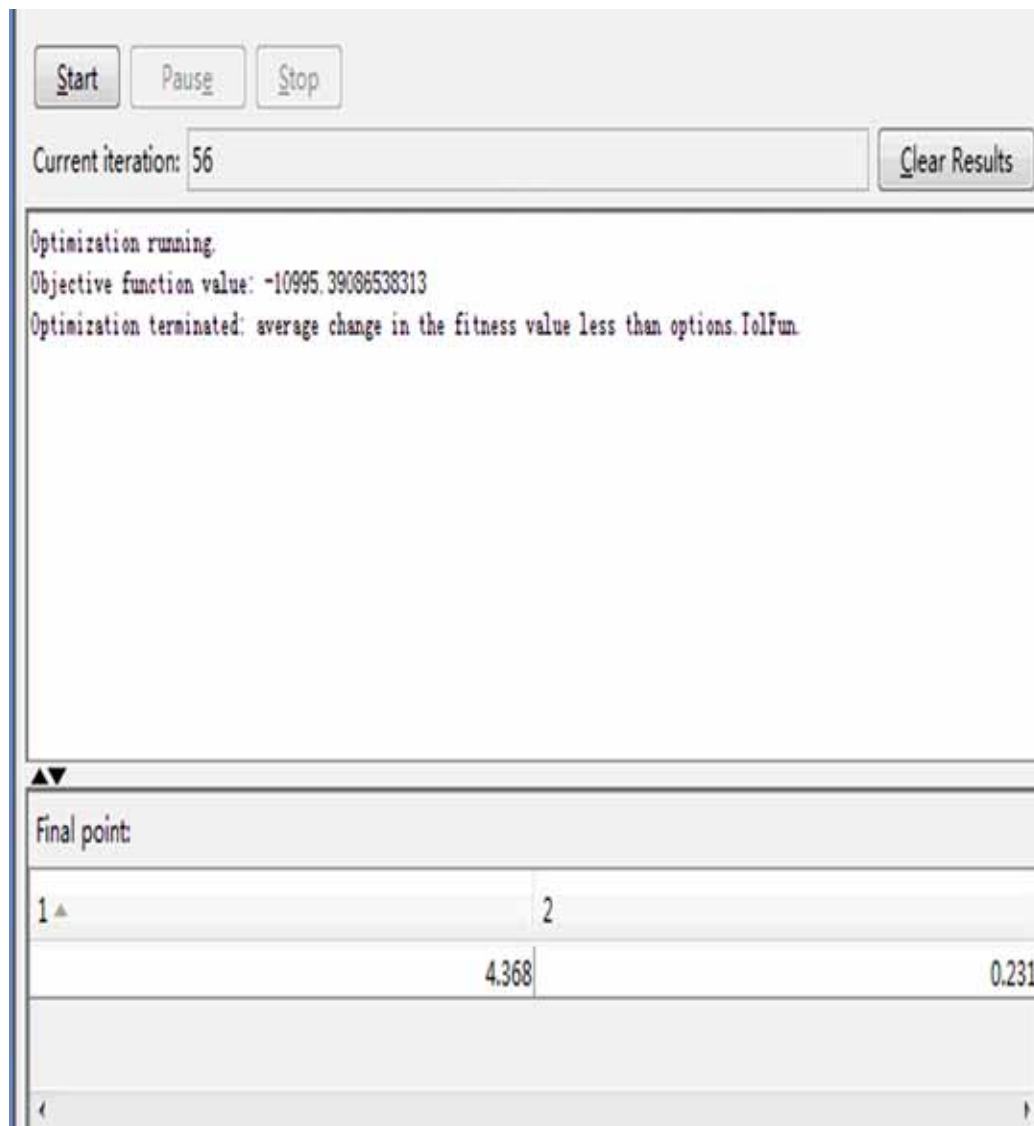


Fig. (5). Optimization results.

maximum. In this way, the optimized pendulum gets the width  $b = 4.37m$  and the thickness  $\delta = 0.23m$ .

The pendulum length should be got after calculating the width and thickness. Because the added mass can not be

calculated, it cannot be further optimized. In present work, using the AQWA the calculation was carried out with the density  $\rho = 300kg / m^3$ , the width  $b = 4.37m$ , the thickness  $\delta = 0.23m$  under the lengths of 6m, 7m, 9m, 10m,



**Fig. (6).** Final optimal values.

respectively. The simulation results show that the energy extraction efficiency of the device increases with the increase of the length. When the length is up to 8m, the trend is weakened. If conditions allowed, a longer pendulum should be adopted. The length of the pendulum is chosen 8m in this paper.

## CONCLUSION

A new multi-pendulum wave energy converter is proposed. On the basis of numerical simulation, the pendulum structure is optimized using the genetic algorithm and the optimal structural parameters are obtained. Under this set of parameters, the multi-pendulum device can obtain maximum energy conversion efficiency. The optimization results also show that the genetic algorithm is an appropriate algorithm to use in pendulum structure optimization.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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