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# Monopole Acoustic Emission Generated by a Molten Metal Drop in Cool Water

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**Abstract:** The problem of high-frequency acoustic emission generated by a spherical hot liquid drop in a cool ambient liquid (water) is considered analytically. It is assumed that the acoustic emission is generated by the motion of the vapor˜cold liquid interface, which is described by the Rayleigh equation for a spherical bubble. It is also assumed that the linear relationship between the saturated pressure and the temperature is governed by the Clapeyron-Clausius law, that heat transfer from the hot liquid drop to the vapor film and the liquid takes place by a radiation, and that heat transfer from the water-vapor interface to the ambient liquid takes place by conduction. Solution of the equation obtained gives an approximate radius time relationship that allows us to evaluate the kinetic enthalpy and acoustic pressure in a far field. It is shown that, in the first approximation, the spectral density of the acoustic energy emitted by such a phenomenon varies as the minus 2.7 power of the frequency and decreases by approximately 8.1 decibels per octave with a rise in frequency.

## 1. INTRODUCTION

The study of acoustic emission during the growth and collapse of vapor bubbles is associated with acoustic techniques used to detect the start of boiling. Both experimental and theoretical results may be useful for the detection of acoustic noises in high-voltage power supplies and microwave power tubes for ground and airborne radar transmitters that are thermally controlled by immersion into a dielectric coolant. Acoustic methods may also be useful for monitoring closed thermohydraulic loops and other pressurized systems, such as reactors in which sonic and ultrasonic noises are generated. The Argonne National Laboratory conducted research and development of acoustic surveillance of reactor and power plants to develop an understanding of the sources of unusual sound, and a performance analysis was conducted to show that a combined acoustic neutronic system was feasible and that such a system would provide a rapid means of reactor shutdown [1, 2]. Acoustic emission induced by boiling and cavitating bubbles in ideal and viscous liquids had been studied by a number of authors [3-8]. Theoretical and experimental investigations show that the maximum spectral intensity for isothermal conditions is observed at a natural frequency corresponding to the stage of the Rayleigh bubble growth or collapse. The spectral density of the acoustic energy varies as $S \sim \omega^4$ at low frequencies; as $S \sim \omega^{-2/5}$ for incompressible inviscid liquids [9], and as $S \sim \omega^{-7/3}$ for compressible viscous liquids [10], at high frequencies. A comprehensive survey of boiling noises in reactors and thermal hydraulic loops has been accomplished by Scarton *et al*. [11], As long ago as the middle of the last century, Plesset and Zwick [12] considered the problem of vapor bubble expansion in a superheated liquid and obtained the dependencies of the radius of the vapor bubble on time for different values of water superheating. It should be noted that if the pressure is known, it is possible to determine the acoustic characteristics of the sources of the perturbations.

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When a hot liquid droplet falls into a cool liquid, a vapor film is generated between the hot and cold liquids. The heat flux entering in the system is spent for evaporation and vapor and liquid heating. Among this process the main part of the energy is spent for evaporation because the latent heat evaporation is very high for water (energy needed for heating up 1 mole of water on $50^0$K is equal to 3765 J while energy needed to evaporate that 1 mole of water is equal to 40683 J which is more than an order of magnitude higher). It may be assumed that sound radiation will be induced by an initial pressure pulse that is brought about by motion of water-vapor interface.

The present study was undertaken to evaluate the high-frequency part of the acoustic noise spectrum generated by a hot molten metal drop in cool water, a phenomenon that could hypothetically occur during melt-down of an active zone in a nuclear reactor.

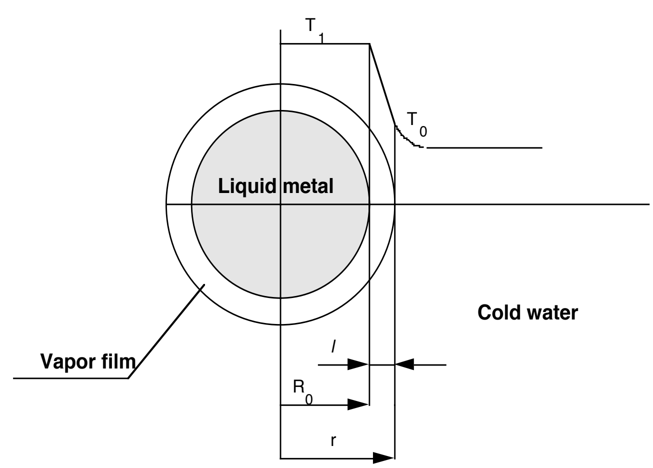


**Fig. (1).** Schematic model.

## 2. PROBLEM STATEMENT

It is assumed that the hot liquid drop is a sphere of radius $R_0$ surrounded by a vapor film of thickness $l$ (Fig. **1**). In real-

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ity, the molten metal drop becomes ellipsoidal [13], but at the initial stages of the process under consideration, these deformations are neglected. Similarly, an influence of buoyancy forces is also neglected. The motion of the spherical vapor – cold liquid interface is described by the Rayleigh equation:

$$r\frac{d^2r}{dt^2}+\frac{3}{2}\left(\frac{dr}{dt}\right)^2=\frac{p_s-p_\infty}{\rho_L} \tag{1}$$

where $r$ is the interface radius, $p_S$ is the vapor pressure, $p_\infty$ is the pressure at infinity, $\rho_L$ is the mass density of the cold liquid, and $t$ is time. It is assumed that surface tension does not play a significant role in the process, i.e., that the drop is sufficiently large. The capillary pressure can be neglected provided that $p_\sigma \approx \dfrac{\sigma}{\lambda} << \rho g \lambda$, where is the scale of motion and $\sigma$ is the surface tension, i.e., if the scale of motion is large in comparison with the capillary constant $\left(\dfrac{\sigma}{g\rho}\right)^{1/2}$, the surface forces do not affect the motion. This will hold true if the droplet diameter exceeds 1-2 mm ($\sigma \approx$ 60 dyne/cm at ~ 100˚C). The right-hand side of equation (1) can be approximately represented by the Clapeyron-Clausius law (which holds true when vapor superheating is not very large)

$$\frac{dp}{dT}=\frac{L}{T(v_0'-v_0)}, \tag{2}$$

where $T$ is the temperature, $L$ is the latent heat of evaporation, $v'_0$ and $v_0$ are the specific volumes of vapor and water (whose dependencies on temperature and pressure are neglected), in the form:

$$\frac{p_s-p_\infty}{\rho_L}\approx A(T-T_0)\cdot \tag{3}$$

In expression (3), $T_0$ is the boiling temperature at the pressure $p_\infty$, and

$$A=\frac{L}{T_0\rho_L(v_0'-v_0)}\cdot$$

Thus, the problem under consideration is reduced to the solution of equation (1) with initial conditions $r = R_0$, $\dot r = \ddot r = 0$ at $t = 0$, where a dot above a letter denotes differentiation with respect to time, and $R_0$ is the radius of the hot drop.

## 3. MOTION OF VAPOR FILM – WATER INTERFACE

It is assumed that the main contribution to the heat flux from the hot drop to the vapor film and ambient water is a result of radiation. The quantity of the heat entering the system is

$$Q=4\pi R_0^2k_r\sigma_{SB}T_1^4 \tag{4}$$

where $k_r$ is the radiation coefficient, $\sigma_{SB}$ is the Stephan-Boltzmann constant, and $T_1$ is the initial temperature of the hot drop. A part of this heat flux goes to water evaporation and heating vapor and water, but, by reasons noted in Introduction, the convective and conductive losses are neglected in comparison with vaporization, although, in principle, they can be taken into account. The mass of vapor (per second) is

$$m=\frac{4}{3}\rho'\left[(R_0+l)^3-R_0^3\right]\approx 4\pi\rho' R_0^2 l$$

where $\rho'=1/v_0'$ is vapor density, $l$ is the vapor film thickness , and the quantity of heat going for evaporation is

$$Q_1\cong\frac{4\pi}{v_0'}LR_0^2 l \tag{5}$$

The heat transfer from the vapor film to the ambient cool liquid is given by:

$$Q_2=4\pi(R_0+l)^2k\left(\frac{\partial T}{\partial r}\right)_r \tag{6}$$

where $k$ is the thermal conductivity coefficient. From equations (4), (5) and (6), it follows that

$$\left(\frac{\partial T}{\partial r}\right)_r=\frac{R_0^2k_r\sigma_{SB}T_1^4-R_0^2L(r-R_0)/v_0'}{kr^2} \tag{7}$$

Substituting equation (3) into equation (1), then differentiating (1) with respect to time ($r\dfrac{d^3r}{dt^3}+\dfrac{d^2r}{dt^2}\dfrac{dr}{dt}+3\dfrac{dr}{dt}\dfrac{d^2r}{dt^2}=A\dfrac{dT}{dt}$, $\dfrac{\partial T}{\partial t}=\dfrac{\partial T}{\partial r}\dfrac{\partial r}{\partial t}$), and using equation (7), we get

$$r\frac{d^3r}{dt^3}+4\frac{d^2r}{dt^2}\frac{dr}{dt}=\left[\frac{A}{kr^2}\left(R_0^2k_r\sigma_{SB}T_1^4+R_0^3L/v_0'\right)-\frac{A}{kr}R_0^2L/v_0'\right]\frac{dr}{dt} \tag{8}$$

By denoting

$$\frac{A}{k}\left(R_0^2k_r\sigma_{SB}T_1^4+R_0^3L/v_0'\right)=F \quad \text{and} \quad \frac{A}{k}R_0^2L/v_0'=G$$

the following equation is obtained:

$$r\dddot r+4\ddot r\dot r-\left(\frac{F}{r^2}-\frac{G}{r}\right)\dot r=0 \tag{9}$$

If equation (9) is multiplied by $r^3$, it will be an equation in full differentials. Integration gives:

$$r^3\ddot r+\frac{1}{2}Fr^2-\frac{1}{3}r^3=C_1 \tag{10}$$

where

$$C_1=\frac{F}{2}R_0^2-\frac{G}{3}R_0^3 \tag{11}$$

If equation (10) is multiplied by $2\dot r r^{-4}$, then the first order equation is obtained:

$$\dot r^2=C_2+\int r^{-4}\left[C_1-\int r^3\left(\frac{F}{r^2}-\frac{G}{r}\right)dr\right]=Fr^{-1}-\frac{2}{3}C_1r^{-3}+\frac{2}{3}G\ln r \tag{12}$$

From equation (12), it follows that

$$t=\int\frac{dr}{\sqrt{Fr^{-1}-\frac{2}{3}C_1r^{-3}+\frac{2}{3}G\ln r}} \tag{13}$$

This integral can not be expressed in elementary functions but it can be simplified taking into account that second and third terms in the radicand are much smaller than first and can be neglected. Then, $t\approx\dfrac{1}{\sqrt{F}}\int r^{1/2}dr$, and integration of this equation leads to

$$r^{3/2} = \frac{3\sqrt{F}}{2}t \quad (14)$$

where integration constant is included into $t$. The radius–time relationship is given by

$$r = (3/2)^{2/3} \sqrt[3]{F}\, t^{2/3} = \alpha t^{2/3} \quad (15)$$

where

$$\alpha = (3/2)^{2/3} \sqrt[3]{F} \quad (16)$$

is introduced for the sake of brevity. For purposes of comparison, it should be noted that a radius of the vapor bubble in a superheated liquid according to Plesset and Zwick [12], increases proportionally to $t^{1/2}$.

## 4. ACOUSTIC PRESSURE

As follows from the continuity equation [$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{u}) = 0$], the liquid compressibility need not be taken into account, if $\partial \rho / \partial t << \rho \operatorname{div} \boldsymbol{u}$, i.e., $\Delta\rho / \vartheta << \rho u / \ell$ or $\Delta\rho / \rho << \vartheta u / \ell$, where $\vartheta$ and $\ell$ are the time and the distance scales, respectively. The condition permitting the liquid to be considered as incompressible is $\ell / \vartheta c_\infty << 1$ or $\ell / c_\infty << \vartheta$, where $c_\infty$ is the adiabatic sound velocity in undisturbed liquid. The physical meaning of this condition is that the time $\ell / c_\infty$ during which a sound wave propagates through the distance $\ell$ must be small in comparison with the time during which the liquid velocity changes significantly. Therefore, when the condition $\ell / c_\infty << \vartheta$ holds, perturbations in the vicinity of the expanding cavity can be considered as propagating instantaneously, and the liquid may be treated as an incompressible medium. At large distances, where the condition $\ell / c_\infty << \vartheta$ is undoubtedly invalid, the liquid compressibility has to be taken into account. Hence, liquid compressibility may be disregarded in the vicinity of an expanding sphere but must be taken into account for calculation of the sound radiation. For calculation velocities and pressure, the Kirkwood and Bethe hypothesis described by Cole [14], and Gilmore [15], can be used. It bases on an assumption that a function $r\left(h + \frac{u^2}{2}\right)$, where $h(p) = \int_{p_\infty}^{p} \frac{dp}{\rho}$ is the enthalpy and $u$ is the radial velocity, propagates in liquid with the velocity that equals to the sum of the local sound velocity and the liquid particle velocity (in the case of spherical symmetry). Mathematically, it is written as

$$\left[\frac{\partial}{\partial t} + (c+u)\frac{\partial}{\partial r}\right]\left[r\left(h + \frac{u^2}{2}\right)\right] = 0 \quad (17)$$

In spherical coordinates, the Navier-Stokes equation can be written in the form (the term representing the product of kinematic viscosity on $\operatorname{grad}\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is neglected as a second order infinitesimal):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{\partial h}{\partial r} \quad (18)$$

On the basis of equalities $c^2 = \frac{\partial p}{\partial \rho}$ and $dh = \frac{dp}{\rho}$, the continuity equation can be written as

$$\frac{\partial u}{\partial r} + \frac{2u}{r} = -\frac{1}{c^2}\frac{\partial h}{\partial t}. \quad (19)$$

For convenience, equation (17) is written through a full derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$:

$$\left[\frac{d}{dt} + c\frac{\partial}{\partial r}\right]\left[r\left(h + \frac{u^2}{2}\right)\right] = 0 \cdot \quad (20)$$

Carrying out term-by-term differentiation gives

$$r\frac{dh}{dt} + ru\frac{du}{dt} + (c+u)\left(h + \frac{u^2}{2}\right) + rc\frac{\partial h}{\partial r} + rcu\frac{\partial u}{\partial r} = 0 \cdot \quad (21)$$

Derivatives with respect to coordinate can be excluded with a help of equations (18) and (19) that leads to complete equation for cavity dynamics in a spherical case obtained by Gilmore [15], and presented also in the monograph by Naugol'nykh and Roi [16]:

$$\frac{du}{dt}\left(1 - \frac{u}{c}\right) + \frac{3}{2}\frac{u^2}{r}\left(1 - \frac{u}{3c}\right) = \left(1 + \frac{u}{c}\right)\frac{h}{r} + \frac{1}{c}\left(1 - \frac{u}{c}\right)\frac{dh}{dt}, \quad (22)$$

where $u$ is the radial velocity of the cavity or bubble wall, $c$ is the local sound velocity This equation can be simplified if we take into account that the value of the hydrodynamic velocity $u$ remains small in comparison with the local sound speed (Mach numbers $M$ << 1). Then, the values $u/c$ can be neglected in comparison with 1, and $c$ can be replaced by $c_\infty$. Equation (22) may then be rewritten as:

$$\frac{du}{dt} + \frac{3}{2}\frac{u^2}{r} = \frac{h}{r} + \frac{1}{c_\infty}\frac{dh}{dt}, \quad (23)$$

where

$$u = \dot{r} \quad \text{and} \quad h = \frac{c_\infty^2}{n-1}\left[\left(\frac{p+B}{p_\infty + B}\right)^{\frac{n-1}{n}} - 1\right] \quad (24)$$

according to the Tait equation of state $\left(\frac{\rho_L}{\rho_\infty}\right)^n = \frac{p+B}{p_\infty + B}$ (Vogel *et al*. [17]), where n = 7 and B ≈ 300 MPa as a good approximation for water. It is clear that perturbations will propagate with the velocity $c_\infty$ and wave form will not be changed, therefore, shock wave does not develop.

Equation (23) provides the possibility of determining the enthalpy if the law of cavity expansion is known. Substituting equation (16) and its derivatives ($\dot{r} = \frac{2}{3}\alpha^{2/3}t^{-1/3}$, $\ddot{r} = -\frac{2}{9}\alpha^{2/3}t^{-4/3}$) into equation (23) gives a linear first-order equation for the enthalpy:

$$h' + \xi t^{-2/3} h - \zeta t^{-2/3} = 0, \qquad (25)$$

where $\xi = c_\infty \alpha^{-2/3}$ and $\zeta = \frac{4}{9}\alpha^{4/3}$ ; and the initial condition is $h = 0$ at $t = 0$. The solution of equation (25) is

$$h = \zeta\, exp\left(-3\xi t^{1/3}\right)\int_0^t y^{-2/3}\, exp\left(3\xi y^{1/3}\right)dy = \frac{4\alpha^2}{9c_\infty}\left(1 - e^{-3\xi t^{1/3}}\right), \qquad (26)$$

where $y$ is the integration variable. As follows from equation (24), the "reduced" pressure is given by

$$p_* = \frac{p + B}{p_\infty + B} = \left[\frac{(n-1)h}{c_\infty^2} + 1\right]^{\frac{n}{n-1}}, \qquad (27)$$

and, neglecting $p_\infty$ in comparison with $B$ (the pressure in nuclear reactors does not exceed 10 MPa), the acoustic pressure, in the first approximation, may be represented as:

$$p \approx \frac{nB}{c_\infty^2} h, \qquad (28)$$

and, therefore,

$$p \approx \frac{4}{9}\frac{nB\alpha^2}{c_\infty^3}\left(1 - e^{-3\xi t^{1/3}}\right), \qquad (29)$$

that holds true for the initial stages of bubble expansion.

## 5. SPECTRAL DENSITY OF RADIATED NOISE

The Fourier transform of an aperiodic function is

$$p(t) = \int_{-\infty}^{\infty} L(i\omega) e^{i\omega t} dt, \qquad (30)$$

where $L(i\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p(t) e^{-i\omega t} dt$ is the complex function of the frequency, $\omega$. Taking into account that $p(t) = 0$ at $-\infty < t < 0$, we may write the function $L(i\omega)$ as:

$$L(i\omega) = \frac{2}{9\pi}\frac{nB\alpha^2}{c_\infty^3}\int_0^\infty \left(1 - e^{-3\xi t^{1/3}}\right) e^{-i\omega t} dt \qquad (31)$$

If the exponent in the integrand is expanded into a series and only the two first terms are retained, then the following integral is obtained:

$$L(i\omega) \cong \frac{8}{27\pi}\frac{nB\alpha^{10/3}}{c_\infty^3}\int_0^\infty t^{1/3} e^{-i\omega t} dt\,. \qquad (32)$$

Since this integral diverges, the integral $\Im = \int_0^\infty t^{1/3} e^{-\varepsilon t - i\omega t} dt$ has to be calculated, and then, in the result, it must be supposed that $\varepsilon \to 0$. According to formula 3.351.5 from Gradshteyn and Ryzhik, [18], this integral is

$$\Im = \Gamma\left(\frac{4}{3}\right)\left(\varepsilon^2 + \omega^2\right)^{-2/3} \exp\left(-i\frac{4}{3}\arctan\frac{\omega}{\varepsilon}\right), \qquad (33)$$

where $\Gamma$ (4/3) $\approx$ 0.893 is a gamma function, and when $\varepsilon \to 0$,

$$\Im = \Gamma\left(\frac{4}{3}\right)\omega^{-4/3} \exp\left(-i\frac{2\pi}{3}\right) \qquad (34)$$

The modulus of $L$ gives the amplitude harmonic components of the pressure impulse distribution. The spectral density of the acoustic energy is given by the modulus square of $L$, $S(\omega) = |L(i\omega)|^2$; therefore, with an accuracy up to a numerical multiplier, the spectral density is obtained in the form:

$$S(\omega) \sim \omega^{-8/3} \qquad (35)$$

## 6. CONCLUDING REMARKS

In this study, the theoretical analysis of the high-frequency part of the acoustic emission spectrum relied on a number of rough approximations (neglecting the liquid compressibility, buoyancy and capillary forces, deviations from spherical form, and heat losses) that certainly limit its applicability. However, the analytical result does allow the asymptotic behavior of this spectrum to be evaluated, and this could be useful for comparison with experimental data. According to Howe [19], the acoustic pressure at large distances from a spherically expanding flame can be approximately estimated as $p(x,t) \approx A_1 [d\Omega/dt]_{t-|x|/c_\infty}$, where $A_1$ is some function of the temperature and the radius, and $\Omega$ is the flame area at time $t$. In this case, the temporal dependence of this area on an expanding sphere radius is given by $d\Omega/dt = 8\pi r\dot{r}$, and, if $r \sim t^{2/3}$, the acoustic pressure is proportional to $t^{1/3}$, which is in agreement with the integrand in equation (32). The result obtained shows that the spectral density of the acoustic energy varies with the rise in frequency as a minus 8/3 power of the frequency and decreases approximately by 10log $2^{-2.7}$ = -8.1 decibels per octave. This value is substantively higher than in the case of spherical waves propagating in an inviscid liquid [20], We may therefore assume that the attenuation of ultrasonic components will be very fast.

## REFERENCES


[1] Carey W. The detection of sodium vapor bubble collapse in a liquid metal fast breeder reactor, Progress in Nuclear Energy, Vol. 1, Pergamon Press, New York, 1977; 437-468.

[2] Carey W. The acoustic background noise of a liquid metal fast breeder reactor, J Acoust Soc Am 1979; 65 (6), 1461-1466.

[3] Aoki T, Welty Y.R. Frequency distribution of boiling generated sound, J of Heat Transfer, 1970; 542-544.

[4] Finch R.D, Neppiras E.A. Vapor bubble dynamics, J Acoust Soc Am 1973; 53, No 5, 1402-1410.

[5] Haig C.P, Ponter A.B. Sound emission and heat transfer in low pressure pool boiling, Int. J. Heat and Mass Transfer, 1969; 12, No 4, 413-428.

[6] Keller J.B, Miksis M. Bubble oscillations of large amplitude, J Acoust Soc Am 1992; 68, No 2, 628-663.

[7] Lezzi A, Prosperetti A. Bubble dynamics in a compressible liquid. Part 2. Second order theory. J Fluid Mech, 1987; 185, 299-321.

[8] Longuet-Higgins M.S, Monopole emission of sound by asymmetric bubble collapse, J. Fluid Mech. 1989; 201, 543-565.

[9] Likhterov L, Elperin T, High frequency asymptotics of the acoustical noise spectrum in nucleate pool boiling, Int Commun in Heat and Mass Transfer 1990; 17, No 2, 179-188,.

[10] Likhterov L. High frequency asymptotics of acoustical nucleate pool boiling noise spectrum in compressible viscous liquids. Int J Heat and Mass Transfer 1993; 36, No 16, 3931-3935.

[11] Scarton H, McDonald J, Carey W. A review of the acoustic detection of boiling in nuclear reactors. Proceedings, Noise and Fluids Engineering, ASME, New York; 1977; 109-123.

[12] Plesset MS, Zwick SA. The growth of vapor bubbles in superheated liquids, J Appl Phys 1954; 25, No 4, 493-500.

[13] Matsumura K, Nariai H. Self-triggering mechanism of vapor explosions for a molten tin and water system. J Nucl Sci Technol 1996; 33, No 4, 298-306.

[14] Cole RH. Underwater Explosions. Princeton University Press, Princeton, New Jersey; 1948.

[15] Gilmore F.R. The growth and collapse of a spherical bubble in a viscous compressible liquid. Hydrodynamic Laboratory Rep. *26-4*. California Institute of Technology; 1952.

[16] Naugol'nykh K.A, Roi N.A. Electrical Discharges in Water. Nauka, Moscow; 1971 (in Russian).

[17] Vogel A, Busch S, Parlitz U. Shock wave emission and cavitation bubble generation by picosecond and nanosecond optical breakdown n water. J Acoust Soc Am 1996; 100(1), 148-165.

[18] Gradshteyn I.S, Ryzhik I.M. Table of Integrals, Series and Products. Academic Press, Inc, New York; 1980.

[19] Howe M.S. Acoustics of Fluid-Structure Interactions. Cambridge University Press; 1998.

[20] Likhterov L, Berman A, Spectrum of high-frequency acoustic noise in inviscid liquid - linear approximation for spherical waves. J. Vibration and Acoustics 2003; 125, 249-251.

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