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Research on the Maximal Bearing Capacity Calculation of a New Double Ring Reducer Based on MATLAB

Qiang Xu*[,1], Qi-sheng Xu[1] and Dao-Yi Xu[2]

[1]Department of Mechanical Engineering, Hefei University, Hefei, 230601, China

[2]Anhui Xiao Xiao Science and Technology Limited Company, Jixi 510660, China

Abstract: The bearing capacity of a new double ring reducer increases with the increase of load because of the elastic deformation of the gear tooth. In order to solve the problem of its bearing capacity quantificationally, the concept of the maximal bearing capacity is put forward. Starting with that the single tooth bending stress is up to the bending fatigue strength, a mathematical model to determine the normal backlash of the gear teeth has been established, the maximal deformation of the single tooth has been determined. The mathematical relationships have also been setup between the normal backlashes of the tooth pairs, the maximal deformation and number of contact points, a corresponding MATLAB program is designed. The maximal bearing capacity of the reducer has been estimated through examples and proved by experiment. The results show that the calculation method is more effective and fully considers the factors that the elastic deformation of the gear tooth can increases its bearing capacity, so the structure of the reducer is more compact, which establishes the theory foundation for designing the reducer.

Keywords: Double ring reducer, engaging point, maximal deformation, normal backlash.

1. INTRODUCTION

For a three-ring reducer and a four-ring reducer, the machining precision is hard to be guaranteed because three or four-phase drive ring plates are at 120 or 90 degree angles from each other, which causes the plates interfere each other, resulting in vibration, noise and heating. A new double ring reducer (Patent China, ZL01206843.8) was invented to overcome the shortcomings of the three or four ring reducer and carry forward their advantages. A new double ring reducer is widely applied in mine, metallurgy, chemical industry, aviation and space flight etc. because it is of many advantages, such as with compact structure, bigger reduction ratio, easy insuring machining precisions, less force of planetary gear bearings [1, 2].

A new double ring reducer is the special planetary transmission mechanism in which internal gears do translation, and make the external gear rotate. In transferring loads, the tooth deformation is more than clearance between some teeth profiles, the untouched teeth are made to mesh, and improving greatly the bearing capacity of the mechanism. Many Chinese scholars have done many researches on the theoretical calculation of the gap of teeth of internal matins gears, and the relation between contact tooth pair number and load. The bigger the load transmitted, the more the number of teeth pairs mating simultaneously. When the strength is checked, although the concept of the elastic mesh effective coefficient is introduced, it has brought the confusions to designers that the coefficient is variable [3-6]. This paper presents the concepts of the maximal bearing capacity of the gear reducer. The calculation of the clearance between the tooth profiles is taken as starting, the maximal allowed deformation of single tooth is analyzed, and the maximal bearing capacity of the reducer is calculated. If the load of the reducer is less than the maximal bearing capacity of the reducer, it is thought that the reducer can meet the demand of bending fatigue strength. The method improves design efficiency, reduces the cost, and fully considers that the elastic deformation has influence on increasing the bearing capacity.

2. THE ESTABLISHMENT OF MATHEMATICAL MODELS OF THE NORMAL BACKLASH OF THE TEETH PAIRES

The structure diagram of a new double ring reducer is shown in Fig. (**1**); the high-speed level is composed of a pair of helical cylinder gears a b, the low-speed level is a planetary transmission with small tooth number difference. H is an active crankshaft, Z_2 is an inner planetary gear, and Z_1 is an external gear. The inner planetary gears do translation by work principles of two-crank mechanism, a rotation motion is input from the gear a, the motion is output from the external gear Z_1 by bigger reduction ratio.

For the special internal gear pairs of the double ring reducer with small teeth difference, under the action of the external load F_n, because of elastic deformation of gear teeth, there are several simultaneously meshing points of K_0, K_1, K_2….. K_i, the relation between the loads F_{ni} withstood by each pair of teeth and the normal backlash Jni of the tooth pair is as follows [4]:

$$\begin{cases} F_{n0}=C_n\delta_{n0} \\ F_{ni}=C_n(\delta_{n0}-j_{ni}) \\ Z_H=i+1 \\ F_n=\sum_{i=0}^{i}F_{ni} \end{cases} \tag{1}$$

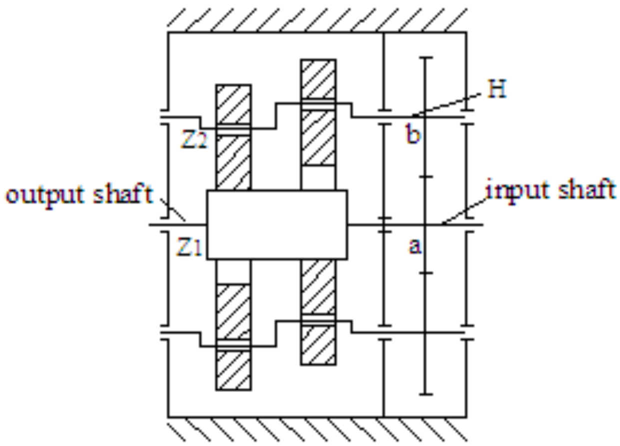

Fig. (1). The structure diagram of a new double ring reducer.

Fig. (**2**) describes the backlash of the tooth pairs before they mesh. B_1B_2 is the actual meshing line. The center O_1 of external gear is the origin of external gear coordinate system, the center O_2 of internal gear is the origin of internal gear coordinate system. When the extended lines of the internal and external tooth profiles mesh at the point K'_i (virtual points and their normal backlash before meshing engaging point), the distance between the tooth top C of external gear and the tooth profile of internal gear is the shortest. While the engaging point K_0 on the meshing line is transferring load, the gear teeth occur deformation, the internal gear is forced to rotate angle ϕ additionally, the point D of the internal tooth profile contacts the tooth tip C of the external gear, which can be the contact point K_i. If α_{a1} is the tip circle's pressure angle of external gear, ϕ_1 (the angle PO_1C) is [6].

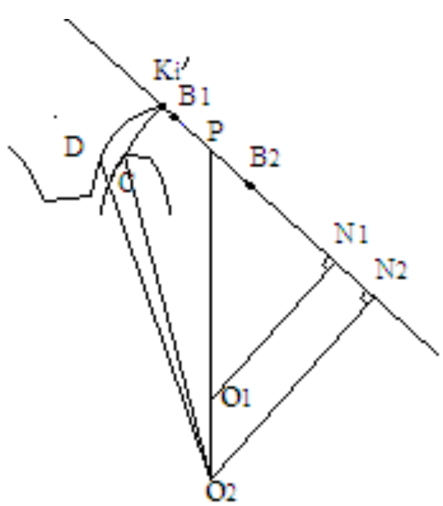

Fig. (2). The relationship between the positions of virtual contact.

$$\phi_1=\text{inv}\alpha_{a1}-\frac{K'_iN_1}{r_{b1}}+\alpha' \tag{2}$$

Here α' is the actual engagement angle of internal gear pair with small teeth difference.

The distance between the point C and the center O_1 of the external gear is the addendum circle radius r_{a1}, the coordinates of the point C in the coordinate system of the external gear are

$$\begin{aligned} x_{c1}&=r_{a1}\sin\phi_1 \\ y_{c1}&=r_{a1}\cos\phi_1 \end{aligned} \tag{3}$$

If a' is the actual centre distance of internal gear pairs with small teeth difference, the coordinates of the point C in the coordinate system of the internal gear are

$$\begin{aligned} x_{c2}&=x_{c1} \\ y_{c2}&=y_{c1}+a' \end{aligned} \tag{4}$$

As a result, the center distance between the point C and the point O_2 is

$$r_{c2}=\sqrt{x_{c2}^2+y_{c2}^2} \tag{5}$$

The distance r_{D2} between the point D of the internal tooth profile and the centre O_2 is also r_{C2}, namely the pressure angle of the point D is as follows

$$\alpha_{D2}=\arccos\left(\frac{r_{b2}}{r_{D2}}\right) \tag{6}$$

ϕ_2 is the angle PO_2D, So [6]

$$\phi_2=\text{inv}\alpha_{D2}-\frac{K'_iN_2}{r_{b2}}+\alpha' \tag{7}$$

The coordinate of the point D in the coordinate system of the internal gear is

$$\begin{aligned} x_{D2}&=r_{D2}\sin\phi_2 \\ y_{D2}&=r_{D2}\cos\phi_2 \end{aligned} \tag{8}$$

The distance between the point D and the point C, namely the normal backlash J_{ni} of the engaging point is as follows:

$$\overline{CD}=\sqrt{(x_{c2}-x_{D2})^2+(y_{c2}-y_{D2})^2} \tag{9}$$

Fig. (**3**) describes the backlash of the tooth pairs after their meshing. B_1B_2 is the actual meshing line. When the extended lines of the internal or external tooth profiles mesh at the point K'_i (virtual engaging point), the distance between the tooth top C of internal gear and the tooth profile of external gear is points and their normal backlash after meshing the shortest. While the engaging point K_0 on the meshing line is transferring load, the gear teeth occur deformation, the internal gear is forced to rotate angle ϕ additionally, the tooth tip C of the internal gear contacts the point D of the external tooth profile, which can be the contact point K_i. If α_{a2} is the tip circle's pressure angle of internal gear, ϕ_2 (the angle PO_2C) is [6]

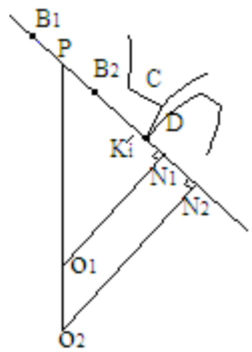

Fig. (3). The relationship between the positions of virtual contact.

$$\phi_2=\text{inv}\alpha_{a2}-\frac{K_i'N_2}{r_{b2}}+\alpha' \quad (10)$$

Here α' is the actual engagement angle of internal gear pair of less tooth difference.

The distance between the point C and the center O_2 of the internal gear is the addendum circle radius r_{a2}, the coordinates of the point C in the coordinate system of the internal gear are

$$x_{c2}=r_{a2}\sin\phi_2 \qquad y_{c2}=r_{a2}\cos\phi_2 \quad (11)$$

If a' is the actual centre distance of internal gear pairs with small teeth difference, the coordinates of the point C in the coordinate system of the external gear are

$$x_{c1}=x_{c2} \qquad y_{c1}=y_{c2}-a' \quad (12)$$

As a result, the center distance between the point C and the point O_1 is

$$r_{c1}=\sqrt{x_{c1}^{\ 2}+y_{c1}^{\ 2}} \quad (13)$$

The distance r_{D1} between the point D of the external tooth profile and the centre O_1 also is r_{C1}, namely the pressure angle of the point D is as follows:

$$\alpha_{D1}=\arccos\left(\frac{r_{b1}}{r_{D1}}\right) \quad (14)$$

ϕ_1 is the angle PO_1D, So [7]

$$\phi_1=\text{inv}\alpha_{D1}-\frac{K_i'N_1}{r_{b1}}+\alpha'$$

The coordinate of the point D in the coordinate system of the external gear is

$$x_{D2}=r_{D2}\sin\phi_1 \qquad y_{D2}=r_{D2}\cos\phi_1 \quad (15)$$

The distance between the point D and the point C, namely the normal backlash Jni of the engaging point Ki is as follows:

$$\overline{CD}=\sqrt{(x_{c1}-x_{D1})^2+(y_{c1}-y_{D1})^2} \quad (16)$$

From the above formula, the CD size is related with the position of virtual engaging point K_i', the engaging angle α', and the addendum circle diameter.

3. DETERMINATION OF THE MAXINAL DEFORMATION OF THE SINGLE GEAR

If the transmitting power of a double ring reducer is P, the motor rotating speed n_D, and the transmission ratio i, the torque of the external gear is

$$T=9.55\times10^6\frac{P}{n_D i} \quad (17)$$

If two translational internal gears are considered, the normal force F_n on external gear is calculated as

$$F_n=\frac{T}{mZ_1\cos\alpha} \quad (18)$$

As Fig. (**4**) shows, the point B_1 is the starting point of the actual meshing line. At the point B_1 the tooth root of the translational internal gear and the tooth top of external gear begin to enter into meshing, the point B_2 is the point to terminate and withdraw from the meshing line, and B_1B_2 is the actual meshing line. Suppose K_0 is any meshing point on B_1B_2, K_1', K_2' … K_i' are virtual engaging points on the meshing line, the index odd number is the virtual engaging points before entering the actual meshing line, and the index even number is the virtual engaging points after quitting the actual meshing line.

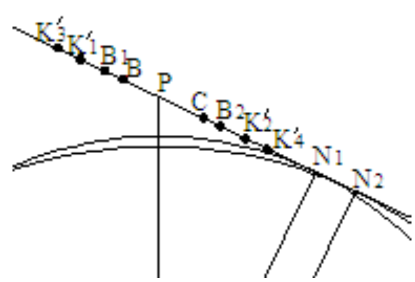

Fig. (4). The name and positions of the different engaging points.

The gear teeth of internal gear pairs with small teeth difference bend in the same direction, the difference among their curvature radiuses are very small. Therefore, soft surface tempering gear is always used. Generally, the touch strength of gear is not calculated, only checking the bending strength [7]. BC is the meshing line of single pair of gears, its upper or lower bound points are the load action point producing maximal bending moment, the maximal bending deformation generate at same time. Suppose K is loading coefficient, Y_F is the tooth form factor, Ys is stress correction coefficient, and Yε is contact ratio coefficient. When the tooth root bears the allowable bending stress $[\sigma_F]$, the maximal affordable normal force is [8]

$$F_{0\max} = \frac{bm[\sigma_F]}{KY_F Y_S Y_\varepsilon} \tag{19}$$

and the allowable bending stress is

$$[\sigma_F] = \frac{\sigma_{F\lim} Y_N}{S_F} \tag{20}$$

Here $\sigma_{F\lim}$ is the bending fatigue strength of the tooth root of the experiment gear. Y_N is the life coefficient for computing the bending strength. S_F is the safety factor for computing the bending strength.

The value of $F_{0\max}$ is related to the points B, C, it is primarily to due to the different allowable bending stresses $[\sigma_F]$ and the different tooth form factor Y_F. Under the effect of $F_{0\max}$, the gear tooth will produce a more normal deformation. It includes contact deformation, bending deformation and shear deformation of the gear tooth, and the deformation of the gear tooth root. The meshing stiffness of the gear differs depending on the position of engaging points. To simply the problem, the average meshing stiffness of the gear tooth is regarded as the meshing stiffness. From Ref. [9], the maximal normal elastic deformation is

$$\delta_{n0} = \frac{F_{0\max}}{bC_r} \tag{21}$$

Here Cr is the average meshing stiffness of the gear tooth. For the meshing gear subjected to the medium load, it takes approximately 20 N/mm▪um.

4. CALCULATION OF THE MAXIMAL BEARING CAPACITY

If the tooth profile coefficient of the external gear Z_1 is less than that of the internal gear Z_2, the external gear tooth at the point B is easier to produce fatigue failure. Therefore, while the gear teeth mesh at the point B, and the stress of the external gear tooth root reaches flexural ultimate strength, the deformation of it is also the biggest. Conversely, at the point C, for the inner gear tooth, we can draw a conclusion that is coincident with the above conclusion. As Fig. (**4**) shows, B_1B_2 is the actual meshing line; BC is the meshing line of single gear pair, B_1B and CB_2 is that of the double gear pairs. So

$$\begin{cases} B_1B_2 = \frac{1}{2} m \cos\alpha [Z_1(\tan\alpha_{a1} - \tan\alpha') + Z_2(\tan\alpha' - \tan\alpha_{a2})] \\ N_1N_2 = \frac{1}{2} m (Z_2 - Z_1)\cos\alpha \tan\alpha' \end{cases} \tag{22}$$

If P_b is a pitch, the length of the meshing line of the double gear pairs is as follows:

$$B_1B = CB_2 = (\varepsilon - 1)P_b \tag{23}$$

The meshing line length of single gear pair is

$$BC = (2-\varepsilon)P_b \tag{24}$$

$$\begin{aligned} B_1N_1 &= B_1N_2 - N_1N_2 \\ &= r_{a2}\sin\alpha_{a2} - \frac{1}{2} m(Z_2 - Z_1)\cos\alpha \tan\alpha' \end{aligned} \tag{25}$$

So,

$$BN_1 = B_1N_1 + BB_1 \tag{26}$$

In order to meet various restrictions, modified gears must be adopted for the double ring reducer. The engagement angle and the total modification coefficient are as follows

$$\alpha' = \arccos(\frac{a}{a'}\cos\alpha) \tag{27}$$

$$x_\Sigma = x_2 - x_1 = \frac{z_2 - z_1}{2\tan\alpha}(inv\alpha' - inv\alpha) \tag{28}$$

Here a is the standard centre distance, a' is the actual centre distance.

For the internal gear pairs with small teeth difference, the actual engagement angle is greater than the standard pressure angle as the actual centre distance is greater than the standard centre distance.

While the gear teeth mesh at the point B, the distance between adjacent engaging points is πmcosα. According to the formula (1), it is determined that how many tooth profiles mesh, the maximal carrying forces at different meshing points are calculated and compared, therefore the maximal carrying capacity of the double ring reducer is computed, the concrete computation steps are given as follows:

1) Input the calculation parameter of the double ring reducer: module m, teeth numbers Z_1 and Z_2, the tooth width b, addendum coefficients X_1 and X_2, other parameters related to bending strength of gear.

2) Compute the engagement angle a', the contact ratio ε, the pressure angle of addendum circle, the actual centre distance a', and the values of B_1B_2, BN_1, N_1N_2, $F_{0\max}$ and δ_{n0}, order i=1.

3) Judge whether i is an odd number or an even number. If i is an odd number, $K_i'N_1$ and $K_i'N_2$ are

$$\begin{aligned} K_i'N_1 &= BN_1 + (\frac{i-1}{2} + 1)P_b \\ K_i'N_2 &= K_i'N_1 + N_1N_2 \end{aligned} \tag{29}$$

If i is an even number, $K_i'N_1$ and $K_i'N_2$ are

$$\begin{aligned} K_i'N_1 &= BN_1 - (\frac{i-2}{2} + 1)P_b \\ K_i'N_2 &= K_i'N_1 + N_1N_2 \end{aligned} \tag{30}$$

If $K_i'N_1 < 0$, order $\delta_{n0} = J_{ni-1}$, compute $F_{0\max}$, switch to step 2.

4) According to formula (2) - formula (16), compute angle ϕ_1, angle ϕ_2, and the minimal tooth profile clearance J_{ni}.

5) If $\delta_{n0} > J_{ni}$, compute F_i, order i=i+1, go to step 3.

6) Using the formula (1), compute F_{0max}, the concrete program flow chart is showed in Fig. (**5**).

It is necessary to state that virtual engaging point cannot exceeds the limit engaging point, otherwise the interference is produced. Therefore, $K_i'N_1$ must be greater than zero. Meanwhile change BN_1 to CN_1or PN_1, namely K_iN_1 whose engaging point is at C or P can be found. The goal of calculating the force F_{nmax} at P is to compare it with F_{nmax} at B or C to determine the maximal carrying capacity of the double ring reducer, although its tooth root stress cannot reaches bending fatigue strength.

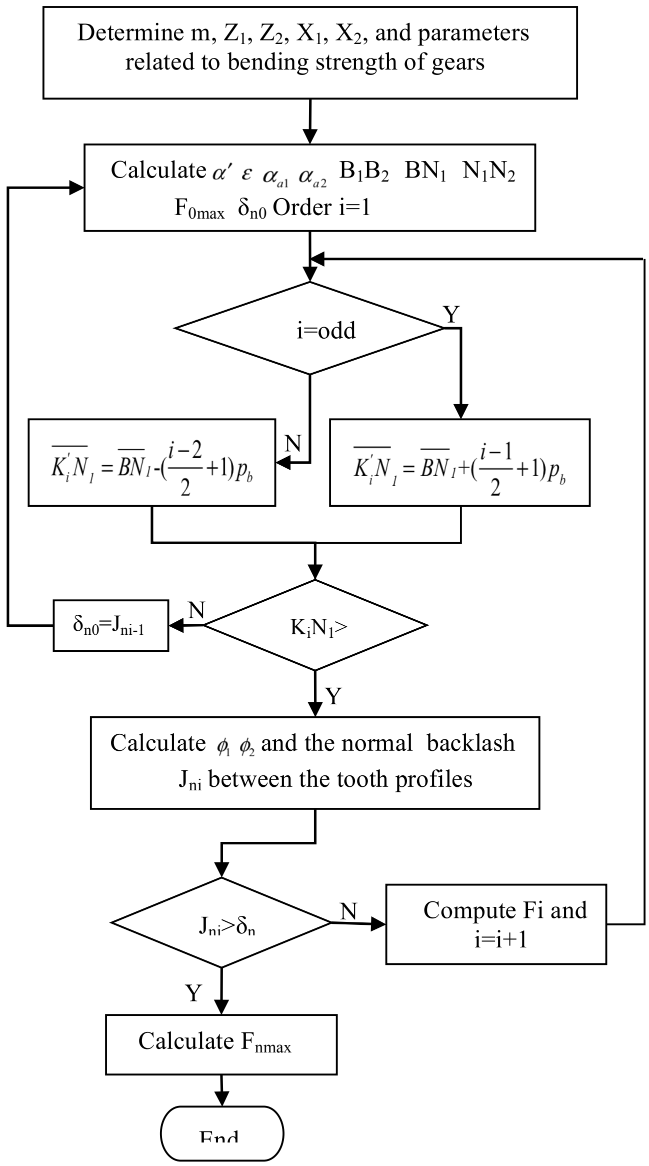

Fig. (5). The program flow block diagram.

5. ANALYSIS OF THE CACULATION EXAMPLE

According to the literature [10], the transmission power of the double ring reducer is 5.1 KW, the transmission ratio is 1:43.5, and the rotating speed of the motor is 1500 rpm. The design parameters are the gear modules m=2, the teeth of the driven external gear Z_1=56, the teeth of the driving internal gear Z_2=58, the gear width b =95. The addendum coefficient is 0.7, the radical clearance coefficient is 0.25, the modification coefficient of the internal gear x_2=1.68, the modification coefficient of the external gear x_1=1.37, the actual centre distance *a*=2.42. The teeth of the shaping cutter Z_0=38, its addendum coefficient is 1.25, its modification coefficient is x_0=0.42. Materials of internal and external gears are selected from 40 Cr, 42 Cr Mu and are quenched. By the design, the material strength of the external gear is close to that of the internal gear, the waste of material is avoided.

Table 1. The maximal bearing capacity in different engaging points.

Engaging Positions	Normal Backlash (um)				δ_{n0} (um)	F_{nmax} (N)
	J_3	J_1	J_2	J_4		
B	34.52	8.05	0	10.94	4.36	16650
P	20.61	2.05	2.12	23.33	4.36	16900
C	10.06	0	8.67	40.82	4.36	16650

While the engaging point is on the point B, P, C, the different clearances between tooth pairs and the maximal load-carrying capacities are shown in Table **1**.

From above Table **1**, we find that the simultaneously meshing teeth pairs are 2 pairs or 3 pairs when the engaging point is on the meshing line of single gear pair. The carrying capacity at the point P is the biggest, but considering the bending fatigue strength, the allowable maximal carrying capacity of the double ring reducer is 16650 N. In order to transfer the above load, its carrying force is1322.2 N according to formula (17) and (18) and is lower than the allowable maximal carrying capacity, and can meet the requirement of design. At the same time, from Table **1**, the smaller the distance from the meshing line of single gear pair is, the smaller the value of the minimal clearance of the tooth pair is, the easier is the tooth pairs to cause elastic contact and bear load.

The two double ring reducers are designed according to the above-mentioned design parameters. During the process of design, the fatigue life of the two helical gears a, b and other components is made higher than that of internal gear pairs with few teeth difference intentionally. By means of the open experimental equipment of the reducer assembly, the fatigue test is conduct. The first task is the empty running-in of the prototypes, and then the prototypes work in fatigue and full load. During the process of the fatigue test, the input torque of the reducer is 35.4 Nm, and its input speed is 2500 rpm, the test results of the assembly are presented in Table **2**.

Table 2. Fatigue life test data of double ring reducers.

	Test Time (Minute)	Recycle Times (Internal Gear)	Failure Mode
001	91	113750	The tooth-breaking of gear Z2
002	82	102500	The tooth-breaking of gear Z1

According to the theoretic calculations of this paper, if the reducer retains continuous running over 70 minutes under circumstances of the above-mentioned input torque and speed, the reducer is qualified. If there had not been the multi-tooth mesh effects, the gear tooth should have fractured because the input torque is 284% higher than the rated torque. The hypothesis does not match the test results; it proves that above-mentioned theory is correct. The test time is larger than the theoretic calculating time and is discrete; this may be caused by using mean value of normal meshing stiffness and the form and position error of gear tooth surface.

Table 3. The effects of the change of centre distance on normal backlash.

Centre Distance (mm)	Normal Backlash (um)				Fmax (N)
	J3	J1	J2	J4	
2.42	20.61.	2.05	2.12	23.33	16900
2.4	19.78	1.91	1.97	22.48	17417

Being satisfied with the various limitations, the centre distance is turned to a=2.4, the modification coefficient of the internal gear x2=1.528, the modification coefficient of the external gear x1=1.231. When the engaging point is on the point P, the comparison of the clearances between tooth pairs and the maximum load-carrying capacity are shown in Table **3**.

From above Table **3**, if the centre distance decreases, the smaller the clearances between tooth pairs become, the easier is the tooth pairs to cause elastic contact, and the carrying capacity is raised.

CONCLUSION

Through the above-mentioned analysis, such conditions can be drawn as follows

1) Starting with that the tooth root stress reaches the bending fatigue strength, the mathematical models of relationships among the normal clearance of each tooth pair and the position of virtual engaging point and the maximal carrying capacity is established, the MATLAB program is compiled.

2) Through MATLAB program, it is easy to estimate the maximal carrying capacity of the double ring reducer. The design efficiency is increased greatly, it is considered fully that the elastic deformation can enhance the carrying capacity, and the manufacturing costs are reduced greatly.

3) The normal stiffness among contact points is regarded as the contact, and it has not been considered that the positions of different engaging points have influence on the stiffness change. The calculated results are somewhat different from the actual conditions, and the further study is required.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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