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Optimal Design of Large Circular Vibrating Screen Based on Multiple Frequencies Constraints and Analytical Sensitivity Methods

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Abstract: Multiple frequencies constraints and analytical sensitivity methods were used in the structural optimization design of the side plate of 4YK1860 vibrating screen to reduce the damage of side plate caused by high dynamic stress. The mass of side plate of the vibrating screen was taken as object function, mode frequencies were regarded as state parameters, size of the plate was used as the constraint parameter and an optimization criterion which is applicable to solve this kind of problem was given. Meanwhile, sensitivity characteristics of variable parameters were considered during the process of optimization. The results show that the optimization program with embedded analytical sensitivity calculation methods is of a higher solving precision and with a more stable calculating process. After structure optimization, the thickness of side plate to withstand large impact is increased, the thickness of stiffener and peripheral angle is decreased, the mass of side plate is reduced by 7.75% and a better optimization effect is achieved. The analysis of the optimal vibrating screen presents that the mass of vibrating screen decreases by 3.5%, elastic deformation frequency which is close to the working frequency increases by 8.8% and the dynamic stress decreases by 48%. The natural frequencies of vibrating screen are improved and modal frequency is far from the working frequency of the exciting motor, stress concentration of vibrating screen is weakened and thus the structural stiffness and reliability of vibrating screen are enhanced.

Keywords: Analytical sensitivity, Large circular vibrating screen, Multiple frequencies constraints, Optimum design.

INTRODUCTION

Side plate and beam of vibrating screen reach the fatigue limit of their material always more quickly than the other parts. It can be found that in practical engineering, with the vibrating screen scrap, the most damaged parts are the side plate and the beam [1]. Another common fatigue failure appears on stress concentration points of the vibrating screen, for instance, joint part of bearing, spring box, welded area, etc. The side plate and beam of vibrating screen bear large inertial force besides the exciting force being in the operating condition. The screen body does not have rigid vibration but has elastic vibration in working frequency. It produces large dynamic stress on the beam and side plate when the screen body is in elastic vibration condition and the stress concentration results in dynamic response. So it appears as a fatigue failure in the structure of the beam and side plate [2-4]. According to the problem stated above, the aim of the large vibrating screen optimization is to have reasonable structure improvements, enhanced fatigue strength, decreased dynamic stress and extended service life of vibrating screen. Therefore, structural dimension optimization of large circular vibrating screen 4YK1860 has been studied in this paper based on multiple frequencies

constraints and analytical sensitivity methods and the optimal structure of vibrating screen was obtained.

ANALYTICAL SENSITIVITY METHOD

Structure parameters that have the most effect on the system in the process of structure optimization can be found by sensitivity analysis, thus improving the optimization efficiency. The general optimization design software uses difference method and semi-analytical method to calculate the sensitivity. These two methods affect the difference step and increase the time taken for optimization analysis which reduces the computational efficiency; moreover, the optimization results are not obvious. According to references [5-7], the analytical formulas of sensitivity based on objective function and constraint conditions are put forward. The mass of structure is the objective function with frequency as the main form of constraint. Analytical sensitivity method of objective function is given as:

$$\frac{\partial W}{\partial p_i} = \frac{\partial \sum w^e}{\partial p_i} = \sum \frac{\partial w^e}{\partial p_i} \tag{1}$$

where, W expresses objective function, w^e expresses element mass and p_i expresses design variable. The method of analytical sensitivity with constraints is:

$$\frac{\partial \lambda_r}{\partial p_i} = \phi_r^T \left(\frac{\partial K}{\partial p_i} - \lambda_r \frac{\partial M}{\partial p_i} \right) \{\phi_r\} \tag{2}$$

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$$\frac{\partial \lambda_r}{\partial p_i} = \phi_r^T \left(\sum \frac{\partial k^e}{\partial p_i} - \lambda_r \frac{\partial m^e}{\partial p_i} \right) \{\phi_r\} \qquad (3)$$

In which, λ_r expresses frequency constraints, ϕ_r expresses modal of corresponding frequency, K expresses general stiffness matrix, M expresses general mass matrix, k^e expresses element stiffness matrix, m^e expresses element mass matrix.

Both the formulas (2) and (3) can be used to calculate the analytical sensitivity, but the method of formula (2) requires assembling partial derivative matrix of design variables about general stiffness matrix and general mass matrix, so a long calculation time and more storage space are needed. According to formula (3), partial derivative of design variables regarding relevant element stiffness matrix and mass matrix initially can be calculated, followed by calculating the contribution that the relevant element makes to frequency sensitivity and finally superimposes the contribution of element to obtain frequency sensitivity. In this way, it omits the multiplication and assembly of large matrix, helps save storage space and improves the computational efficiency.

It can be known from formula (3) that partial derivative of element stiffness matrix and element mass matrix need to be calculated when calculating the frequency sensitivity of the structure. If this function is put in the program of structure optimization analysis, it can calculate the partial derivative matrix of element stiffness matrix and element mass matrix simultaneously by structure analysis.

OPTIMAL PARAMETERS DETERMINATION OF CIRCULAR VIBRATING SCREEN

Before the optimization of the vibrating screen side plate, the optimal parameters of the structure of side plate need to be determined. The structure of side plate with stiffeners, angle stiffeners, mounting hole of shaft and vibration exciter is very complex. Taking the least weight as objective function, frequency optimization problem can be formulated [8].

Objective function:

$$\min w(x) = \sum_{i=1}^{n} \rho_i x_i l_i \qquad (4)$$

Behavior constraint:

$$g_j(x) = G_j(x) - \bar{G}_j \leq 0 \, (j = 1,2,\cdots,m) \qquad (5)$$

Dimension constraint:

$$x_i^l \leq x_i \leq x_i^u \, (i = 1,2,\cdots,n) \qquad (6)$$

where, x expresses vector containing n- x_i, ρ_i expresses density, x_i expresses design variable, l_i expresses element length, $G_j(x)$ expresses vector of deflection, stress and frequency, $\bar{G}_j$ expresses limit value $G_j(x)$ needs to meet, x_i^l and x_i^u express upper and lower limits of element dimensions respectively.

Side thickness h_1, thickness of angle around side plate h_2, and stiffener width b are taken as design variables of objective function in the optimization process. The initial values of h_1, h_2 and b are 15mm, 15mm and 100mm, respectively. The constraints of the design variables are given below.

$$8mm \leq h_1 \leq 20mm \, ; \; 6mm \leq h_2 \leq 20mm \, ;$$
$$50mm \leq b \leq 150mm$$

In order to meet the mechanical structural reliability, modal frequency is taken as constraint considering the dynamic characteristics of vibrating screen. So the first three modal frequencies f_1, f_2 and f_3 are taken as constraints which are obtained by inherent characteristic analysis of vibrating screen side plate before optimization. f_1=7.83Hz, f_2=10.21Hz, f_3=15.20Hz and constraints are set below.

$$3.5Hz \leq f_1 \leq 8Hz \, ; \; 8Hz \leq f_2 \leq 11Hz \, ; \; 15Hz \leq f_3 \leq 20Hz$$

The optimization objective function is mass of side plate with the initial value as 1780 Kg.

OPTIMIZATION CRITERIA

Formula (5) can be expressed as below from multi-frequency constraints aspects [9, 10].

$$g_j(x) = (f_j^2 - \alpha_j \bar{f}_0^2) \leq 0 \qquad (7)$$

where, $\bar{f}_0$ expresses initial base frequency, $\alpha_j \bar{f}_0^2$ expresses square of f_j which is j order expected frequency. Characteristic equation of formula above is this.

$$\{\Psi_j\}^T [K]\{\Psi_j\} - f_j^2 \{\Psi_j\}^T [M]\{\Psi_j\} = 0 \qquad (8)$$

Element stiffness matrix $[K^e]$ and element mass matrix $[M^e]$ can be expressed as below considering nonlinear characteristics of the structure [11].

$$[K^e]_i = [K^e]_i^{(0)} + x_i[K^e]_i^{(1)} + x_i^2[K^e]_i^2 + ... \qquad (9)$$

$$[M^e]_i = [M^e]_i^{(0)} + x_i[M^e]_i^{(1)} \qquad (10)$$

Taking complex formulas from (8) to (10), following can be obtained

$$G_{ij} = \{\tilde{\Psi}\}^T ([K]_i^{(1)} + 2x_i[K]_i^{(2)} - f_j^2[M]_i^{(1)})\{\tilde{\Psi}\} \qquad (11)$$

where, $\{\tilde{\Psi}\}$ expresses modal vector of mass matrix $[M]$ after standardization.

Combining optimized parameters to build Lagrange equation.

$$L(x,\varsigma) = \sum_{i=1}^{n} \rho_i x_i l_i - \sum_{j=1}^{m} \varsigma_j (f_j^2 - \alpha_j \bar{f}_0^2) \qquad (12)$$

where, ς expresses Lagrange operator.

Table 1.　Optimization results of structure.

Optimized Sequence	f_1 /Hz	f_2 /Hz	f_3 /Hz	h_1 /mm	h_2 /mm	b /mm	m_t /kg
Set1	0	0	0	0	0	0	0
Set2	8.61	9.81	16.62	17.86	12.40	88.80	1841.1
Set3	8.04	9.15	14.75	15.69	13.52	111.8	1673.0
Set4	8.10	9.53	16.25	17.87	6.046	96.91	1824.5
Set5	5.88	8.03	10.03	10.48	17.10	108.2	1196.6
Set6	8.02	9.07	15.08	16.09	13.14	64.66	1647.4
Set7	7.97	9.03	15.11	16.15	12.32	62.92	1646.9
Set8	3.78	9.38	11.68	16.22	8.494	53.73	1624.6
Set9	7.96	9.01	15.07	16.10	12.23	62.74	1642.2

Differentiating formula (12) for x_i, and calculating the minimum value of L, the optimization criteria can be expressed as

$$1 = \sum_{j=1}^{m} \varsigma_j \frac{G_{ij}}{x_i^2 \rho_i l_i} \quad (i = 1,2,...,n) \tag{13}$$

Making $\phi_i = \sum_{j=1}^{m} \varsigma_j \frac{G_{ij}}{x_i^2 \rho_i l_i}$, the optimize recursive formula can be obtained.

$$(x_i)_{r+1} = (x_i) r[1 + \frac{1}{v}(\phi_i - 1)]_r \tag{14}$$

where, r expresses cycles and v expresses the step factor.

Solving method of ς_j is complex given by N. S. Khot in reference [12]. Here, an approximate formula is proposed which meets the computing requirements through revised parameters and simplifies the small factor.

$$\varsigma_j = \frac{W}{f_j^2 (1 - \xi_j)} \tag{15}$$

where, $W = \sum_{j=1}^{m} \rho_i x_i l_i + w^*$, w^* expressing element fluctuation mass, $\xi_j = \dfrac{\{\Psi_j\}^T [M]_s \{\Psi_j\}}{\{\Psi_j\}^T [M] \{\Psi_j\}}$

ANALYSIS OF OPTIMIZATION RESULTS

Side Plate Optimization Results

Final optimization results obtained after nine iterations are shown in Table **1**. The results of the ninth sequence shown in Table **1** are optimum, and the objective mass m_t is changed to 1642 Kg from 1799 Kg.

Optimization process of objective m_t is shown in Fig. (**1**). Optimization process of side plate thickness h_1, angle thickness h_2, and stiffeners width are shown in Figs. (**2-4**). Optimization processes of constraints frequency f_1, f_2 and f_3 are shown in Figs. (**5-7**) respectively.
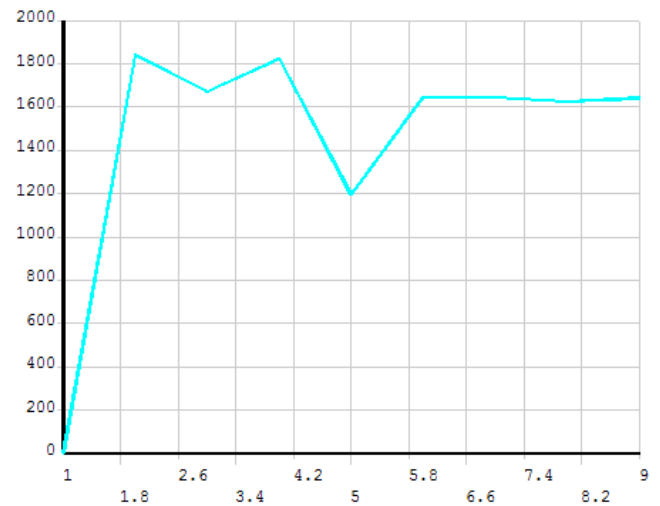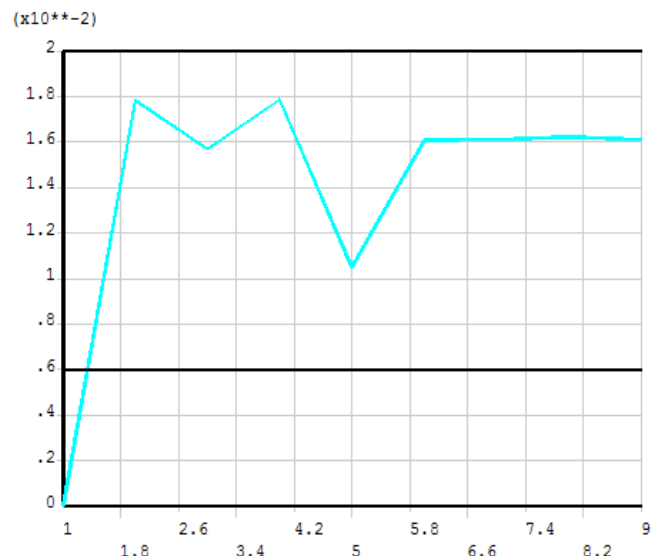

Fig. (1). Optimization process of m_t.

Fig. (2). Optimization process of h_1.
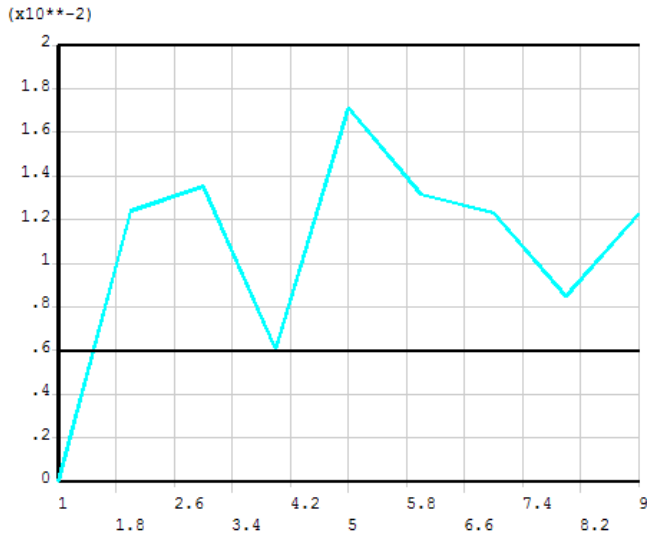
(x10**-2)

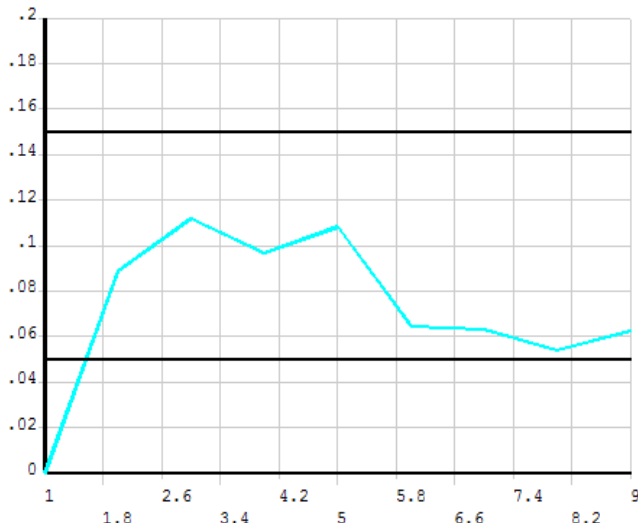**Fig. (3).** Optimization process of h_2.

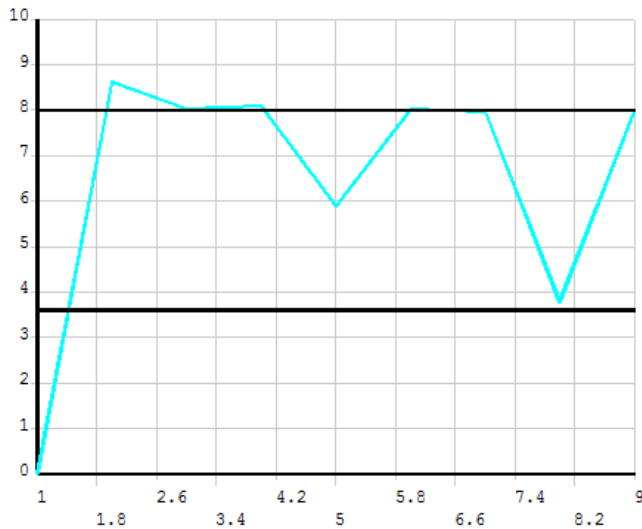**Fig. (4).** Optimization process of b.

Fig. (5). Optimization process of f_1.
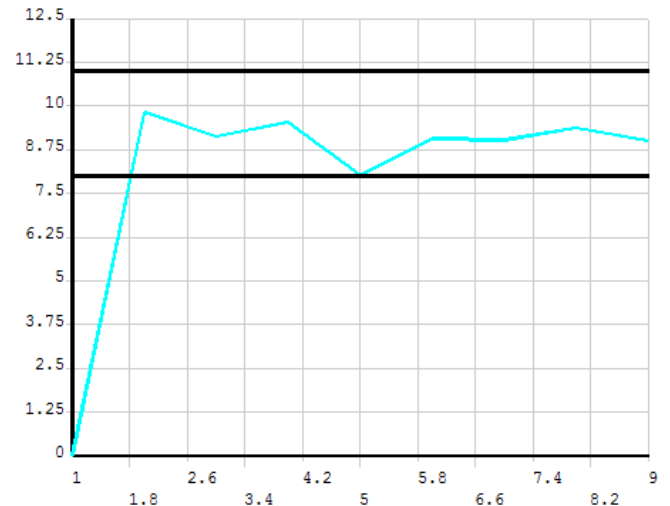

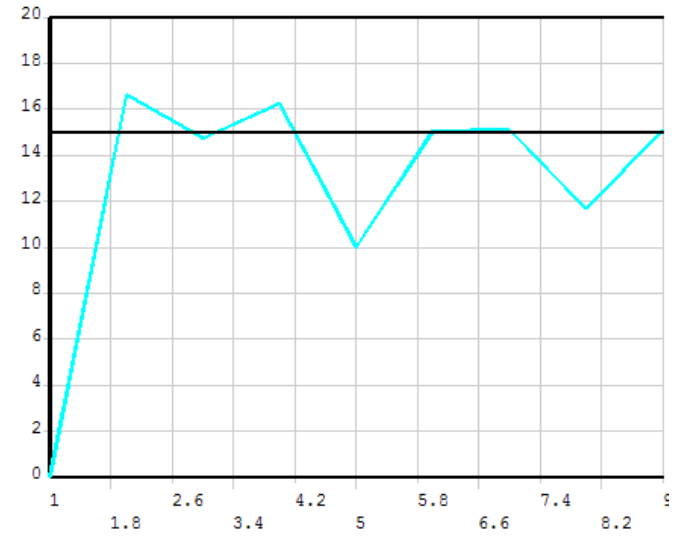**Fig. (6).** Optimization process of f_2.

Fig. (7). Optimization process of f_3.

Table 2.   Parameter optimization results comparison.

Model Parameters	Before	After	Change Range/%
f_1/Hz	7.83	7.9614	1.67
f_2/Hz	10.21	9.0158	11.6
f_3/Hz	15.20	15.067	0.9
h_1/mm	15	16.105	7.3
h_2/mm	15	12.228	2.772
b/mm	100	62.741	37.2
m_t/kg	1780	1642	7.75

Comparison of the results before and after optimization in Table **2** shows that the side plate has a 7.75% mass reduction and all parameters have obvious change. The first order frequency increases by 1.67% meanwhile the second and third order frequency decreases by 11.6% and 0.9% respectively. Thickness of side plate increases by 7.3% based

on 15 mm, thickness of angle iron around side plate decreases by 2.772%, and the thickness of the stiffener reduces to 63 mm from 100 mm. Overall, it achieves the desired objectives and ensures natural frequency having little change.

Evaluation and Analysis of Vibrating Screen After Optimization

Results of Natural Mode Before and After Model Modification

Fig. (**8**) shows the modified structure of the vibrating screen according to the optimization results. The results of dynamical analysis performed again and the first ten natural frequencies and natural mode of the vibrating screen calculated are shown in Table **3**.
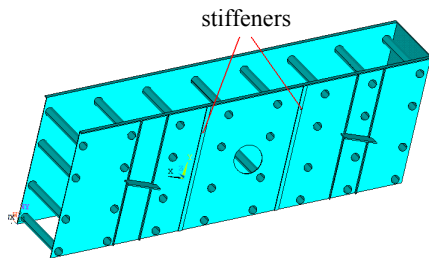

Fig. (8). Structure model.

Table 3.   Model Shapes of new model.

Order	Frequency/Hz	Modes
1	0.75149	Rigid rotation around z axis
2	1.4068	Rigid translation along x direction
3	2.9427	Rigid rotation of feed end and discharge end around centroid axis
4	4.1851	Rigid translation along y direction
5	5.2309	Rigid rotation around x axis
6	22.668	Anisotropic torsion vibration of side plates in xy plane
7	30.686	Reciprocating swing of feed end and discharge end along z direction
8	35.034	Anisotropic swing of side plates along x direction
9	50.083	Fluctuation of screen box along z direction
10	61.237	Reciprocating swing of side plates along z direction

As shown in Table **3** below, frequencies of rigid vibration have a little change, having no effect on the dynamic characteristics. There is great increment in frequencies of bending deformation, and the ninth order frequency increases by nearly 30%, which shows that the overall stiffness of vibrating screen increases. The sixth order frequency which is close to the working frequency 16.7 Hz impacts on deformation greatly and it changes from 20.834Hz to 22.668 Hz. The sixth order vibrating mode after

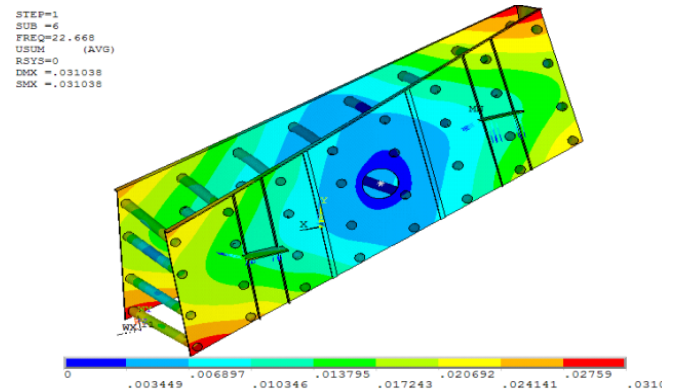optimization is shown in Fig. (**9**), and the displacement of vibration Dmx is 0.31 mm

Fig. (9). Sixth vibrating mode.

Results of Dynamic Response Before and After Model Modification

Extracting the dynamic response chart and displacement chart of vibrating screen under working frequency of 16.7Hz, the dynamic stress of screen box is 34.7MPa which is more than the allowable stress and concentrates in the central of side plate. After model modification, making the same dynamic analysis under the same conditions, the dynamic response chart and displacement chart are shown in Figs. (**10**, **11**). The maximum stress of the improved model is 18MPa and is less than the allowable stress of 24.5MPa. Large displacement also appears in the discharge end under working frequency, and the value decreases to 0.10 mm from 0.17 mm. In general, stress concentration of vibrating screen improved.
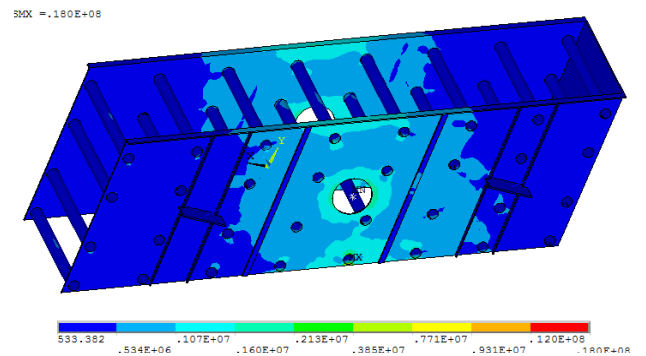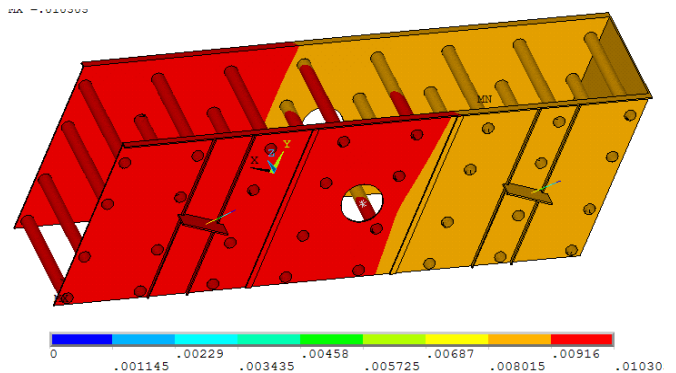

Fig. (10). Model dynamic stress charts.

Fig. (11). Displacement charts.

Table 4. Relevant parameters of circular vibrating screen.

Optimized Order	f_1/Hz	f_2/Hz	f_3/Hz	m_t/kg	Dynamic Stress/MPa
Before	20.834	29.735	33.026	9870	34.7
After	22.668	30.686	35.034	9515	18
Change range/%	8.8	3.2	6.1	3.5	48

After comparing the parameters of the circular vibrating screen before and after optimization, from the results (as shown in Table **4**), it can be observed that the first three frequencies of elastic deformation increased by 8.8%, 3.2% and 6.1% . The dynamic stress decreased by 48% after optimization. Rigidity of vibrating screen after size optimization improved, elastic deformation of screen body reduced, stress concentration weakened, stress distribution became more uniform and mass of structure decreased. In short, the stability of structure improved while the material and cost were saved.

CONCLUSION

(1) Using embed analytic sensitivity method by considering sensitivity of variable parameters into size optimization process of the vibrating screen side plate, the calculation efficiency improved and stale optimization results were obtained.

(2) Taking the mass of circular vibrating screen side plate as objective function, the minimum mass of side plate was obtained after meeting multiple frequencies conditions. Single side plate mass decreased by 7.75% after optimization and the material and cost were saved.

(3) By evaluating vibrating screen after optimization, the mass decreased by 3.5%, frequency of elastic deformation close to working frequency increased by 8.8%, and the dynamic stress reduced by 48%. Natural frequencies were raised, rigidity of plate was improved, stress concentration weakened, mode frequencies were observed to be far away from working frequency and the stability of structure improved.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

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