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The Study of Natural Convection Heat Transfer in a Partially Porous Cavity Based on LBM

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Abstract: In the present work, the natural convection problem in a two-dimensional square cavity partially filled with porous media is simulated by lattice Boltzmann method (LBM). The flow field and temperature field of fluid in the cavity are obtained by numerically simulating. A comprehensive study of natural convection heat transfer is carried out for various values of the vertical porous layer dimensionless width D, of Rayleigh number, of Darcy number, and of porosity. Research results show that in the case of $D < 0.1$ or $D > 0.9$, the hot wall average Nusselt number is sensitive to porous layer width. Under the conditions of high Rayleigh number and high Darcy number, the effect of natural convection becomes distinct and the change of porosity with high Darcy number has obvious influence on heat transfer.

Keywords: Lattice Boltzmann method, porous media, natural convection heat transfer.

1. INTRODUCTION

The phenomena of energy, momentum and mass transfer in porous media exist in all the field of industrial and agricultural. Natural convection heat transfer in porous media is a benchmark problem due to its wide application background, such as heat pipe, heat insulating material, petroleum reserve, groundwater decontamination, thermal drying process and casting solidification, etc. Ingham and Pop [1, 2] elaborated the natural convection problem in porous media in their works. An excellent and comprehensive review has been given by Nield and Bejan [3]. The natural convection issue in a partially porous cavity has been studied by D. Gobin, B. Goyeau and Beckermann [4, 5], and the mathematical descriptions of this problem were based on one-domain and two-domain formulation of the conservation equations, respectively.

As a promising numerical method, lattice Boltzmann method [6] has became a new tool to simulate fluid motion and model complex physical phenomena after decades of development. Different from the traditional method of computational fluid mechanics, LBM is not based on the macroscopic continuous equation but grounded on the fluid microscopic model and mesoscopic dynamic equations. Then, the evolution mechanism in accord with physical laws is constructed to calculate. LBM can be used to simulate the fluid flows and heat transfer due to its simple implements, good concurrency, simple boundary treatment etc. Meanwhile, it has been widely applied in the study of porous media by international scholars. Takeshi Seta *et al*. [7] applied lattice Boltzmann method to analyze the performance of natural convection heat transfer in porous media cavity for different Rayleigh number, Darcy number and porosity. Yan Weiwei *et al*. [8, 9] implemented LBM to simulate the flow field and the temperature field in a cavity filled with porous medium, especially researched the influence of porosity of the medium vary from place to place on the nature convection.

This paper uses the coupled lattice Boltzmann model proposed by Guo *et al.* [10] to solve the natural convection heat transfer problem in a partially porous cavity at representative elementary volume scale. The velocity field and temperature field are modeled by different lattice Boltzmann equations and they are coupled through Boussinesq approximate equation. On this basis, a comprehensive study of natural convection heat transfer is carried out for various values of the vertical porous layer dimensionless width D, of Rayleigh number, of Darcy number, and of porosity.

2. PHYSICAL MODEL AND GOVERNING EQUATIONS

2.1. Physical Model

In this paper, the physical model is a two-dimensional square cavity of length L that partially filled with vertical porous layer, as shown in Fig. (**1**). For the model, the vertical surfaces are held at constant temperature T_L and T_H ($T_H > T_L$), respectively. The two horizontal walls are adiabatic. The actual width of the porous layer is d and the dimensionless width of the layer is $D(d/L)$. The origin of coordinate system is the lower left corner of the cavity. The model takes horizontal direction as the x direction and the opposite direction of gravity as the y direction.

The initial conditions and boundary conditions are as follows:

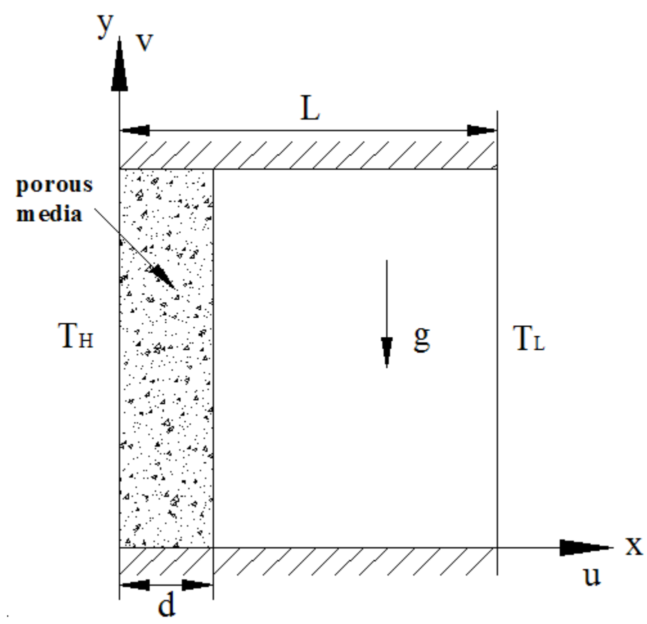

Fig. (1). Schematic of physical model of enclosure cavity.

$t = 0,\ u = v = 0,\ T = 0$;

$x = 0,\ u = v = 0,\ T = T_H$;

$x = L,\ u = v = 0,\ T = T_L$;

$y = 0,\ u = v = 0,\ \partial T/\partial n \quad 0$;

$y = L,\ u = v = 0,\ \partial T/\partial n \quad 0$;

2.2. Governing Equations

In the study, we assume that the configuration of porous media in the cavity is homogeneous, rigid and isotropic. We also assume that the fluid is incompressible and viscous fluid flow can be described by Brinkman-Forchheimer model and it meet the Boussinesq hypothesis. At this point, the flow continuity equation and Brinkman-Forchheimer equation can be written as the following form:

$$\nabla \cdot \boldsymbol{u} \quad 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\left(\frac{\boldsymbol{u}}{\varepsilon}\right) = -\frac{1}{\rho}\nabla(\varepsilon p) + v_e \nabla^2 \boldsymbol{u} + \boldsymbol{F} \tag{2}$$

where ε is the porosity of porous media; ρ is the density of fluid; $\boldsymbol{u}$ and p are the average speed of fluid volume and pressure, respectively; v_e is the effective kinematic viscosity coefficient; $\boldsymbol{F}$ is the total body force due to the presence of porous media and other external force fields, which can be written as the following form:

$$\boldsymbol{F} = -\frac{\varepsilon v}{K}\boldsymbol{u} - \frac{\varepsilon F_\varepsilon}{\sqrt{K}}|\boldsymbol{u}|\boldsymbol{u} + \varepsilon \boldsymbol{G} \tag{3}$$

On the right side of the equation (3), the first item is the frictional resistance of fluid and porous media skeleton, the second one is the inertia due to the presence of porous medium. v is the kinematic viscosity of the fluid; K and F_ε represent permeability and geometric function, respectively; $\boldsymbol{G}$ is the volume force caused by external forces. If $\boldsymbol{G}$ is caused only by gravity, the influence of gravity can be expressed as equation (4) under the Boussinesq hypothesis.

$$\boldsymbol{G} = -\boldsymbol{g}\beta\left(T - T_m\right) \tag{4}$$

where $\boldsymbol{g}$ is the gravitational acceleration; β is the thermal expansion coefficient; T_m is the average temperature of the system; The geometric function F_ε and the permeability K have relationship with the porosity ε, respectively. For the porous media that is made of solid particles, F_ε and K can be expressed based on Ergun's [11] empirical formula as:

$$F_\varepsilon = \frac{1.75}{\sqrt{150\varepsilon^3}},\ K = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2} \tag{5}$$

where d_p is the diameter of the solid particles.

In equation (2), the generalized Navier-Stokes equation (2) will be degraded to the standard Navier-Stokes equation when the porosity $\varepsilon \to 1$, namely, in the absence of the porous media.

The heat transfer problem always involves in fluid flow in actual applications. If we ignore the compression work and viscous heat dissipation, it can meet local thermodynamic equilibrium condition between the fluid and solid, and then the energy equation of convection heat transfer in the porous media can be expressed as:

$$\sigma \frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \nabla \cdot \left(\alpha_m \nabla T\right) \tag{6}$$

where T is the average volume temperature of fluid; The formula $\sigma = \varepsilon + (1-\varepsilon)\rho_s c_{ps} / \rho_f c_{pf}$ represents the ratio between the heat capacities of the solid and fluid phase; $\rho_s, \rho_f, c_{ps}, c_{pf}$ are the density and capacity of the solid and fluid phase, respectively; α_m denotes the effective thermal diffusivity.

In order to represent the characters of natural convection heat transfer in porous media, we can introduce several dimensionless numbers: the Darcy number $Da = K/L^2$, the viscosity ratio $J = v_e/v$, the Prandtl number $\Pr = v/\alpha_m$ and the Rayleigh number $Ra = g\beta\Delta T L^3 / v\alpha_m$.

where L is the cavity length; $\Delta T = T_H - T_L$ is the temperature difference between the hot and cold side walls.

3. LATTICE BOLTZMANN MODEL

For the natural convection heat transfer problem in porous media in this paper, we use the double distribution function model to study the fluid flow field and temperature field. Meanwhile, the D2Q9 model is employed and the lattice Boltzmann evolution equations [10] can be expressed as follows:

$$f_i\left(\boldsymbol{x}+\boldsymbol{e}_i\delta_t, t+\delta_t\right)-f_i\left(\boldsymbol{x},t\right)=-\frac{f_i\left(\boldsymbol{x},t\right)-f_i^{eq}\left(\boldsymbol{x},t\right)}{\tau_v}+\delta_t F_i \quad (7)$$

$$g_i\left(\boldsymbol{x}+\boldsymbol{e}_i\delta_t, t+\delta_t\right)-g_i\left(\boldsymbol{x},t\right)=-\frac{g_i\left(\boldsymbol{x},t\right)-g_i^{eq}\left(\boldsymbol{x},t\right)}{\tau_t} \quad (8)$$

where $i=0:8$; f_i is the distribution function; f_i^{eq} is the corresponding equilibrium distribution function; g_i is the temperature distribution function and g_i^{eq} is the equilibrium temperature distribution function; τ_v and τ_t are the velocity non-dimensional relaxation time and the temperature relaxation time, respectively. Equation (7) recovers the continuity and the momentum Eqs. (1) and (2). Equation (8) describes the evolution of the internal energy and leads to Eq. (6).

Usually the speed configuration of D2Q9 model is defined as follows:

Usually the speed configuration of D2Q9 model is defined as follows:

$$\boldsymbol{e}_i=\begin{cases}(0,0) & i=0\\ c\left(\cos\left[(i-1)\frac{\pi}{2}\right],\sin\left[(i-1)\frac{\pi}{2}\right]\right) & i=1,2,3,4\\ \sqrt{2}\left(\cos\left[(2i-1)\frac{\pi}{4}\right],\sin\left[(2i-1)\frac{\pi}{4}\right]\right) & i=5,6,7,8\end{cases} \quad (9)$$

where lattice speed $c=\delta_x/\delta_t$; δ_x and δ_t are time step and grid step. Generally the grid spaces on the directions of x and y are the same $\delta_x=\delta_y$.

On the basis of the continuous Boltzmann equation, we can get the equilibrium distribution function according to discrete the time and space. It is defined as:

$$f_i^{eq}=\omega_i\rho\left[1+\frac{\boldsymbol{e}_i\cdot\boldsymbol{u}}{c_s^2}+\frac{\boldsymbol{uu}:\left(\boldsymbol{e}_i\boldsymbol{e}_i-c_s^2\boldsymbol{I}\right)}{2\varepsilon c_s^4}\right] \quad (10)$$

$$g_i^{eq}=\omega_i T\left[\sigma+\frac{\boldsymbol{e}_i\cdot\boldsymbol{u}}{c_s^2}\right] \quad (11)$$

where $c_s=c/\sqrt{3}$ is the speed of sound; The values of the weight are given by $\omega_0=4/9$, $\omega_i=1/9\left(i=1\sim 4\right)$, $\omega_i=1/36\left(i=5\sim 8\right)$.

In Eq. (7), the forcing term can be given by:

$$F_i=\omega_i\rho\left(1-\frac{1}{2\tau_v}\right)\left[\frac{\boldsymbol{e}_i\cdot\boldsymbol{F}}{c_s^2}+\frac{\boldsymbol{uF}:\left(\boldsymbol{e}_i\boldsymbol{e}_i-c_s^2\boldsymbol{I}\right)}{\varepsilon c_s^4}\right] \quad (12)$$

The corresponding effective viscosity and the effective thermal conductivity in macro equation are given by:

$$\nu_e=c_s^2\left(\tau_v-\frac{1}{2}\right)\delta_t,\ \alpha_m=\sigma c_s^2\left(\tau_t-\frac{1}{2}\right)\delta_t \quad (13)$$

The macroscopic quantities, fluid density and internal energy are defined as:

$$\rho=\sum_i f_i,\ T=\sum_i g_i/\sigma \quad (14)$$

The speed of the fluid is calculated by using a temporary speed, which can be written as:

$$\boldsymbol{u}=\frac{\boldsymbol{\upsilon}}{c_0+\sqrt{c_0^2+c_1\left|\boldsymbol{\upsilon}\right|}} \quad (15)$$

where parameters c_0, c_1 and $\boldsymbol{\upsilon}$ are given by:

$$c_0=\frac{1}{2}\left(1+\varepsilon\frac{\delta_t}{2}\frac{\nu}{K}\right),\ c_1=\varepsilon\frac{\delta_t}{2}\frac{F_\varepsilon}{\sqrt{K}},\ \boldsymbol{\upsilon}=\sum_i f_i\frac{\boldsymbol{e}_i}{\rho}+\frac{\delta_t}{2}\varepsilon\boldsymbol{G} \quad (16)$$

Both the equilibrium distribution of lattice Boltzmann model and the forcing term contain porosity. When the porosity is equal to one, they can become the standard form. Therefore, in this model, not only the effect of porous media but also the feature of fluid free flow are considered. Both strictly obey the momentum transfer in different regions. The interface between porous medium and free fluid can automatically satisfy the continuity conditions, which avoids the problem of interface slip.

4. RESULTS AND ANALYSIS

4.1. Method Validation

In order to test the reliability of the model and method, the lattice Boltzmann method was used to simulate natural convection in a two-dimensional square cavity filled with porous medium. The vertical surfaces of the cavity are held at constant hot temperature and cold temperature, respectively. The horizontal walls are adiabatic and impermeable. To evaluate the calculation results, the comparison of average Nusselt numbers on the hot wall between the existing literature data and the LBM results is tabulated in Table **1**. Meanwhile, it shows good quantitative agreement.

4.2. The Influence of Vertical Porous Layer Width on Natural Convection Heat Transfer

In order to study the effect of variation in the width of the vertical porous layer on natural convection heat transfer in the cavity, computational parameters are set as follows: $Ra=10^6, Da=10^{-4}, \Pr=1.0, J=1, \sigma=1, \varepsilon=0.5$.

Table 1. Comparison of average Nusselt numbers on the hot wall with other numerical results.

Ra	ε	Da	Pr	Literature Data [12]	LBM Result[a]	LBM Result[b]	Calculated Value of this Paper
10^3	0.4	10^{-4}	1.0	1.010	——	1.007	1.001
10^5	0.4	10^{-4}	1.0	1.067	1.066	1.063	1.058
10^6	0.4	10^{-4}	1.0	2.550	2.602	2.544	2.573

[a]Corresponds to the reference [8], [b] Corresponds to the reference [7].

Fig. (**2**) is the fluid streamlines and isotherms diagram when steady-state is reached under different *D*. It can be clearly seen from Fig. (**2**) that at D = 0, namely, in the absence of the porous media, there is a pair of symmetrical distribution of vortex in the cavity. With the increase of porous layer width, the left vertex moves to the domain of pure fluid and has a little reduction, while the right one becomes long and narrow. At D = 1, the cavity is full of porous media and there is only one vertex in the body.

The impact of dimensionless width *D* on natural convection can be observed in Fig. (**3**). On the whole, the heat transfer intensity decreases with the increase of porous layer width. In the case of $D < 0.1$ or $D > 0.9$, the hot wall average Nusselt number is sensitive to porous layer width. A slight increase of porous layer width can result in drastic deceasing of Nusselt number. However, with increasing porous layer width, there is subtle variation in Nusselt number for $0.1 < D < 0.9$, which means the central region has little contribution to the natural convection heat transfer.

4.3. The Influence of Rayleigh Number on Natural Convection Heat Transfer

Fig. (**4**) shows the effects of different Rayleigh numbers on the average Nusselt number. The value of the Rayleigh numbers are 10^3,10^4,10^5,10^6 and 10^7, respectively. As it can be seen from Fig. (**4**), with increasing *Ra*, the average Nusselt number on the hot wall jumps significantly. This is because the greater the Rayleigh number, the stronger the motivation of natural convection and vortex intensity. Conversely, at the low Rayleigh numbers (10^3 to 10^4), the change of hot wall average Nusselt number is not obvious. However, at a relative small Darcy number, the change of Rayleigh number has little influence on the intensity of natural convection.

Fig. (**5**) shows the vertical velocity and temperature comparison along the horizontal line at mid-height for different Rayleigh numbers. When the value of *Ra* is small, the velocity is almost zero in the porous media region and the free fluid region. With increasing *Ra*, the fluid flow in free space gradually enhances and the velocity boundary layer gradually thins. Along with the further increase of Rayleigh number, natural convection heat transfer in porous media becomes stronger. At $Ra = 10^3$, the temperature profile is almost linear distribution, since only conduction occurs. With increasing *Ra*, the temperature is almost flat in the bulk region, but the temperature gradient changes greatly near the vertical walls. Meanwhile, because of the existence of thin porous layer in the left side, the temperature boundary near the left wall is thicker than the right.

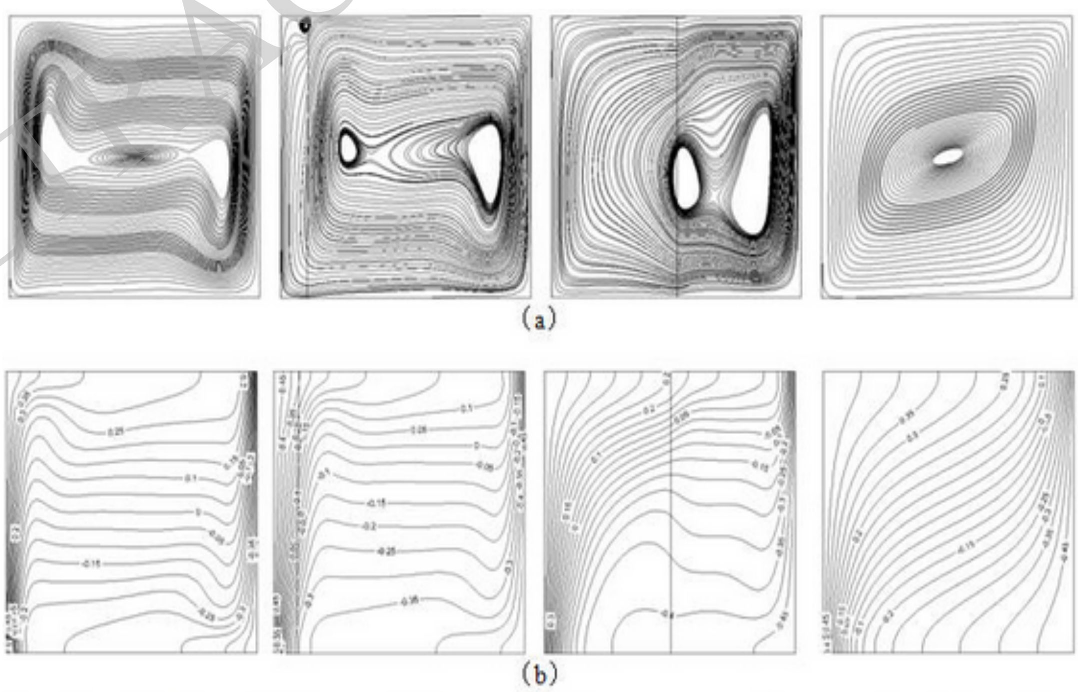

Fig. (2). Streamlines (**a**) and isotherms (**b**) for a variable D. $Da = 10^{-4}$, $\Pr = 1.0$, $Ra = 10^6$, $\varepsilon = 0.5$; from left to right, D=0,0.1, 0.5,1.0.

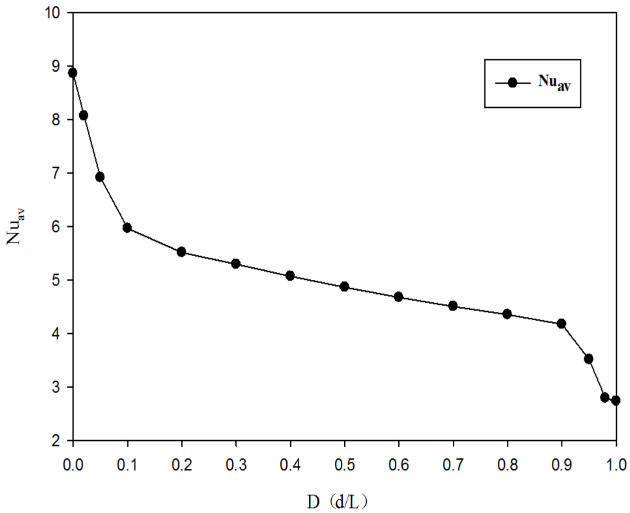

Fig. (3). Nusselt number as function of the dimensionless width D.

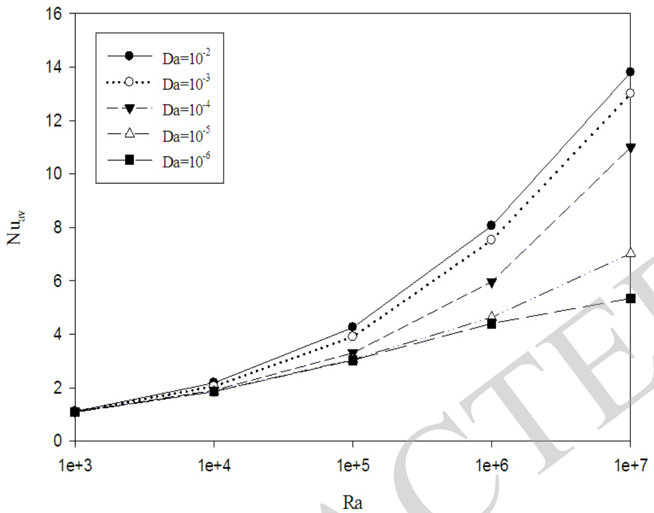

Fig. (4). The effects of Rayleigh number on the Nusselt number for different Darcy numbers, $D = 0.1$, $\varepsilon = 0.5$.

4.4. The Influence of Darcy Number on Natural Convection Heat Transfer

Fig. (**6**) shows the effects of different Darcy numbers on average Nusselt number. With increasing Darcy number, the average Nusselt number on hot wall increases greatly. However, when the Darcy number is in the range of 10^{-5}, the curve remains horizontal and the Nusselt number has a little growth, which is because the heat transfer mainly rely on solid heat conduction. At $Da > 10^{-5}$, the Nusselt number increases rapidly, which can be explained by the coexistence of natural convection and heat conduction in the porous media cavity.

4.5. The Influence of Porosity on Natural Convection Heat Transfer

Fig. (**7**) shows the influence of the porosity on heat transfer for different Darcy numbers. Here we can see that the variation of porosity has little impact on the average Nusselt number of the hot wall at low Da. At $Da \geq 10^{-3}$, the influence of the porosity on natural convection is significant.

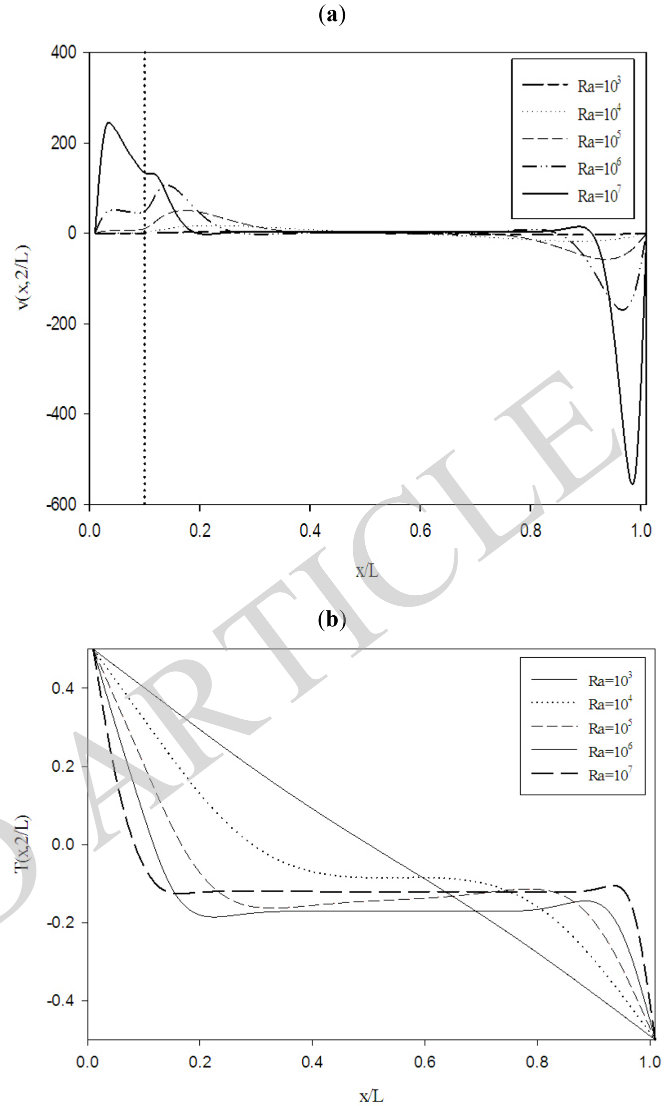

Fig. (5). Vertical velocity distribution (**a**) and temperature distribution (**b**) and at mid-height of cavity for different Rayleigh numbers.

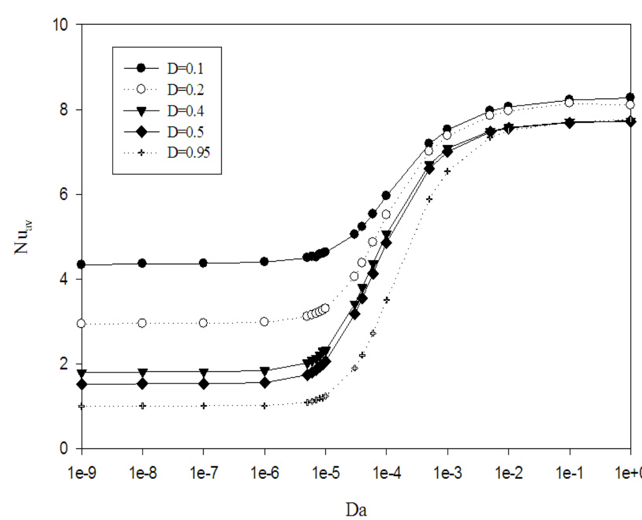

Fig. (6). The effects of Darcy number on the Nusselt number, $Ra = 10^6$, $\varepsilon = 0.5$.

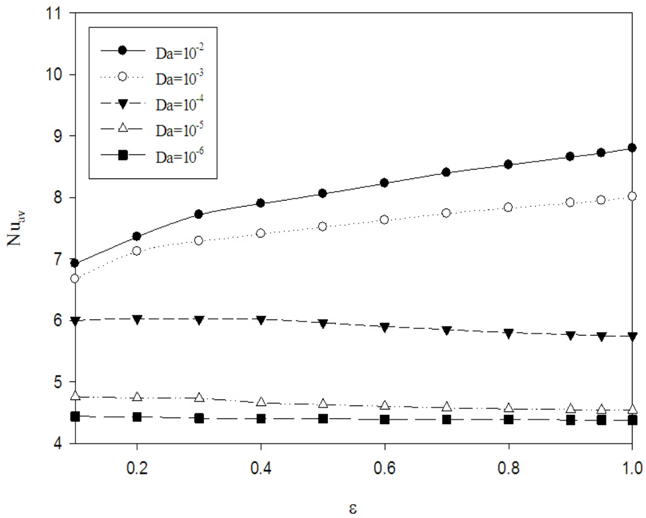

Fig. (7). Nusselt number as function of the porosity for different Darcy numbers, $Ra = 10^6$, $D = 0.1$.

CONCLUSION

A systematic numerical study of natural convection in the cavity partially filled with porous media has been presented by using the lattice Boltzmann method. Based on the results, following main conclusions can be achieved:

1. The lattice Boltzmann method and the proposed model are capable of solving natural convection in porous media at the representative elementary volume scale.
2. In the case of $D < 0.1$ or $D > 0.9$, the hot wall average Nusselt number is sensitive to porous layer width. However, at $0.1 < D < 0.9$, the change of thickness has little influence on heat transfer.
3. The Rayleigh number and Darcy number have a great impact on natural convection, of which the effect of the former is more distinct than the latter. Furthermore, in the case of high Darcy number, the flow and heat transfer in compound cavity enhance significantly with the increase of porosity.

CONFLICT OF INTEREST

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work.

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