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Effect of the Rotation on the Radial Vibrations in a Non-Homogeneous Orthotropic Hollow Cylinder

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Abstract: In the present paper, the redial harmonic vibrations of hollow cylinders of orthotropic material as affected by the angular velocity are discussed on the basis of the linear theory of elasticity. The one-dimensional equation of elastodynamics has been solved in terms of radial displacement. Three different boundary conditions, the free, fixed and mixed ones are considered. The determination of the eigenfrequencies of radial harmonic vibrations under-different boundary conditions is presented. Numerical results are given and illustrated graphically for each case. Comparisons are made with previous results given in the literature in the absence of rotation and non-homogeneity.

Keywords: Elastodynamics, radial vibrations, non-homogeneous material, orthotropic material, rotation.

1. INTRODUCTION

In the past, accidental failure of rotating cylinders due to flexural vibrations has frequently occurred in rotodynamic machinery such as steam turbines and gas turbines. The analysis of the dynamic problems of elastic bodies is an important and interesting research field for engineers and scientists. Influences of Rotation, Magnetic Field, Initial Stress and Gravity on Rayleigh Waves in a Homogeneous Orthotropic Elastic Half-Space is investigated by Abd-Alla *et al.* [1-2]. S.R. Mahmoud [3] studied the wave propagation in cylindrical poroelastic dry bones. M. Abd-Alla and Mahmoud [4] solved magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model.

Free vibrations in elastic homogeneous isotropic cylinders have been studied by Abd-Alla and Abo-Dahab [5], Buchanan and Chua [6], Galmudi and Dvorkin [7] and Gladwell and Vijay [8]. However, little attention has been given to the problem of wave propagation in orthotropic circular cylinders. Some problems of the three-dimensional elastic theory for the axisymmetric free vibrations of hollow circular cylinders have been studied and analyzed by Hutchinson and El-Azhari [9]. Cowin and Fraldi [10] have investigated a dynamic problem of singularities associated with the curvilinear anisotropic elastic symmetries. Ebenezer *et al*. [11] have investigated forced vibrations of solid elastic cylinders. Chou and Achenbach [12] have provided a three-dimensional solution to vibrations of orthotropic cylinders. Hutchinson [13] has investigated the free vibrations problems of solid cylinders. In addition, Markus and Mead [14] have presented an analytical method for investigating the dispersion behavior of axisymmetric and asymmetric wave's motion in orthotropic cylinders. Buchanan and Liu [15] have analyzed of the free vibrations of thick walled isotropic toroidal shells. Sharma and Kumar [16] have performed an asymptotic analysis of wave motion in transversely isotropic plates. Buchanan and Yii [17] have studied the effect of symmetrical boundary conditions on the vibrations of thick hollow cylinders. Tarn [18] has studied a state space formalism of the free vibrations problems of anisotropic cylinders. Wang and Williams [19] have studied vibrational modes of thick cylinders of finite length, Zhou, *et al.* [20] have investigated *3D* vibrations analysis of solid and hollow circular cylinders via Chebyshev-Ritz method and Mofakhamia, *et al.* [21] have studied finite cylinder vibrations with different end boundary conditions.

2. FORMULATION OF THE PROBLEM

In this section, we derive the analytical formulation of the problem in cylindrical coordinates (r, θ , z) with the z-axis coinciding with the axis of the cylinder. We consider the strains to be symmetric about the z-axis. The only unknown of the problem is the radial displacement $\vec{u} = (u, 0, 0)$, with u being a function of r and t *since*, the circumferential displacement $u_\theta = 0$ and the longitudinal displacement $u_z = 0$. The stress-strain relations for a cylindrically orthotropic elastic body are given by Lekhnitskii [22]. The stress-displacement relations for a cylindrically orthotropic material in one dimension are,

$$\begin{aligned} \sigma_{rr} &= c_{11}\frac{\partial u}{\partial r} + c_{12}\frac{u}{r}, \\ \sigma_{\theta\theta} &= c_{12}\frac{\partial u}{\partial r} + c_{22}\frac{u}{r}. \end{aligned} \tag{2.1}$$

The dynamical equation in the r direction, (taking the rotation term about the z-axis as a body force) is given by:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \rho\Omega^2 u = \rho\frac{\partial^2 u}{\partial t^2} \tag{2.2}$$

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where, Ω is the uniform angular velocity and ρ is the density of the cylinder material, The elastic constants c_{ij} in (2.1) and the density ρ of the non-homogeneous material in (2.2) are of the form:

$$c_{ij} = \alpha_{ij}\, r^{2m} \quad at \quad i = 1,2 \ ; \ j = 1,2,3, \qquad \rho = \rho_0\, r^{2m} \tag{2.3}$$

where, α_{ij} and ρ_0 are the values of c_{ij} and ρ in the homogeneous case, respectively, and m is a rational number.

Substituting equations (2.1) and (2.3) into equation (2.2) we obtain:

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{(2m+1)}{r}\frac{\partial u}{\partial r} + \frac{(\alpha_{12}(2m+1)-\alpha_{22})}{\alpha_{11} r^2} u + \frac{\rho_0 \Omega^2}{\alpha_{11}} \qquad u = \frac{\rho_0}{\alpha_{11}}\frac{\partial^2 u}{\partial t^2}. \tag{2.4}$$

which is the governing equation of the problem in terms of the radial displacement u

3. SOLUTION OF THE PROBLEM

In this section, we obtain the analytical solution of the above problem for a cylinder of inner radius a and outer radius b and different boundary conditions (free-fixed-mixed) by assuming harmonic vibrations. Thus, the function u in (2.4) is assumed to be of the form:

$$u(r,t) = U(r)e^{-i\omega t} \tag{3.1}$$

where, ω is the frequency of vibrations. Substituting equation (3.1) into equation (2.4) we obtain the equation:

$$\frac{d^2U}{dr^2} + \frac{(2m+1)}{r}\frac{dU}{dr} + \left[\frac{(2m+1)-\alpha_{22})}{\alpha_{11}r^2} + \frac{\rho_0}{\alpha_{11}}(\Omega^2+\omega^2)\right]U = 0. \tag{3.2}$$

Furthermore, assuming that,

$$U(r) = r^{-m}\phi(r). \tag{3.3}$$

equation (3.2) becomes:

$$r^2\frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} + [\lambda^2 r^2 - n^2]\phi = 0 \tag{3.4}$$

where,

$$\lambda^2 = \frac{\rho_0}{\alpha_{11}}(\Omega^2+\omega^2), \qquad n^2 = \frac{m^2\alpha_{11} + \alpha_{22} - 2m\alpha_{12}}{\alpha_{11}}.$$

Equation (3.4) is Bessel's equation, with a general solution of the form:

$$\phi(r) = A\ J_n(\lambda r) + B\ Y_n(\lambda r) \tag{3.5}$$

where, A and B are arbitrary constants and $J_n(\lambda r)$ and $Y_n(\lambda r)$ denote Bessel's functions of the first and second kind of order n, respectively.

Substituting from equations (3.5) and (3.3) into equation (3.1), the complete solution of equation (2.4) becomes:

$$u(r,t) = r^{-m}e^{-i\omega t}[AJ_n(\lambda r) + BY_n(\lambda r)]. \tag{3.6}$$

Substituting from equations (2.3) and (3.6) into equation (2.1), the components of the stresses σ_{rr} and $\sigma_{\theta\theta}$ are obtained as

$$\sigma_{rr} = r^{-m}e^{-i\omega t}\left\{\begin{array}{l} A\left[\lambda\alpha_{11}J_{n-1}(\lambda r) + \dfrac{\alpha_{12}-\alpha_{11}(m+n)}{r}J_n(\lambda r)\right] \\ +B\left[\lambda\alpha_{11}Y_{n-1}(\lambda r) + \dfrac{\alpha_{12}-\alpha_{11}(m+n)}{r}Y_n(\lambda r)\right]\end{array}\right\}, \tag{3.7}$$

$$\sigma_{\theta\theta} = r^{-m}e^{-i\omega t}\left\{\begin{array}{l} A\left[\lambda\alpha_{12}J_{n-1}(\lambda r) + \dfrac{\alpha_{22}-\alpha_{12}(m+n)}{r}J_n(\lambda r)\right] \\ +B\left[\lambda\alpha_{12}Y_{n-1}(\lambda r) + \dfrac{\alpha_{22}-\alpha_{12}(m+n)}{r}Y_n(\lambda r)\right]\end{array}\right\}. \tag{3.8}$$

4. BOUNDARY CONDITIONS AND FREQUENCY EQUATION

In this case, we are going to obtain the frequency equation of the problem for various boundary conditions of the hollow cylinder.

We consider the following transformations:

$$W_1 = \sqrt{W^2\left(\frac{\alpha_{12}}{\alpha_{11}}\right) + \frac{\rho_0}{\alpha_{11}}\bar{\Omega}^2}, \qquad \omega = \frac{W}{b}\sqrt{\frac{\alpha_{12}}{\rho_0}},\ \bar{\Omega} = \frac{\Omega}{b},\ \ h = \frac{a}{b},\ \ \lambda = \frac{W_1}{b} \tag{4.1}$$

Which will make all the quantities dimensionless in equations (4.1) and (3.8), where, W_1 denotes the dimensionless frequency.

4.1. Free Surface Traction

In this case, we are going to obtain the frequency equation for the boundary condition, which specify that the inner and outer surfaces of the hollow cylinder are free of stresses.

$$\sigma_{rr} = 0 \quad at \quad r = a, \qquad \sigma_{rr} = 0 \quad at \quad r = b. \tag{4.2}$$

Substituting equation (4.1) into equation (3.7) and using equation (4.2), we obtain-two homogeneous linear equations in A and B of the from:

$$\left\{\begin{array}{l} A\left[\alpha_{11}\dfrac{W_1}{b}J_{n-1}(W_1 h) + \dfrac{\alpha_{12}-\alpha_{11}(m+n)}{a}J_n(W_1 h)\right] \\ +B\left[\alpha_{11}\dfrac{W_1}{b}Y_{n-1}(W_1 h) + \dfrac{\alpha_{12}-\alpha_{11}(m+n)}{a}Y_n(W_1 h)\right]\end{array}\right\} = 0,$$

$$\left\{\begin{array}{l} A\left[\alpha_{11}\dfrac{W_1}{b}J_{n-1}(W_1)+\dfrac{\alpha_{12}-\alpha_{11}(m+n)}{b}J_n(W_1)\right] \\ +B\left[\alpha_{11}\dfrac{W_1}{b}Y_{n-1}(W_1)+\dfrac{\alpha_{12}-\alpha_{11}(m+n)}{b}Y_n(W_1)\right]\end{array}\right\}=0. \tag{4.3}$$

The condition for nonzero solutions of (4.3) produces the frequency equation in the form of a second order determinant as:

$$\begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix}=0 \tag{4.4}$$

where,

$$\begin{aligned} D_{11} &= \alpha_{11}hW_1\ J_{n-1}(hW_1)+(\alpha_{12}-\alpha_{11}(m+n))\ J_n(hW_1), \\ D_{21} &= \alpha_{11}W_1\ J_{n-1}(W_1)+(\alpha_{12}-\alpha_{11}(m+n))\ J_n(W_1) \\ D_{12} &= \alpha_{11}hW_1\ Y_{n-1}(hW_1)+(\alpha_{12}-\alpha_{11}(m+n))\ Y_n(hW_1), \\ D_{22} &= \alpha_{11}W_1\ Y_{n-1}(W_1)+(\alpha_{12}-\alpha_{11}(m+n))\ Y_n(W_1). \end{aligned} \tag{4.5}$$

From equation (4.4), we deduce the frequency equation in the form

$$D_{11}\ D_{22}-D_{12}\ D_{21}=0 \tag{4.6}$$

which represents an implicit equation in W_1. By solving this equation, we can obtain the eigenfrequencies W.

4.2. Surfaces Fixed

In this case, we are going to obtain a frequency equation for the boundary condition, which specify that the inner and outer surfaces of the hollow cylinder are fixed, implying zero displacements, i.e.

$$\begin{aligned} u(r,t)&=0 \qquad at \qquad r=a, \\ u(r,t)&=0 \qquad at \qquad r=b \end{aligned} \tag{4.7}$$

From (3.6) and (4.7), we obtain two homogeneous linear equations in A and B

$$\begin{aligned} A\ J_n(\lambda a)+B\ Y_n(\lambda a)&=0, \\ A\ J_n(\lambda b)+B\ Y_n(\lambda b)&=0. \end{aligned} \tag{4.8}$$

The condition for nonzero solutions of (4.8) produces the frequency equation in the form of a second order determinant as

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix}=0 \tag{4.9}$$

where,

$$\begin{aligned} d_{11}&=J_n(hW_1), \\ d_{12}&=Y_n(hW_1), \\ d_{21}&=J_n(W_1), \\ d_{22}&=Y_n(W_1). \end{aligned}$$

From equation (4.9), we deduce the frequency equation in the form

$$J_n(W_1)\ \ Y_n(hW_1)-J_n(hW_1)\ \ Y_n(W_1)=0 \tag{4.10}$$

which represents an implicit equation in W_1. By solving this equation, we can obtain the eigenfrequencies W.

4.3. Inner Surface Fixed and Outer Surface Free

In this case, we are going to obtain a frequency equation for the boundary condition, which specify that the inner surface of the hollow cylinder is fixed and outer surface is free, i.e.,

$$\begin{aligned} u(r,t)&=0 \qquad at \qquad r=a, \\ \sigma_{rr}&=0 \qquad at \qquad r=b. \end{aligned} \tag{4.11}$$

From equations (3.6), (3.7) and (4.11), we obtain two homogeneous linear equations in A and B:

$$\begin{aligned} &A\ J_n(\lambda a)+B\ Y_n(\lambda a)=0\ , \\ &A\left[\lambda\alpha_{11}J_{n-1}(\lambda b)+\frac{\alpha_{12}-\alpha_{11}(m+n)}{b}J_n(\lambda b)\right]. \\ &+B\left[\lambda\alpha_{11}Y_{n-1}(\lambda b)+\frac{\alpha_{12}-\alpha_{11}(m+n)}{b}Y_n(\lambda b)\right]=0 \end{aligned} \tag{4.12}$$

The condition for nonzero solutions of (4.12) produces the frequency equation in the form of a second order determinant as:

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}=0$$

where,

$$m_{11}=J_n(hW_1),$$

$$m_{12}=Y_n(hW_1),$$

$$m_{21}=\alpha_{11}W_1J_{n-1}(W_1)+(\alpha_{12}-\alpha_{11}(m+n))J_n(W_1),$$

$$m_{22}=\alpha_{11}W_1Y_{n-1}(W_1)+(\alpha_{12}-\alpha_{11}(m+n))Y_n(W_1).$$

From equation (4.12)', we deduce the frequency equation in the form:

$$\begin{aligned} &J_n(hW_1)[\alpha_{11}W_1Y_{n-1}(W_1)+(\alpha_{12}-\alpha_{11}(m+n))Y_n(W_1)] \\ &-Y_n(hW_1)[\alpha_{11}W_1J_{n-1}(W_1)+(\alpha_{12}-\alpha_{11}(m+n))J_n(W_1)]=0. \end{aligned} \tag{4.13}$$

By solving this equation, we can obtain the eigenfrequencies W.

4.4. Inner Surface Free and Outer Surface Fixed

In this case, we are going to obtain a frequency equation for the boundary condition, which specify that the inner surface of the hollow cylinder is free and outer surface is fixed, i.e.,

$$\sigma_{rr} = 0 \quad at \quad r = a, \qquad u(r,t) = 0 \quad at \quad r = b. \tag{4.14}$$

From equations (3.6), (3.7) and (4.14), we obtain two homogeneous linear equations in *A* and *B*:

$$AJ_n(\lambda b) + BY_n(\lambda b) = 0,$$

$$A\left[\lambda\alpha_{11} J_{n-1}(\lambda a) + \frac{\alpha_{12} - \alpha_{11}(m+n)}{a} J_n(\lambda a)\right] + B\left[\lambda\alpha_{11} Y_{n-1}(\lambda a) + \frac{\alpha_{12} - \alpha_{11}(m+n)}{a} Y_n(\lambda a)\right] = 0. \tag{4.15}$$

The condition for nonzero solutions of (4.15) produces the frequency equation in the form of a second order determinant as:

$$\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} = 0 \tag{4.15)'}$$

where,

$$M_{11} = J_n(W_1),$$
$$M_{12} = Y_n(W_1),$$
$$M_{21} = \alpha_{11} h W_1 J_{n-1}(hW_1) + (\alpha_{12} - \alpha_{11}(m+n)) J_n(hW_1),$$
$$M_{22} = \alpha_{11} h W_1 Y_{n-1}(hW_1) + (\alpha_{12} - \alpha_{11}(m+n)) Y_n(hW_1).$$

From equation (4.15)', we deduce the frequency equation in the form:

$$J_n(W_1)[\alpha_{11} hW_1 Y_{n-1}(hW_1) + (\alpha_{12} - \alpha_{11}(m+n))Y_n(hW_1)] - Y_n(W_1)[\alpha_{11} hW_1 J_{n-1}(hW_1) + (\alpha_{12} - \alpha_{11}(m+n))J_n(hW_1)] = 0. \tag{4.16}$$

which represents an implicit equation in W_1. By solving this equation numerically, we can obtain the eigenfrequencies *W*.

5. NUMERICAL RESULTS AND DISCUSSION

Here, we shall obtain the eigenfrequencies of the problem considered by solving equations (4.6), (4.10), (4.13), and (4.16) numerically. Since these equations are an implicit functional relation of W_1 and h we proceed to find the variation of natural frequency with ratio h . A fortran program to evaluate the roots *W* of the above equations versus different values of h for the first mode was made. We have adopted the following iterative procedure for numerical computations. For a fixed value of h , we evaluate the determinetal, equations (17), (20), (23) and (26), for various values of the unknown quantity *W* commencing with the initial value near zero and each time adding a fixed but small increment to that unknown quantity until the value of the determinant changes its signs. Then the half-interval method is applied to locate the root correct to a chosen number of decimal places. With this root as the initial value, the procedure is repeated to find the next root, etc.

For a given geometry and elastic constants of the cylinder, the frequency equation is essentially an implicit transcendental function for the frequency parameter *W*. Thus, for a fixed values of h , the frequency equation for different cases (free - fixed - mixed) is a function of *W* only. Values of *W* were chosen as 0, 0.5, 1,0 and 1.5. The results of frequency versus the ratioh are plotted in Figs. (**1-11**) on the basis of data for orthotropic material. As an illustrative example, the elastic constants for an orthotropic material (Cobalt) are used here (Hearman[23]):

$$\alpha_{13} = 7.289 \quad GPa,$$
$$\frac{\alpha_{11}}{\alpha_{13}} = 2.34, \quad \frac{\alpha_{12}}{\alpha_{13}} = 0.93, \quad ,$$
$$\frac{\alpha_{22}}{\alpha_{13}} = 8.18, \quad \rho_0 = 8.93 \quad g/cm^3.$$

Figs. (**1-11**) show the natural frequency *W* increases with increasing ratio h. It is shown in Fig. (**1**) that dispersion curves at point 0.2 and it is at small values of rotation, while Fig. (**7**) shows the dispersion curves at point 0.3, 0.4 and at large values of rotation. The variations of the frequency *W* are due to the effect of rotation and non-homogeneity. In addition, the influence of non-homogeneity and rotation on frequency *W* is very pronounced. These results are specific for the frequency considered, but other frequency *W* may have different trends, because of the dependent of the results on the mechanism of the material. Figs. (**1-3**) show the natural frequency of free-surfaces rotating cylinder with variable

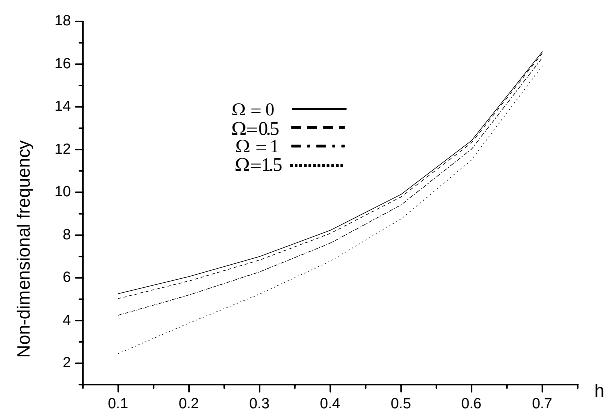

Fig. (1). Frequency *W versus* the ratio *h* of non-homogeneous material (free traction surfaces) , *n = 0.*

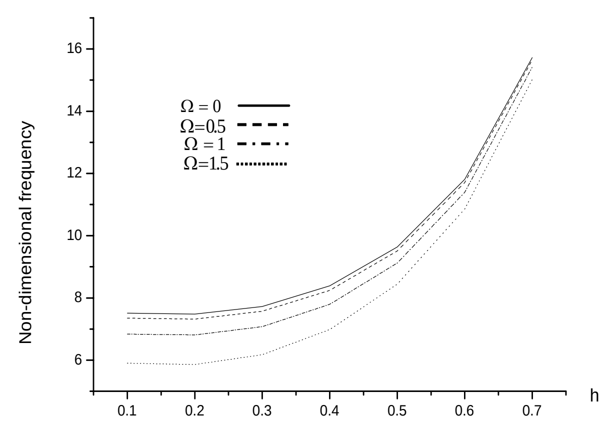

Fig. (2). Frequency *W versus* the ratio *h* of non-homogeneous material (free traction surfaces) , *n = 1.*

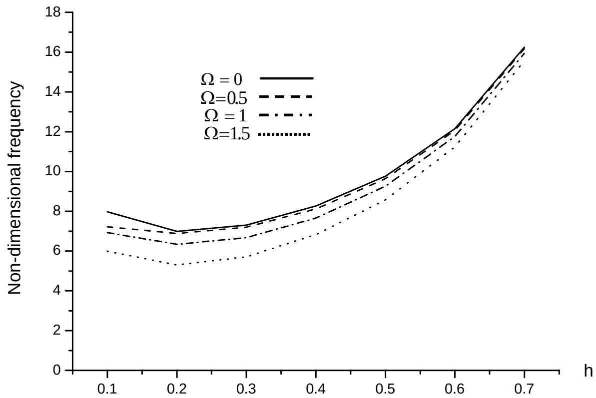

Fig. (3). Frequency *W versus* the ratio *h* of homogeneous material (free traction surfaces), *n = 1*.

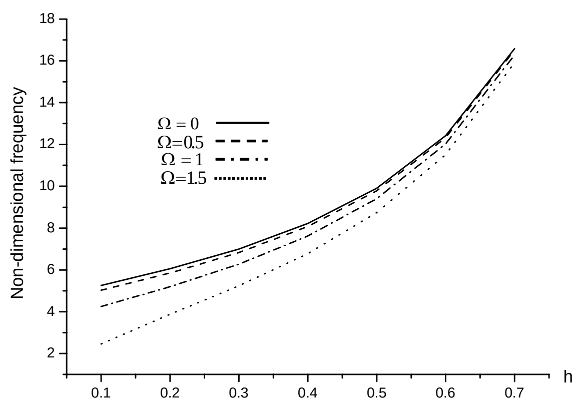

Fig. (4). Frequency *W versus* the ratio *h* of non-homogeneous material (fixed surfaces), *n = 0*.

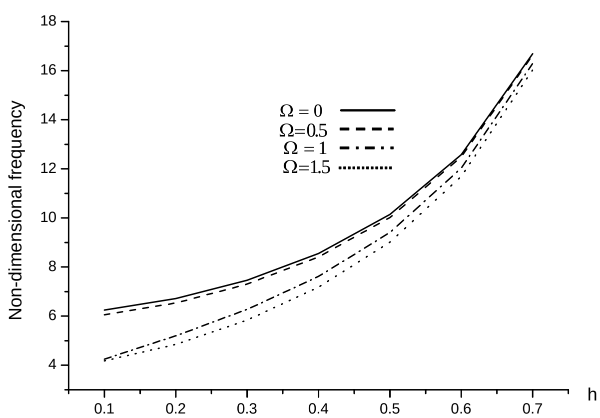

Fig. (5). Frequency *W versus* the ratio *h* of non-homogeneous material (fixed surfaces), *n = 1*.

ratio h, the natural frequency of the cylinder with increasing ratio are the largest. Figs. (**4-6**) present the natural frequency of fixed-surfaces rotating cylinder with linearly varying frequency versus the ratio h. In general, the natural frequency becomes larger monotonically with an increase of the ratio h. Figs. (**7-11**) show the variation of the natural frequency of mixed-surfaces rotating cylinder. It is clear from Figs. (**7-11**) that the natural frequency becomes largest with the relative ratio h and with the effect of rotation are decreases.

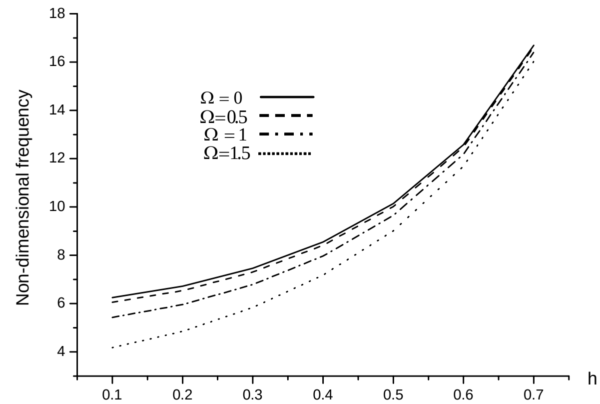

Fig. (6). Frequency *W versus* the ratio *h* of homogeneous material (fixed surfaces), *n = 1*.

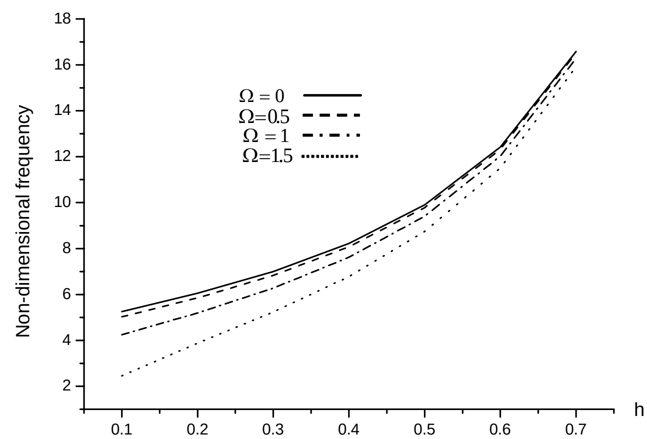

Fig. (7). Frequency *W versus* the ratio *h* of non-homogeneous material (inner fixed surfaces and outer free surfaces), *n = 0*.

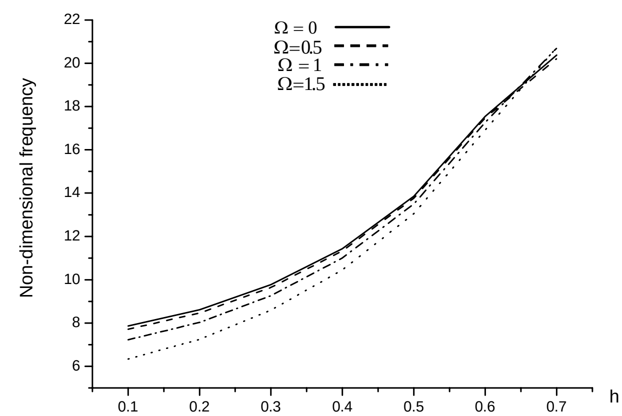

Fig. (8). Frequency *W versus* the ratio *h* of non-homogeneous material (inner fixed surfaces and outer free surfaces), *n = 1*.

Next consideration is given to the relative change in the frequencies over a range of aspect (i.e., a/b) ratio of cylinder. This allows one to observe the frequency response for different rotation values of cylindrical shape. The results are presented as the variation in the dimensionless frequency *W* as a function of the aspect ratio of the cylinder, and are shown in Fig. (**1**) All values were obtained by using half-interval method. There is a good agreement between these results and those of Charalambopoulos *et al*. [24].

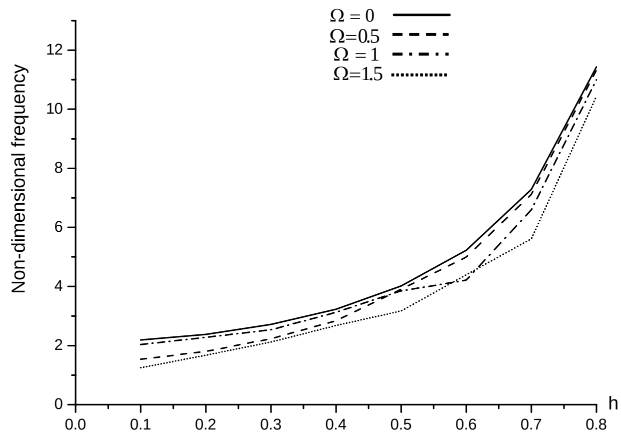

Fig. (9). Frequency *W versus* the ratio *h* of homogeneous material (inner fixed surfaces and outer free surfaces), *n = 1*.

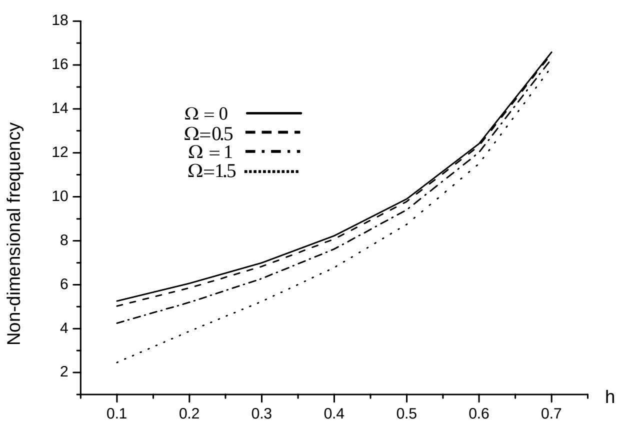

Fig. (10). Frequency *W versus* the ratio *h* of non-homogeneous material For mixed B.C , *n = 0.*

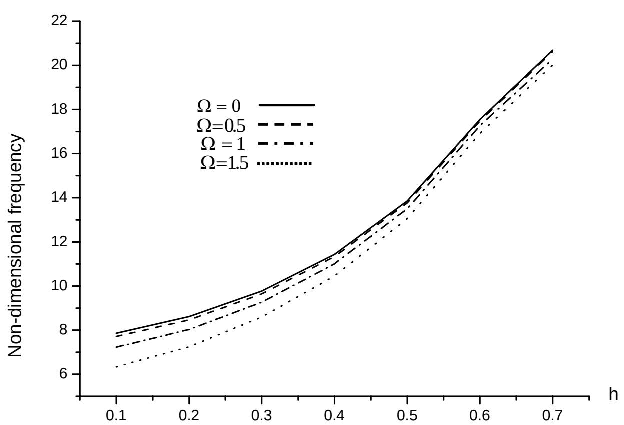

Fig. (11). Frequency *W versus* the ratio *h* of non-homogeneous material (inner free surfaces and outer fixed surfaces), *n = 1*.

CONCLUSION

Harmonic vibrations of infinite elastic cylinder have been studied using a half-interval method. The governing equations in cylindrical coordinates are recorded for future reference.The frequency equations have been obtained under the effects of rotation and non-homogeneity. Numerical results are given and illustrated graphically. To examine the effects of rotation and non-homogeneity, variations of the frequency W with the ratio h of non-homogeneous materials have been shown graphically and they are compared with those for the material in the absence of rotation and non-homogeneity. It is found that the frequency decreases with rotation increases for all cases and with the high values of the ratio h is increases.

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