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# Rapid Heating Induced Vibration of a Laminated Shell with the GDQ Method

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**Abstract:** The generalized differential quadrature (GDQ) method is used to obtain the computational results of two-layer cross-ply laminated shell under rapid uniform heat induced vibration. In the thermally insulated vibration analyses of a laminated shell, the sudden uniform heat input over its lower surface only and temperature load is linear of axial coordinate. The GDQ method provides a method for calculating the time response of axial, circumferential and normal displacements.

**Keywords:** Generalized differential quadrature, GDQ, Laminated shell, Rapid uniform heat, Insulated vibration.

## 1. INTRODUCTION

When a multilayered shell is used in thermally insulated vibration problem, there is a normal flow of heat through the thickness direction of the shell structure. In 1974, Jadeja and Loo made the heat induced vibration study of a rectangular plate under sinusoidal heating loads [1]. In 1981, Chung had studied the free vibration problem of circular cylindrical shells [2]. In 1988, Bert, Jang and Striz had used the generalized differential quadrature (GDQ) method to study free vibration of the structure components [3]. In 1997, Shu and Du used the GDQ method to make the research of clamped and simply supported boundary conditions of beams and plates under free vibration [4]. In 1998, Hua and Lam used the GDQ method to study the frequency characteristics of a thin rotating cylindrical shell [5]. In 2000 and 2003, Hong and Jane used the numerical GDQ method to study the thermal bending and thermal vibration of laminated plates, respectively [6-8]. In 1973, Dym made the research for the Vibrations of Circular Cylinders [9].

In 2006, Yilbas and Kalyon made the analytical research of heating and cooling with laser for multilayer assembly. They found that the temperature rises rapidly in the heating cycle, but temperature decay is gradual in the cooling cycle for the two-layer steel and copper assembly [10]. In 2006, Maki, Ishiguro, Mori and Makino present the experimental results and studied the thermo-mechanical treatment by using the resistance heating for aluminum alloy sheets [11]. In 2004, Huang and Tan made the finite element research of both external moment and heating on the thermally restrained steel columns. They studied the effects of both rapid and slow heating for the columns [12]. In 1957, Boley and Barber [13] and in 1958 Brull [14], they made the rapid heating dynamic response studies for the beams and plates. In 2007, Hong made the study about thermal vibration of magnetostrictive material in laminated plates with the GDQ method. The time responses of center displacement and stresses with and without velocity control have been obtained, respectively [15]. Recently, there are seldom studies about the thermal vibration of multilayer shell in open literature or commercial software. In this paper, we study the thermally induced vibration of a laminated shell with two edges clamped condition under rapid heating loads. The GDQ method is used to obtain some numerical time response displacements.

## 2. DYNAMIC EQUILIBRIUM DIFFERENTIAL EQUATIONS

We consider a thin generally orthotropic multilayered shell under vibration and thermal effect, the thermoelastic stress-strain relationship of the $k^{th}$ layer including thermal strain can be given as in the equations of [16, 17]. From the Love's theory for thin multilayered shell under the pulsating axial load $N_a$, the thermally dynamic equilibrium differential equations for a stress field in the $k^{th}$ layer can be expressed as in the forms of [5, 16]. Considering the behaviors of a multilayered shell constructed of specially orthotropic layers, we can rewritten the following equilibrium differential matrix equations in terms of displacement components $u, v$ and $w$ [16]:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \tag{1}$$

where

$$L_{11} = A_{11} \frac{\partial^2}{\partial x^2} + \frac{A_{66}}{R^2} \frac{\partial^2}{\partial \theta^2} - \rho_t \frac{\partial^2}{\partial t^2}$$

$$L_{12} = \left(\frac{B_{12}}{R^2} + \frac{A_{12}}{R} + \frac{A_{66}}{R} + 2\frac{B_{66}}{R^2}\right) \frac{\partial^2}{\partial x \partial \theta}$$

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$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - B_{11}\frac{\partial^3}{\partial x^3} - (\frac{B_{12}}{R^2} + 2\frac{B_{66}}{R^2})\frac{\partial^3}{\partial x \partial \theta^2}$$

$$L_{21} = (\frac{B_{12}}{R^2} + \frac{A_{12}}{R} + \frac{A_{66}}{R} + \frac{B_{66}}{R^2})\frac{\partial^2}{\partial x \partial \theta}$$

$$L_{22} = (A_{66} + \frac{3B_{66}}{R} + 2\frac{D_{66}}{R^2})\frac{\partial^2}{\partial x^2}$$

$$+ (\frac{2B_{22}}{R^3} + \frac{A_{22}}{R^2} + \frac{D_{22}}{R^4})\frac{\partial^2}{\partial \theta^2}$$

$$- \rho_t \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial x}(N_a \frac{\partial}{\partial x})$$

$$L_{23} = (\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3})\frac{\partial}{\partial \theta}$$

$$- (\frac{2B_{66}}{R} + \frac{B_{12}}{R} + \frac{2D_{66}}{R^2} + \frac{D_{12}}{R^2})\frac{\partial^3}{\partial x^2 \partial \theta}$$

$$- (\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4})\frac{\partial^3}{\partial \theta^3}$$

$$L_{31} = -\frac{A_{12}}{R}\frac{\partial}{\partial x} + B_{11}\frac{\partial^3}{\partial x^3} + (\frac{B_{12}}{R^2} + 2\frac{B_{66}}{R^2})\frac{\partial^3}{\partial x \partial \theta^2}$$

$$L_{32} = -(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3})\frac{\partial}{\partial \theta}$$

$$+ (\frac{2B_{66}}{R} + \frac{B_{12}}{R} + \frac{4D_{66}}{R^2} + \frac{D_{12}}{R^2})\frac{\partial^3}{\partial x^2 \partial \theta}$$

$$+ (\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4})\frac{\partial^3}{\partial \theta^3}$$

$$L_{33} = 2\frac{B_{12}}{R} - D_{11}\frac{\partial^4}{\partial x^4} - (2\frac{D_{12}}{R^2} + \frac{4D_{66}}{R^2})\frac{\partial^4}{\partial x^2 \partial \theta^2}$$

$$- \frac{D_{22}}{R^4}\frac{\partial^4}{\partial \theta^4} + 2\frac{B_{22}}{R^3}\frac{\partial^2}{\partial \theta^2} - \frac{A_{22}}{R^2}$$

$$- \rho_t \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial x}(N_a \frac{\partial}{\partial x})$$

$$b_1 = \frac{\partial \overline{N}_x}{\partial x} + \frac{1}{R}\frac{\partial \overline{N}_{x\theta}}{\partial \theta}$$

$$b_2 = \frac{\partial \overline{N}_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial \overline{N}_{\theta}}{\partial \theta} + \frac{1}{R}\frac{\partial \overline{M}_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial \overline{M}_{\theta}}{\partial \theta}$$

$$b_3 = \frac{\partial^2 \overline{M}_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 \overline{M}_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial^2 \overline{M}_{\theta}}{\partial \theta^2} - \frac{\overline{N}_{\theta}}{R}$$

$\rho_t = \int_{-h/2}^{h/2} \rho dz$ in which $\rho$ is the density, $h$ is the thickness of shell. Where $u, v$ and $w$ are displacement components in the directions of axial $x$, circumferential $\theta$ and normal $z$, respectively. $R$ is the mean radius.

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\ (1, z, z^2) dz$$

$$(\overline{N}_x, \overline{M}_x) = \int_{-h/2}^{h/2} (\overline{Q}_{11}\alpha_x + \overline{Q}_{12}\alpha_\theta + \overline{Q}_{16}\alpha_{x\theta}) \Delta T(1, z) dz$$

$$(\overline{N}_\theta, \overline{M}_\theta) = \int_{-h/2}^{h/2} (\overline{Q}_{12}\alpha_x + \overline{Q}_{22}\alpha_\theta + \overline{Q}_{26}\alpha_{x\theta}) \Delta T(1, z) dz$$

$$(\overline{N}_{x\theta}, \overline{M}_{x\theta}) = \int_{-h/2}^{h/2} (\overline{Q}_{16}\alpha_x + \overline{Q}_{26}\alpha_\theta + \overline{Q}_{66}\alpha_{x\theta}) \Delta T(1, z) dz$$

where $\alpha_x$ and $\alpha_\theta$ are the coefficients of thermal expansion, $\alpha_{x\theta}$ is the coefficient of thermal shear, $\Delta T$ is the temperature difference between the laminate and curing area, $\overline{Q}_{ij}$ is the transformed reduced stiffness,

## 3. DYNAMIC DISCRETIZED EQUATIONS

The GDQ method can be represented for example, the $m^{th}$-order derivative of one-dimensional function $f(x, t)$ at the $i^{th}$ discrete point $x = x_i$ in the $x$ direction on a grid is given as follows [3, 4]:

$$\frac{\partial^m f(x,t)}{\partial x^m}\Big|_{x=x_i} = \sum_{j=1}^{N} C_{i,j}^{(m)} f(x_j, t),\ i = 1, 2, ..., N \tag{2}$$

where $C_{i,j}^{(m)}$ is the weighting coefficient related to the $m^{th}$-order derivative and $N$ is the number of the total discrete grid points used in the $x$ direction.

The following displacement components for the vibration case are used:

$$u = U(x)\cos(n\theta + \omega t)$$

$$v = V(x)\sin(n\theta + \omega t)$$

$$w = W(x)\cos(n\theta + \omega t) \tag{3}$$

where $\omega$ (rad/sec) is the natural circular frequency and $n$ is an integer for the circumferential wave number of the multilayered shell.

The following non-dimensional parameters are introduced:

$$X = x/L,\quad Z = z/h,\quad U = U(x)/L,\quad V = V(x)/R,$$
$$W = W(x)/h \tag{4}$$

where $L$ is the length of shell.

For two edges are clamped, symmetric ($B_{ij} = 0$), orthotropic ($A_{16} = A_{26} = 0$, $D_{16} = D_{26} = 0$, $\alpha_{x\theta} = 0$) of laminated shell under temperature loading, we applying one-dimensional GDQ method to discretize the equilibrium differential equation (1), then the dynamic discretized equations can be rewritten in matrix form as follows [16]:

$$[DM]\{SUVW\} + [FM]\{UVW\} = \{F\} \tag{5}$$

where

$$\{SUVW\} = \left\{ \sum_{l=2}^{N-1} A_{i,l}^{(3)} U_l \ \sum_{l=2}^{N-1} A_{i,l}^{(2)} U_l \ \sum_{l=2}^{N-1} A_{i,l}^{(1)} U_l \ \sum_{l=2}^{N-1} A_{i,l}^{(2)} V_l \right.$$

$$\left. \sum_{l=2}^{N-1} A_{i,l}^{(1)} V_l \ \sum_{l=3}^{N-2} C_1 W_l \ \sum_{l=3}^{N-2} C_4 W_l \ \sum_{l=3}^{N-2} C_7 W_l \ \sum_{l=3}^{N-2} C_8 W_l \right\}^t$$

$$\{UVW\} = \{U_i \quad V_i \quad W_i\}^t$$

$$\{F\} = \{F_1 \quad F_2 \quad F_3\}^t$$

The elements of $3\times 9$ matrix $[DM]$, $3\times 3$ matrix $[FM]$ are as follows:

$$DM_{11} = 0$$

$$DM_{12} = (R^2 / L)\cos(n\theta + \omega t)$$

$$DM_{13} = DM_{14} = 0$$

$$DM_{15} = nR^2[(\frac{B_{12}}{R} + A_{12} + A_{66}\ \frac{2B_{66}}{R})/(L\cdot A_{11})]$$
$$\cos(n\theta + \omega t)$$

$$DM_{16} = DM_{17} = 0$$

$$DM_{18} = \{[A_{12}R + n^2(B_{12} + 2B_{66})]h/(L\cdot A_{11})\}\cos(n\theta + \omega t)$$

$$DM_{19} = -[B_{11}hR^2/(L^3\cdot A_{11})]\cos(n\theta + \omega t)$$

$$DM_{21} = DM_{22} = 0$$

$$DM_{23} = -nR^2[(\frac{B_{12}}{R^2} + \frac{A_{12}}{R} + \frac{A_{66}}{R} + \frac{B_{66}}{R^2})/A_{11}]$$
$$\sin(n\theta + \omega t)$$

$$DM_{24} = R^3[(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2} + N_a)/(L^2 A_{11})]$$
$$\sin(n\theta + \omega t)$$

$$DM_{25} = DM_{26} = 0$$

$$DM_{27} = [nhR^2(\frac{2B_{66}}{R} + \frac{B_{12}}{R} + \frac{2D_{66}}{R^2} + \frac{D_{12}}{R^2})/(L^2 A_{11})]$$
$$\sin(n\theta + \omega t)$$

$$DM_{28} = DM_{29} = 0$$

$$DM_{31} = \frac{B_{11}R^2}{L^2 A_{11}}\cos(n\theta + \omega t)$$

$$DM_{32} = 0$$

$$DM_{33} = -[(\frac{A_{12}}{R} + n^2\ \frac{B_{12} + 2B_{66}}{R^2})R^2/A_{11}]\cos(n\theta + \omega t)$$

$$DM_{34} = [nR^3(\frac{2B_{66}}{R} + \frac{B_{12}}{R} + \frac{4D_{66}}{R^2} + \frac{D_{12}}{R^2})/(L^2 A_{11})]$$
$$\cos(n\theta + \omega t)$$

$$DM_{35} = 0$$

$$DM_{36} = -\frac{D_{11}hR^2}{L^4 A_{11}}$$

$$DM_{37} = [(\frac{2B_{12}}{R} + n^2\ \frac{2D_{12} + 4D_{66}}{R^2} + N_a)hR^2/(L^2 A_{11})]$$
$$\cos(n\theta + \omega t)$$

$$DM_{38} = DM_{39} = 0$$

$$FM_{11} = (-n^2 A_{66}/A_{11} + f^{*2})L\cos(n\theta + \omega t)$$

$$FM_{12} = FM_{13} = 0$$

$$FM_{21} = 0$$

$$FM_{22} = [-n^2R^3(\frac{2B_{22}}{R^3} + \frac{A_{22}}{R^2} + \frac{D_{22}}{R^4})/A_{11} + f^{*2}R)]$$
$$\sin(n\theta + \omega t)$$

$$FM_{23} = \{[-n(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}) - n^3(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4})]hR^2/A_{11}\}$$
$$\sin(n\theta + \omega t)$$

$$FM_{31} = 0$$

$$FM_{32} = \{[-n(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}) - n^3(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4})]R^3/A_{11}\}$$
$$\cos(n\theta + \omega t)$$

$$FM_{33} = \{[(-n^4\ \frac{D_{22}}{R^4} - n^2\ \frac{2B_{22}}{R^3} - \frac{A_{22}}{R^2})hR^2/A_{11}] + f^{*2}h\}$$
$$\cos(n\theta + \omega t)$$

in which

$$C_8 = A_{i,l}^{(3)} - \frac{A_{i,2}^{(3)}\cdot AXK1 + A_{i,N-1}^{(3)}\cdot AXKN}{AXN}$$

$$C_7 = A_{i,l}^{(1)} - \frac{A_{i,2}^{(1)}\cdot AXK1 + A_{i,N-1}^{(1)}\cdot AXKN}{AXN}$$

$$C_4 = A_{i,l}^{(2)} - \frac{A_{i,2}^{(2)}\cdot AXK1 + A_{i,N-1}^{(2)}\cdot AXKN}{AXN}$$

$$C_1 = A_{i,l}^{(4)} - \frac{A_{i,2}^{(4)}\cdot AXK1 + A_{i,N-1}^{(4)}\cdot AXKN}{AXN}$$

$$AXN = A_{N,2}^{(2)}\cdot A_{1,N-1}^{(2)} - A_{1,2}^{(2)}\cdot A_{N,N-1}^{(2)}$$

$$AXK1 = -(A_{1,l}^{(2)}\cdot A_{N,N-1}^{(2)} - A_{1,N-1}^{(2)}\cdot A_{N,l}^{(2)})$$

$$AXKN = -(A_{1,2}^{(2)}\cdot A_{N,l}^{(2)} - A_{1,l}^{(2)}\cdot A_{N,2}^{(2)})$$

and frequency parameter $f^* = \omega R\sqrt{\rho_t / A_{11}}$, $F_1 = b_1R^2/A_{11}$, $F_2 = b_2R^2/A_{11}$, $F_3 = b_3R^2/A_{11}$.

## 4. COMPUTATIONAL RESULTS

We use the following coordinate for the grid point of the GDQ method:

$$x_i = 0.5[1-\cos(\frac{i-1}{N-1}\pi)]L,\ i = 1,2,...,N \quad (6)$$

The convergence studies of the frequency parameter $f^*$ have been made by the GDQ method [16] and made in series solution of Chung method [2]. In the case of rapid heating induced vibration of an insulated two-layer shell, the material properties of cross-ply ($0^\circ/90^\circ$) laminated shell is considered as follows:

Inner layer:

$E_1/E_2 = 25$, $G_{12}/E_2 = 0.5$, $\nu_{12} = 0.15$, $\alpha_x = 6\times10^{-6}$, $\alpha_\theta/\alpha_x = 1$, $\rho_1 = 0.087 lb/in^3$

Outer layer:

$E_1/E_2 = 40$, $G_{12}/E_2 = 0.6$, $\nu_{12} = 0.27$, $\alpha_x = 6.5\times10^{-6}$, $\alpha_\theta/\alpha_x = 1$, $\rho_2 = 0.283 lb/in^3$

The upper surface and all edges of the shell are considered to be thermally insulated. The temperature $T_0$ depends only on $Z$ and $t$, where $Z = z/h$. $T_0 = T_0(Z,t)$ can be found as in the following equation [18, 19]:

$$T_0 = \frac{hq_0}{\kappa}[\frac{\beta t}{\pi^2} + \frac{1}{2}(\frac{z}{h}+\frac{1}{2})^2 - \frac{1}{6} - \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{(-1)^j}{j^2}e^{-j^2\beta t}\cos j\pi(\frac{z}{h}+\frac{1}{2})] \quad (7)$$

where $\beta = \pi^2\kappa/h^2$, $\kappa$ is the coefficient of thermal conductivity, $q_0$ is the heat flux.

For simplification, we considering the vibration case for the condition of expansion strain distribution which is independent of $x$ and $\theta$ and an even function of $z$ ($\overline{M}_x = \overline{M}_\theta = \overline{M}_{x\theta} = 0$), using $\overline{N}_x = \overline{N}_{x\theta} = -N_a R^2/A_{11}, (\overline{N}_\theta = 0)$ as the thermally initial expansion load which is independent of $x$. We assume that the temperature difference $\Delta T$ is linearly dependent on $x$, thus $\Delta T = T_0 x$. The temperature $T_0 = T_0(\frac{1}{2},t)$ is applied in the sudden uniform input over the lower surface $Z = 1/2$ of inner layer only. Also for grid point $N = 20$, $R/h = 500$, $L/R = 10$, $h_1 = h_2$, wave number $n = 4$, $f = 0.080783$, natural circular frequency $\omega = 0.029892$ /sec under clamped-clamped boundary condition. We study the time responses by the effect of temperature load $\Delta T$ on the displacement of two layer insulated shell at circumferential coordinate $\theta = 1$ radian.

Figs. (**1-3**) show that the time responses of axial, circumferential and normal displacement $\overline{u} = u/L$, $\overline{v} = v/R$ and $\overline{w} = w/h$ along $X$, respectively under $q_0 = 2\ Btu/\sec\cdot in^2$ and $\kappa = 0.002963\ Btu/\sec\cdot in\cdot{}^\circ F$. We find that there are two maximum magnitudes of $\overline{u}$ locate nearly at length positions $X = 0.3156$ and $X = 0.6838$ at each time of $t = 0,3,5,9$ sec. The magnitudes of $\overline{u}$ is decreasing with the time. The maximum magnitude of $\overline{v}$ locates at middle length $X = 0.5$ at each time of $t = 0,3,5,9$ sec. The magnitudes of $\overline{v}$ is increasing with the time. The maximum magnitude of $\overline{w}$ also locates at middle length $X = 0.5$ at each time of $t = 0,3,5,9$ sec, and the $\overline{w}$ magnitude is the dominant one. The magnitudes of $\overline{w}$ is decreasing with the time.

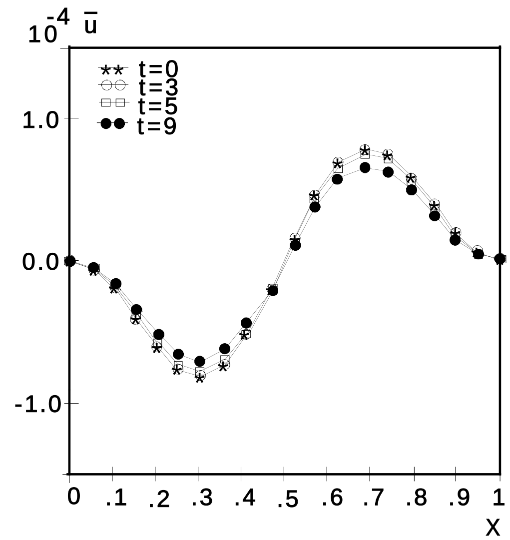


**Fig. (1).** Time response of axial displacement $\overline{u}$ along $X$.

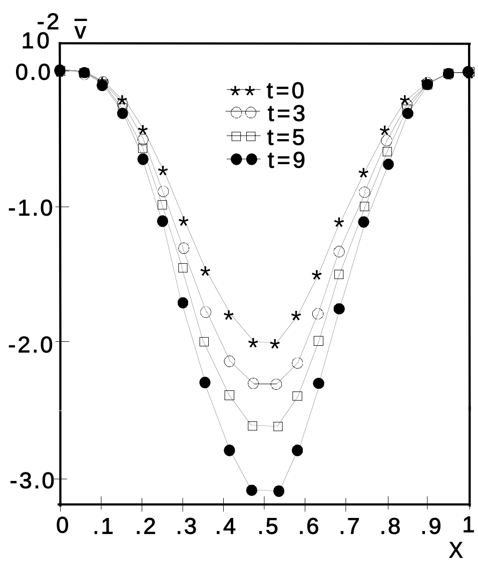


**Fig. (2).** Time response of circumferential displacement $\overline{v}$ along $X$.

## 5. CONCLUSIONS

In the study of the rapid heating induced vibration of an insulated two-layer shell, the computational GDQ method

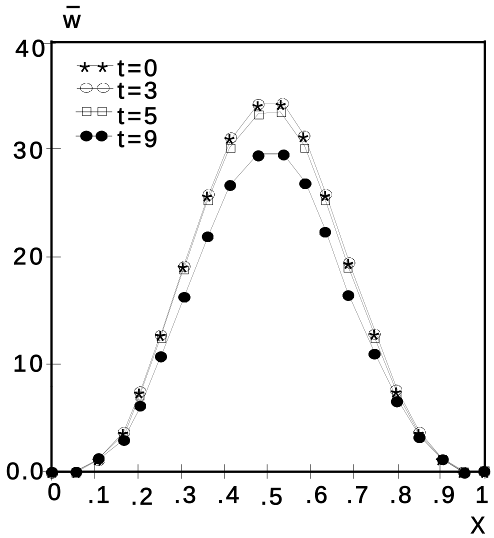


**Fig. (3).** Time response of normal displacement $\overline{w}$ along $X$.

provides a method for calculating the displacements. Numerical GDQ results show that: (1) Normal displacement $\overline{w}$ along $X$ is the dominant one for the two edges clamped condition. (2) Both the magnitudes of displacement $\overline{w}$ and $\overline{u}$ are decreasing with the time, but the magnitudes of displacement $\overline{v}$ is increasing with the time.

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