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Evaluation of Pressure-Strain Models in Compressible Mixing Layer

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Abstract: The experimental results of Goebel and Dutton for compressible mixing layer are used to examine the performance of the corrections proposed by Adumitroaie *et al*. and Marzougui *et al*. to the Launder, Reece and Rodi model for the pressure-strain correlation. These models are considered with the explicit dilatational terms of Sarkar for the study of two cases in which Mc = 0.69 and 0.86. The obtained results show that the predictions of Marzougui *et al*. model are in good agreement with the experimental results compared with the predictions of Adumitroaie *et al*.

1. INTRODUCTION

The comprehension of compressibility effects on turbulence is fundamental for many industrial applications, such as combustion, environment and aerodynamics. These effects became significant when the mean flow is strongly deformed or when the turbulent kinetic energy contained in the dilatational fluctuations is important. It is well known that the growth rate of turbulent kinetic energy is critically reduced with increasing turbulent Mach number.

Several explanations of compressibility have been offered. The DNS results of Erlebacher *et al*. [1], Sarkar [2] and Simone *et al*. [3] for compressible turbulent flow have been performed to clarify the compressibility effect with special focus on the dilatational terms such as the dilatation dissipation and the pressure-dilatation correlation. Sarkar *et al*. [4, 5] have proposed a turbulence models for $\overline{p^{'}d^{'}}$ and ε_c. These models are obtained from an asymptotic analysis that is formally valid for small turbulent Mach number. Zeman [6] proposes that the dilatational part of the total dissipation becomes progressively important as the turbulent Mach number increases due to the appearance of eddy shocklets. The turbulence model of Zeman for the dilatational dissipation is proportional to the solenoidal dissipation and a function of the turbulent Mach number. On the other hand the studies of Vreman *et al*. [7] and Sarkar [2] confirmed that the dilatational terms couldn't be regarded as essential in causing the reduced growth rate of turbulent kinetic energy. They pointed out that compressibility was found to affect the production term *via* the pressure-strain correlation.

Actually, the modeling of the pressure-strain correlation in compressible turbulence articulates on the simple extension of models established in incompressible turbulence as mentioned in the works of Adumitroaie *et al*. [8], Hamba [9] and Marzougui *et al*. [10]. Adumitroaie *et al*. have developed a compressible correction depending on the magnitude of the turbulent Mach number to the Launder, Reece and Rodi model [11]. For Hamba, the compressibility effect is reproduced in term of the parameter of normalized pressure-variance.

The idea of Marzougui *et al.* to extend the incompressible model of Launder, Reece and Rodi is based on the energy concept. The proposed corrections concern the C_1, C_3 and C_4 coefficients, which became in compressible turbulence a function of the turbulent Mach number. The applications of the proposed corrections show good agreement with DNS results for homogeneous shear flow of Sarkar [2] for cases A1, A2 and A3 in which the nonlinear effects are important.

In this paper, we propose to examine the pressure-strain models proposed by Marzougui *et al*. and Adumitroaie *et al*. for compressible mixing layer. Two values of the convective Mach number have been considered, which are Mc = 0.69 and 0.86. This range of convective Mach numbers spans the region of significant compressibility effects.

2. TURBULENCE MODELS

In incompressible turbulence situation, various models for the pressure-strain correlation have been proposed. The simplest model is due to Launder, Reece and Rodi [11].

$$\begin{aligned}\Phi_{ij}^{*} = &- C_1\overline{\rho}\;\varepsilon_s b_{ij} + C_2\overline{\rho}k\langle S_{ij}\rangle \\ &+ C_3\;\overline{\rho}\;k\left[\;b_{ik}\langle S_{jk}\rangle + b_{jk}\langle S_{ik}\rangle - \frac{2}{3}b_{mn}\langle S_{mn}\rangle\,\delta_{ij}\;\right] \\ &+ C_4\;\overline{\rho}\;k\left[b_{ik}\langle\Omega_{jk}\rangle + b_{jk}\langle\Omega_{ik}\rangle\right] \qquad (1)\end{aligned}$$

Where

C_1, C_2, C_3 and C_4 are constants that take on the values of 3, 0.8, 1.75 and 1.31.

$\langle S_{ij}\rangle = \frac{1}{2}(\langle u_i\rangle_{,j} + \langle u_j\rangle_{,i})$, $\langle\Omega_{ij}\rangle = \frac{1}{2}(\langle u_i\rangle_{,j} - \langle u_j\rangle_{,i})$ and

$$b_{ij}\left(= \frac{\langle u_i^{''} u_j^{''}\rangle}{2k} - \frac{1}{3}\delta_{ij}\right)$$

denote respectively, the mean strain, the mean vorticity and the anisotropy tensor.

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We recall that this model remains until our days the most popular model. Its celebrity is due to its implementation, which is relatively simple compared to other literature models. For this raison the majority of authors agree to consider it as a good support for any compressible correction.

2.1. Adumitroaie *et al.* Model

Adumitroaie *et al*. proposed a model due to a conjunction between the traditional techniques used by Launder, Reece and Rodi to integrate the Poisson's equation for the pressure and the continuity constraint. This model is given by [8]:

$$\Phi^*_{ij} = -C_1 \bar{\rho}\,\varepsilon\, b_{ij} + \bar{\rho} k\left(\frac{4}{5} + \frac{2}{5} d_1\right)\langle S_{ij}\rangle$$
$$+ 2\bar{\rho}\, k(1 - C_3 + 2d_2)\left(b_{ip}\langle S_{pj}\rangle + b_{pj}\langle S_{ip}\rangle - \frac{2}{3} b_{pq}\langle S_{pq}\rangle\delta_{ij}\right)$$
$$- 2\bar{\rho}\, k(1 - C_4 - 2d_2)\left[b_{ip}\langle\Omega_{pj}\rangle - \langle\Omega_{ip}\rangle b_{pj} - \frac{4}{3} d_2\langle S_{pp}\rangle b_{ij}\right] \quad (2)$$

The d_i are determined by the pressure-dilatation model. For the Sarkar model, one requires that $d_1 = \frac{8\chi_3 M_t^2}{9}$, $d_2 = \frac{\chi_1 M_t}{2\sqrt{3}}$

2.2. Marzougui *et al.* Model

The contribution of Marzougui *et al*. [10] appears in the correction of the C_i coefficients, which became in a compressible turbulence situation a function of the turbulent Mach number. The suggested method is based on proportionality relations between, the ratio of compressible and incompressible components of the pressure strain correlation and the ratio relating the compressible and incompressible growth rate of the turbulent kinetic energy. This method generates a pressure strain model parameterized according to the turbulent Mach number.

$$\Phi^*_{ij} = -C_1 \frac{(1 - 0.44\, M_t^2)^2}{(1 + \alpha M_t^2)} \bar{\rho}\;\varepsilon_s b_{ij} + C_2 \bar{\rho} k\langle S_{ij}\rangle$$
$$+ C_3(1 - 1.5\, M_t^2)\bar{\rho}\, k\left[b_{ik}\langle S_{jk}\rangle + b_{jk}\langle S_{ik}\rangle - \frac{2}{3} b_{mn}\langle S_{mn}\rangle\delta_{ij}\right]$$
$$+ C_4(1 - 0.5\, M_t)\bar{\rho}\, k\left[b_{ik}\langle\Omega_{jk}\rangle + b_{jk}\langle\Omega_{ik}\rangle\right] \quad (3)$$

3. APPLICATION OF THE TURBULENT MODELS FOR COMPRESSIBLE MIXING LAYER

3.1. Governing Equations

The above turbulence models for the pressure-strain correlations were used to simulate stationary compressible mixing layers. The governing equations are the Reynolds averaged Navier-Stokes equations with the additional equations for the energy, the Reynolds stresses and the turbulent dissipation.

$$\partial_k \bar{\rho}\langle u_k\rangle = 0 \quad (4)$$

$$\partial_k \bar{\rho}\langle u_i\rangle\langle u_k\rangle = -\partial_i \bar{p} + \partial_k\left(-\overline{\rho u_i'' u_k''} + \bar{\tau}_{ik}\right) \quad (5)$$

$$\partial_k(\bar{\rho} C_v\langle T\rangle\langle u_k\rangle) = -\overline{p' d'} + \bar{\rho}\varepsilon - \partial_k Q_k \quad (6)$$

$$\tau_{ik} = -\frac{2}{3}\mu u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \quad (7)$$

$$Q_k = \bar{\rho} C_v\langle u_k'' T''\rangle \quad (8)$$

The turbulent heat flux is simply modeled using the gradient transport model:

$$\bar{\rho}\langle u_k'' T''\rangle = -\left(C_T \bar{\rho}\frac{k}{\varepsilon_s}\langle u_k'' u_j''\rangle\partial_j\langle T\rangle\right) \quad (9)$$

A Reynolds stress transport equation model was used to determine the stress $\overline{\rho u_i'' u_j''}$

$$\partial_k \overline{\rho u_i'' u_j''}\langle u_k\rangle = P_{ij} + T_{ijk} + \Phi_{ij} + D_{ij} \quad (10)$$

where the symbols

$$P_{ij} = -\overline{\rho u_j'' u_k''}\partial_k\langle u_i\rangle - \overline{\rho u_i'' u_k''}\partial_k\langle u_j\rangle \quad (11)$$

$$T_{ijk} = \partial_k\left(-\overline{\rho u_i'' u_j'' u_k''} + \overline{u_j''\tau_{ik}} + \overline{u_i''\tau_{jk}} - \overline{p u_i''}\delta_{jk} - \overline{p u_j''}\delta_{ik}\right) \quad (12)$$

$$\Phi_{ij} = \overline{p'\partial_j u_i'' + p'\partial_i u_j''} = \Phi^*_{ij} + \frac{2}{3}\overline{p' d'}\,\delta_{ij} \quad (13)$$

$$D_{ij} = -\overline{\tau_{ik}\partial_k u_j''} - \overline{\tau_{jk}\partial_k u_i''} \quad (14)$$

represent production, transport, redistribution and dissipation, respectively.

Except for P_{ij} , the terms (12)-(14) require modeling. The dissipation term is determined by using an isotropic dissipation model:

$$D_{ij} = -\frac{2}{3}\bar{\rho}(\varepsilon_s + \varepsilon_c) \quad (15)$$

where:

ε_s represents the turbulent dissipation arising from the traditional energy cascade which is solenoïdal, å$_c$ represent the turbulent dissipation arising from compressible modes. The transport equation for the solenoïdal dissipation is of the form [12]:

$$\partial_k \bar{\rho}\varepsilon_s\langle u_k\rangle = -C_{\varepsilon_1}\bar{\rho}\frac{\varepsilon_s}{k}\langle u_i'' u_k''\rangle\partial_k\langle u_i\rangle - C_{\varepsilon_2}\bar{\rho}\frac{\varepsilon_s^2}{k} + \partial_k\left(C_\varepsilon\bar{\rho}\frac{k}{\varepsilon_s}\langle u_k'' u_m''\rangle\partial_m\varepsilon_s\right) \quad (16)$$

The diffusion term is given by the Daly and Harlow model [13]:

$$T_{ijk} = -C_s\frac{k}{\varepsilon}\bar{\rho}\langle u_k'' u_l''\rangle\partial_l\langle u_i'' u_j''\rangle \quad (17)$$

Finally, to close the above equations, turbulence models were used to determine the dilatational terms and the pressure-strain correlation.

For the dilatational terms, we have retained models of Sarkar *et al*. [4, 5]:

$$\overline{p'd'} = -\alpha_2 \bar{\rho} P M_t + \alpha_3 \bar{\rho} \varepsilon_s M_t^2 \tag{18}$$

$$\varepsilon_c = \alpha M_t^2 \varepsilon_s \tag{19}$$

α_1, α_2 and α and are constants (α_1 = 0.15, α_2 = 0.2, α =0.5)

For the pressure-strain correlation, we used the L. R. R model in its standard form [11] and in its corrected form [8, 10].

3.2. Results and Discussion

The averaged governing equations are solved using a finite difference scheme. The physical domain is a rectangular box defined by the set of point (x, y), in which x represents the streamwise coordinate and y the transverse coordinate. The grid overlaying the computational domain has 6666×41 points. The boundary conditions for the mean velocity and the turbulent kinetic energy are determined from experimental results of Goebel and Dutton [14]. We note here that the only initial profiles, available in the experience of Goebel and Dutton are those for $\langle u''u''\rangle$, $\langle v''v''\rangle$ and $\langle u''v''\rangle$. Consequently, it appears necessary to generate some initial profiles for ε_s, $\langle T\rangle$ and $\bar{\rho}$. The initial profile of the dissipation is determined from the turbulent viscosity model.

$$\varepsilon_s = -C_\mu \frac{k^2}{\langle u''v''\rangle} \frac{\partial}{\partial y}\langle u\rangle, \quad C_\mu = 0.09 . \tag{20}$$

For the initial profile of the temperature, we admitted the following similarity:

$$\frac{\langle u\rangle - U_2}{\Delta U} = \frac{\langle T\rangle - T_2}{\Delta T} \tag{21}$$

The initial profile of the density is deducted from the equation of state perfect gas

$$\frac{\bar{\rho}}{\Delta\rho} = \frac{\Delta T M_2^2 (1 - \frac{U_1}{U_2})^2}{\langle T\rangle M_1^2 (1 - \frac{U_2}{U_1})^2 (1 - \frac{\rho_2}{\rho_1})(1 - \frac{T_1}{T_2})} \tag{22}$$

For $\langle w''w''\rangle$, the same initial profile for $\langle v''v''\rangle$ is chosen.

In this section, we analyze the capacity of the used turbulent models for the pressure-strain correlation to predict the compressibility effects. We have calculated two mixing layers varying the convective Mach number from 0.69 to 0.86. The results presented here are determined at x =150mm, where the turbulence is plainly developed. Comparisons will be made with the experimental results of Goebel and Dutton. Firstly, we start with results obtained from the L. R. R model in its standard form, in order to assess the performance of corrections proposed by Adumitroaie *et al*. and Marzougui *et al*. From the results plotted in Figs. (**1-4**), it is clear that the standard model in conjunction with dilatational terms proposed by Sarkar is unable to predict the compressibility effect on turbulence. Fig. (**1**), shows the computed mean velocity profiles ($U^* = \frac{\langle u\rangle - U_2}{\Delta U}$) obtained at different convective Mach number. The computations do in fact exhibit a small change in the shape of the mean velocity profile as the convective Mach number is raised, this point must be regarded and explained. In graph 2, we present the results for the streamwise turbulence intensity at $M_c = 0.69$ and 0.86 obtained with the standard model of L.R.R and those obtained with the Adumitroaie *et al.* and Marzougui *et al.* models. From these results as one can remark that the Adumitroaie *et al.* model underestimates the streamwise turbulence intensity by about the same amount when we use the standard model of L.R.R. However, the Marzougui *et al*. model is able to predict the reduced of σ_{11} that arises from compressibility effects. The computed spatial evolutions of the transverse turbulence intensity are compared with experimental results in Fig. (**3**). It's clear from these results that the transverse turbulence intensity is overpredicted by Adumitroaie *et al*. model. The results obtained from Marzougui *et al.* model are very close to the experimental results of Goebel and Dutton. These results show the capacity of Marzougui *et al.* model to predict the compressibility effects on turbulence. Fig. (**4**) shows an important diminution in R_{uv} ($= \frac{\langle u''v''\rangle}{(\Delta U)^2}$). These results join the analyses presented in the literature, which

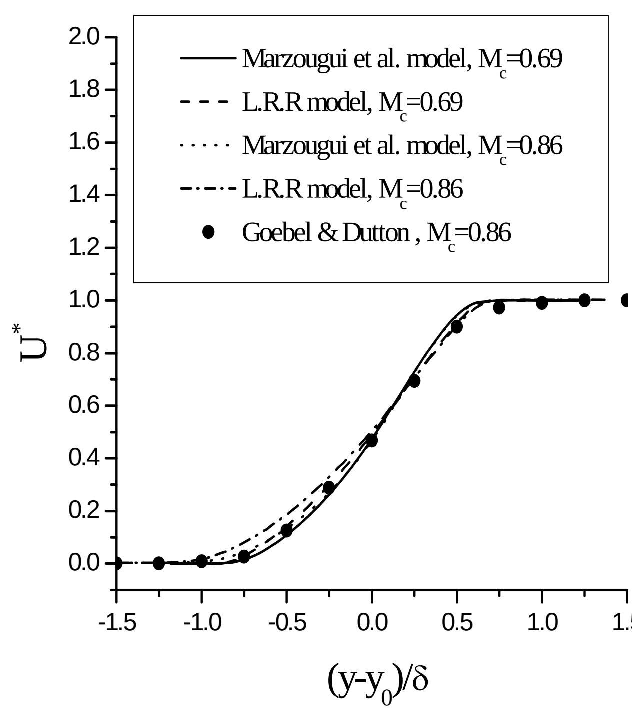

Fig. (1). Similarity profiles of mean stream wise velocity.

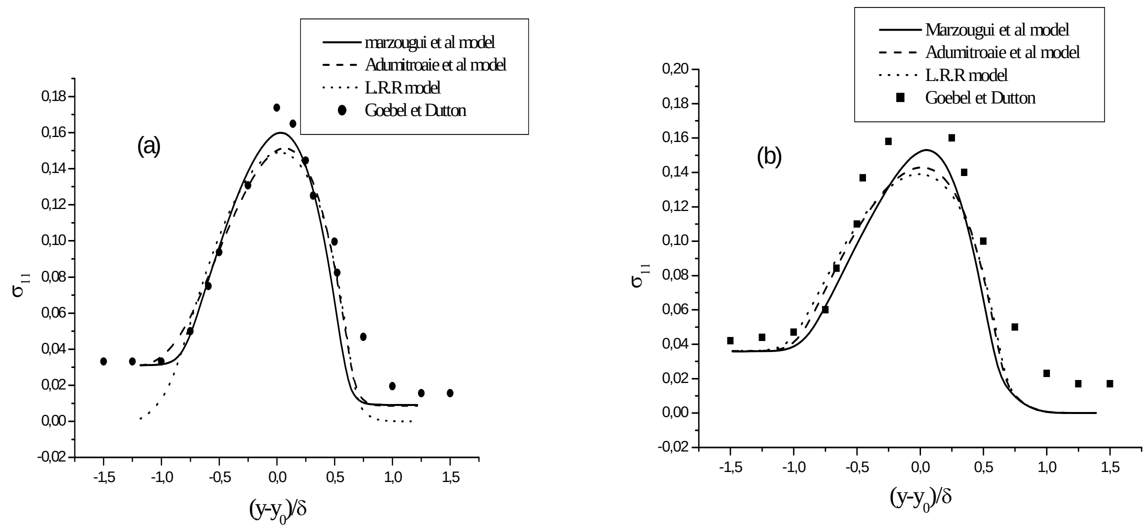

Fig. (2). Similarity profile of longitudinal turbulence intensity: (**a**) $M_c = 0.69$, (**b**) $M_c = 0.86$.

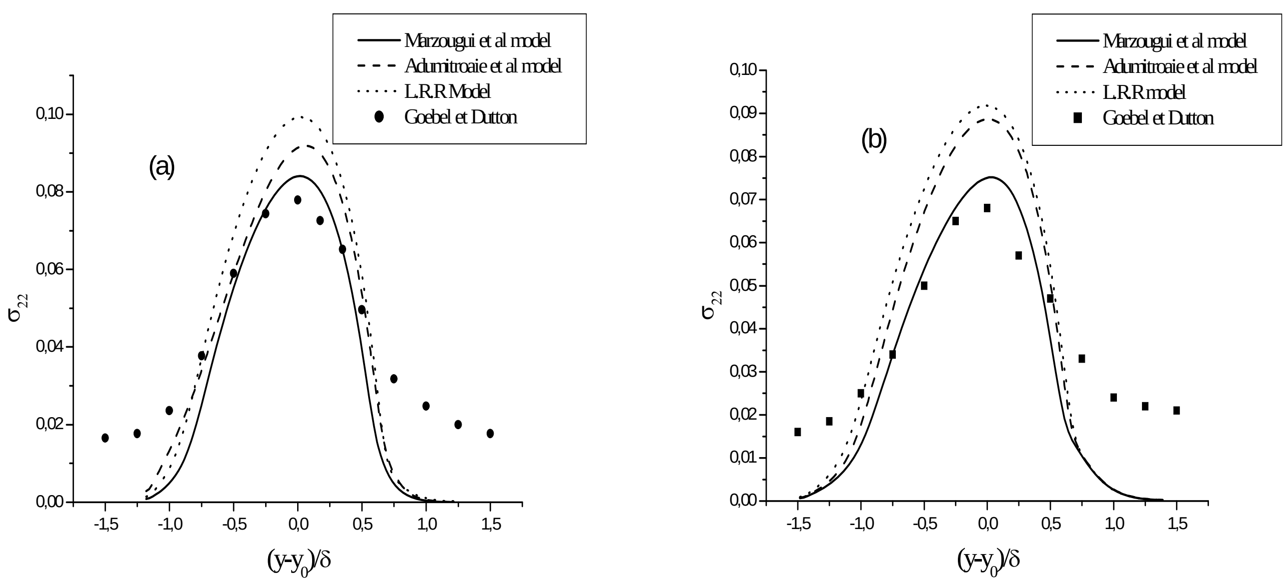

Fig. (3). Similarity profiles of transverse turbulence intensity: (**a**) $M_c = 0.69$, (**b**) $M_c = 0.86$.

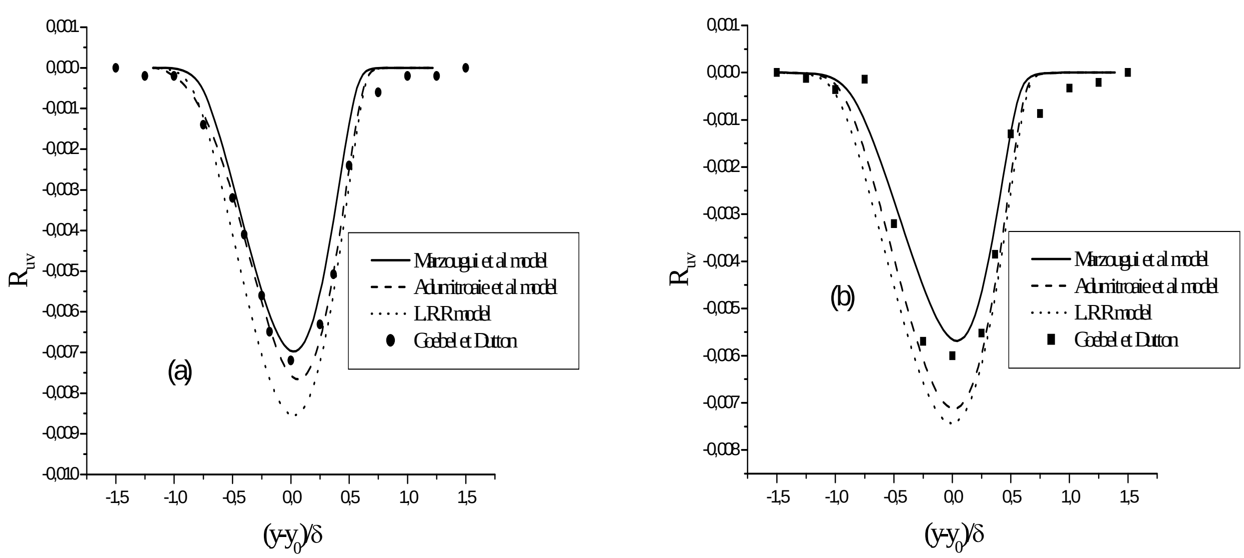

Fig. (4). Similarity profiles of Reynolds stress: (**a**) $M_c = 0.69$, (**b**) $M_c = 0.86$.

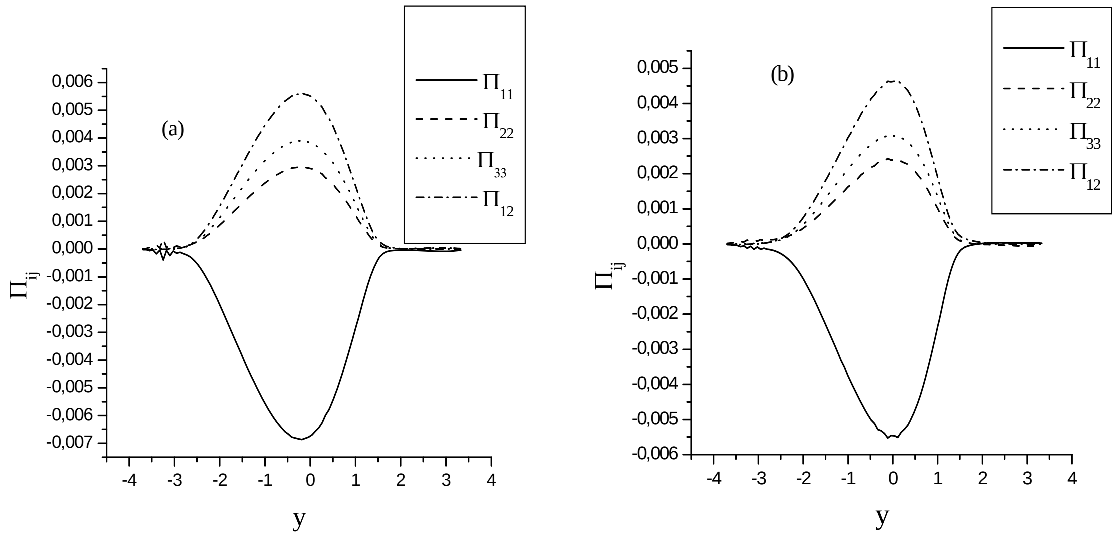

Fig. (5). Similarity profiles of pressure-strain correlation: (**a**) $M_c = 0.69$, (**b**) $M_c = 0.86$.

aim level raised of compressibility effects on the turbulent production. The effect of compressibility on the pressure-strain tensor ($\Pi_{ij} = \dfrac{\Phi^*_{ij}}{\bar{\rho}(U_1 - U_2)^2 \dfrac{d}{dy}\langle u \rangle}$) is also of interest. Fig. (**5a** and **5b**) show that there is a decrease in different components of the pressure-strain correlation from Mc = 0.69 to Mc = 0.86. On Fig. (**6**) are plotted the results for the normalized growth rate. We recall here that the growth rate is given by $\dfrac{d\delta}{dx}$, the width of the mixing layer δ is defined by the transverse distance between the two positions where the mean velocities are equal to $U_1 - 0.1\Delta U$ and $U_2 + 0.1\Delta U$. It can be seen that the proposed models reproduce the decrease of the growth rate with increasing convective Mach number, behavior are very much in accord with experimental studies of compressible mixing layers. As mentioned in the literature that the critical M_c for shocklets formation is still unknown. Li and Fu [15] have pointed out that for temporal compressible mixing layer, the shocks are detected for 2-D simulations when M_c is above 0.7. Sandham and Reynolds [16] observed shocklets in 3-D mixing layer at M_c =1. In 3-D DNS of Vreman *et al*. the shocklets appear at M_c =1.2 for three times (t = 122, 182, 200). These results enable us to think that the numerical methods and the boundary conditions have an influence on the critical value of M_c. The shocklets have not been observed in the present simulation and in the experimental results of Goebel and Dutton for M_c = 0.86. This can be due to the stationary of turbulence and the used numerical method for resolving the above equations.

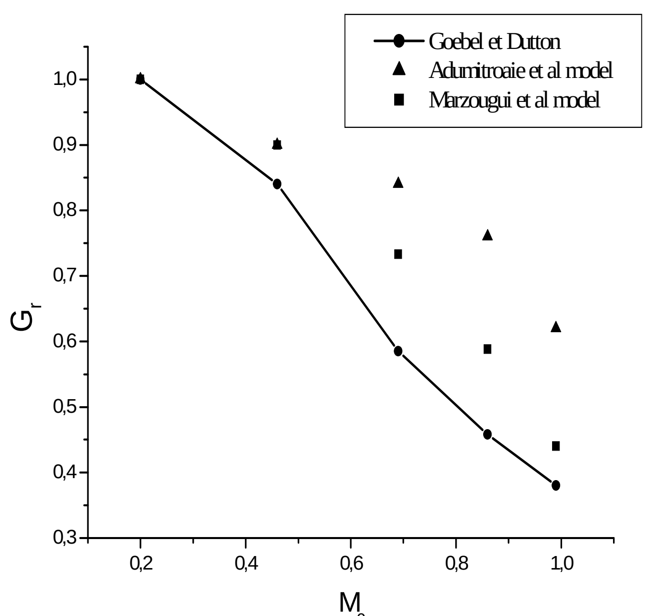

Fig. (6). Normalized growth rate G_r versus convective Mach number.

4. CONCLUSION

In this work, we have studied the compressibility effect on mixing layer. The Adumitraie *et al*. and Marzougui *et al*. models are retained for the pressure strain correlation. Two cases in which M_c = 0.69 and 0.86 are considered. The computational results show that: the Adumitroaie *et al*. model yields poor predictions for compressible mixing layer. This can be explained by the fact that the proposed correction of these authors is based on the continuity constraint. The corrections proposed by Marzougui *et al*. are able to reproduce in a very satisfactory way the compressibility effects on turbulence.

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