Wave Propagation in Nanodoped Films

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Abstract: We analyze the propagation of electromagnetic plane waves through a dielectric film endo-wed with a nano doped permittivity made of a sequence of Dirac delta pulses.

Keywords: Nanodoping, Dielectric film, Dirac pulses, TE, TM fields

1. INTRODUCTION

The propagation of electromagnetic plane waves through homogeneous and periodically stratified films has been the subject of several important works since the first edition of the Born-Wolf book [1] and after Arzéliès' publications [2]. We are interested here in the beha-viour of TE, TM harmonic plane waves incident on the z = 0 face of a dielectric film $0 \le z \le d$ with a nano doped permittivity, the nanodoping being obtained from a sequence of delta Dirac pulses. We also consider succinctly magnetic composite films nano-doped with magnetic hol-low nanospheres.With the light velocity c = 1, the permeability $\mu = 1$, the permittivity ε_1, $n_1 = \sqrt{\varepsilon_1}$ and $\exp(-i\omega t)$ implicit, the components E_y, H_x, H_z of the incident TE wave are [1] in $z \le 0$

$$E_y^{\,i} = A_e\, \psi_i(x,z)\ ,\ H_x^{\,i} = -\,A_e\, n_1\cos\theta_i\, \psi_i(x,z),$$

$$H_z^{\,i} = A_e\, n_1\sin\theta_i\, \psi_i(x,z) \tag{1}$$

$$\psi_i(x,z) = \exp[i\omega n_1(x \sin\theta_i + z \cos\theta_i)] \tag{1a}$$

while , since $\theta_r = \pi - \theta_i$, the reflected field is

$$E_y^{\,r} = R_e\, \psi_r(x,z)\ ,\ H_x^{\,r} = R_e\, n_1\cos\theta_i\, \psi_r(x,z),$$

$$H_z^{\,r} = R_e\, n_1\sin\theta_i\, \psi_r(x,z) \tag{2}$$

$$\psi_r(x,z) = \exp[i\omega n_1(x \sin\theta_i - z \cos\theta_i)] \tag{2a}$$

A_e, R_e are the field amplitudes and we have similarly for the TM field with components H_y, E_x, E_z

$$H_y^{\,i} = -A_m\, n_1\psi_i(x,z),\ E_x^{\,i} = -\,A_m \cos\theta_i\, \psi_i(x,z),$$

$$E_z^{\,i} = A_m \sin\theta_i\, \psi_i(x,z) \tag{3}$$

$$H_y^{\,r} = -R_m\, n_1\psi_r(x,z)\ ,\ E_x^{\,r} = A_m \cos\theta_i\, \psi_r(x,z),$$

$$E_z^{\,r} = R_m \sin\theta_i\, \psi_r(x,z) \tag{4}$$

with the expressions (1a), (2a) of ψ_i, ψ_r.

2. TE, TM FIELDS INSIDE A FILM WITH NANODOPED PERMITTIVITY

We consider a dielectric film with permittivity nano doped according to the relation

$$\varepsilon(z) = \varepsilon_0 + \eta\, \Sigma_{m=1}^{M}\, \delta(z/ z_0 - m)\ ,\ 0 \le z \le d\ ,\ Mz_0 \le d \tag{5}$$

$$= \varepsilon_0 + \eta z_0\, \Sigma_{m=1}^{M}\, \delta(z - mz_0) \tag{5a}$$

ε_0 ,η , $z_0 > 0$ are constant parameters and δ the Dirac distribution.This permittivity has the property to have a first derivative null , the relation $f(x)\, \delta'(x) = -f'(x)\, \delta(x)$ implying $\varepsilon'(z) = 0$

since ηz_0 is constant.

Now, inside the film, the Maxwell equations are with $\mu = c = 1$ and $\exp(-i\omega t)$ implicit

$$\partial_y H_z^{\dagger} - \partial_z H_y^{\dagger} + i\omega\varepsilon(z)\, E_x^{\dagger} = 0\ ,\ \partial_y E_z^{\dagger} - \partial_z E_y^{\dagger} - i\omega\, H_x^{\dagger} = 0$$

$$\partial_z H_x^{\dagger} - \partial_x H_z^{\dagger} + i\omega\varepsilon(z)\, E_y^{\dagger} = 0\ ,\ \partial_z E_x^{\dagger} - \partial_x E_z^{\dagger} - i\omega\, H_y^{\dagger} = 0$$

$$\partial_x H_y^{\dagger} - \partial_y H_x^{\dagger} + i\omega\varepsilon(z)\, E_z^{\dagger} = 0\ ,\ \partial_x E_y^{\dagger} - \partial_y E_x^{\dagger} - i\omega\, H_z^{\dagger} = 0 \tag{6, 7}$$

For the TE field, depending only on x,z, the equations (6, 7) reduce to

$$\partial_z E_y^{\dagger} = -\,i\omega\, H_x^{\dagger}\ ,\ \partial_x E_y^{\dagger} = i\omega\, H_z^{\dagger},$$

$$\partial_z H_x^{\dagger} - \partial_x H_z^{\dagger} + i\omega\, \varepsilon(z)\, E_y^{\dagger} = 0 \tag{8a}$$

and for the TM field

$$\partial_z H_y^{\dagger} = i\omega\varepsilon(z)\, E_x^{\dagger}\ ,\ \partial_x H_y^{\dagger} = -\,i\omega\, \varepsilon(z)\, E_z^{\dagger}\ ,$$

$$\partial_z E_x^{\dagger} - \partial_x E_z^{\dagger} - i\omega\, H_y^{\dagger} = 0 \tag{8b}$$

These fields are consistent with (1-4), just changing $\varepsilon(z)$ into ε_1 and using (1a), (2a).

Eliminating $H_x^{\dagger}$, $H_z^{\dagger}$ from (8a) and taking into account $\varepsilon'(z) = 0$ gives for $E_y^{\dagger}$ the same wave equation as that obtained for $H_y^{\dagger}$ by eliminating $E_x^{\dagger}$, $E_z^{\dagger}$ from (8b)

$$[\partial_x^{\,2} + \partial_z^{\,2} + \omega^2\, n^2(z)]\{\, E_y^{\dagger}\ ,\ H_y^{\dagger}\} = 0\ ,\ n^2(z) = \varepsilon(z) \tag{9}$$

We look for the solutions of Eq.(9) in the form

$$E_y^{\dagger}(x,z) = T_e \exp(i\omega n_0 x \sin\theta^{\dagger})\phi(z) \qquad a)$$

$$H_y^{\dagger}(x,z) = -\,n_0 T_m \exp(i\omega n_0 x \sin\theta^{\dagger})\phi(z) \quad b) \tag{10}$$

$n_0 = \sqrt{\varepsilon_0}$ and T_e, T_m are the field amplitudes. Substituting (10) into (9) gives the differential equation satisfied by $\phi(z)$

$$\partial_z^{\,2}\phi(z) + \omega^2\, [n^2(z) - n_0^{\,2} \sin^2\theta^{\dagger}]\, \phi(z) = 0 \tag{11}$$

The solutions of (11) are discusssed in Appendix A and assuming $\eta << 1$ we get to the $0(\eta^2)$ order

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$$\phi(z) = \phi_0(z) + \eta \sum_{m=1}^{M} \phi_m(z) + 0(\eta^2) \quad (12)$$

with

$$\phi_0(z) = \exp(in_0 \cos\theta^\dagger \omega z) \quad (12a)$$

and

$$\phi_m(z) = (i\omega z_0/2\pi\, n_0 \cos\theta^\dagger) \exp[i\omega n_0 \cos\theta^\dagger \{mz_0 + |z - mz_0|\}] \quad (12b)$$

Now, in the dielectric film, the field reflected on the z = d face has to be taken into account so that according to (10) the components $E_y^\dagger(x,z)$, $H_y^\dagger(x,z)$ are

$$E_y^\dagger(x,z) = \beta^\dagger(x)\,[T_e^1\phi(z) + T_e^2\,\phi(\{d-z\})] \quad a)$$

$$H_y^\dagger(x,z) = -\,n_0\,\beta^\dagger(x)\,[T_m^1\phi(z) + T_m^2\,\phi(\{d-z\})] \quad b) \quad (13)$$

in which

$$\beta^\dagger(x) = \exp(i\omega n_0\, x \sin\theta^\dagger) \quad (14)$$

Substituting (13a) into (8a) gives

$$H_x^\dagger(x,z) = i\,\omega\,\beta^\dagger(x)\,[T_e^1\phi'(z) + T_e^2\,\phi'(\{d-z\})] \quad a)$$

$$H_z^\dagger(x,z) = n_0 \sin\theta^\dagger\,\beta^\dagger(x)\,[T_e^1\phi(z) + T_e^2\,\phi(\{d-z\})] \quad b) \quad (15)$$

Similarly with (13b) substituted into (8b)

$$E_x^\dagger(x,z) = [in_0/\varepsilon(z)]\,\beta^\dagger(x)\,[T_m^1\phi'(z) + T_m^2\,\phi'(\{d-z\})] \quad a)$$

$$E_z^\dagger(x,z) = [n_0^2/\varepsilon(z)] \sin\theta^\dagger\,\beta^\dagger(x)\,[T_m^1\phi(z) + T_m^2\,\phi(\{d-z\})] \quad b) \quad (16)$$

We now have all the ingredients to analyze the electromagnetic plane wave propagation through the nano doped dielectric film.

3. ELECTROMAGNETIC WAVE PROPAGATION

3.1. TE Field

The amplitudes of the TE field must satisfy boundary conditions at z = 0 and z = d. Then, noting first that the Descartes-Snell relation $n_1 \sin\theta_i = n_0 \sin\theta^\dagger$ transforms (14) into

$$\beta^\dagger(x) = \exp(i\omega n_1 x \sin\theta_i) \quad (17)$$

we have at z = 0

$$E_y^i(x,0) + E_y^r(x,0) = E_y^\dagger(x,0)\,,$$

$$H_x^i(x,0) + H_x^r(x,0) = H_x^\dagger(x,0) \quad (18)$$

and, taking into account (1), (1a), (2), (2a) and (13a), (15a) together with (17), we get from (18)

$$R_e + A_e = T_e^1\,\phi(0) + T_e^2\,\phi(d)$$

$$n_1 \cos\theta_i\,(R_e - A_e) = i[T_e^1\,\phi'(0) + T_e^2\,\phi'(d)] \quad (19)$$

Now, to get the TE field E_y^t, H_x^t, H_z^t outside the film for z > d, one has just to change in (1) the amplitude A_e into $A_{t,e}$ so that the boundary conditions at z = d are

$$E_y^t(x,d) = E_y^\dagger(x,d)\,,\ H_x^t(x,d) = H_x^\dagger(x,d) \quad (20)$$

and, still using (1), (1a) and (13a), (15a), (17), we get

$$T_e^1\,\phi(d) + T_e^2\,\phi(0) = \gamma\,A_{t,e}$$

$$i[T_e^1\,\phi'(\omega) + T_e^2\,\phi'(0)] = -n_1\cos\theta_i\,\gamma\,A_{t,e} \quad (21)$$

in which

$$\gamma = \exp(i\omega\, n_1 d \sin\theta_i) \quad (21a)$$

So, we get from (19) and (21) four relations to determine the four unknown amplitudes $R_e, T_e^1\ T_e^2, A_{t,e}$, this set of equations is solved in Appendix B.

3.2. TM Field

The boundary conditions for the TM field are at z = 0

$$H_y^i(x,0) + H_y^r(x,0) = H_y^\dagger(x,0)\,,$$

$$E_x^i(x,0) + E_x^r(x,0) = E_x^\dagger(x,0) \quad (22)$$

Then, using (3), (4) with (1a), (2a) together with (13b), (16a), taking into account (17), we get since $\varepsilon(0) = \varepsilon_o = n_0^2$

$$n_1(R_m + A_m) = n_0\,[T_m^1\,\phi(0) + T_m^2\,\phi(d)]$$

$$n_0 \cos\theta_i\,(R_m - A_m) = i[T_m^1\,\phi'(0) + T_m^2\,\phi'(d)] \quad (23)$$

Now the TM field in z > d has the expression (3) with A_m changed into $A_{t,m}$ so that the boun-dary conditions at z = d are

$$H_y^t(x,d) = H_y^\dagger(x,d)\,,\ E_x^t(x,d) = E_x^\dagger(x,d) \quad (24)$$

implying with γ given by (21a) since $\varepsilon(d) = \varepsilon_o = n_0^2$

$$n_0[T_m^1\,\phi(d) + T_m^2\,\phi(0)] = \gamma n_1\,A_{t,m}$$

$$i[T_m^1\,\phi'(d) + T_m^2\,\phi'(0)] = -\gamma n_0 \cos\theta_i\,A_{t,m} \quad (25)$$

We get from (23), (25) four relations to determine R_m, $,T_{m},^1$ $,T_{m},^2$, $A_{t,m}$ which is made in Ap-pendix B

4. DISCUSSION

High-k dielectrics are used for instance in semiconductor manufacturing process to replace silicon gate dielectrics, allowing a miniutarization of microelectronics component with better performances in thin materials such as dielectric films. Nano doped dielectrics offer the possibility of high-k dielectrics. Here for instance, the mean value of the dielectric constant is

$$\varepsilon = 1/d \int_0^d \varepsilon(z)\,dz \quad (26)$$

that is substituting (5) into (26)

$$\varepsilon = \varepsilon_0 + \eta/\omega d \int_{-\infty}^{\infty}[U(z) - U(z-d)] \sum_m^M \delta(z-mz_0)$$

$$= \varepsilon_0 + \eta/\omega d \sum_m^M [U(mz_0) - U(mz_0-d)]$$

$$= \varepsilon_0 + M\eta/\omega d \quad (26a)$$

taking great values when M/ω is high.

Incidently, the sum in (5) is the truncated series of the Dirac distribution [4, 5]

$$\pi\delta[\sin(\pi z/z_0)] = \sum_n \delta(z/z_0 - n)\,,\ n \text{ integer in } (-\infty, \infty) \quad (27)$$

The matrix technique [1,2] used to analyze the propagation of electromagnetic plane waves through homogeneous and periodically stratified dielectric films is not suitable for TE, TM fields inside a film with the permittivity (5) which is neither homogeneous nor stratified be-cause the dielectric constant is only perturbed by the Dirac pulses at local points. The impor-tance of $\varepsilon'(z) = 0$, to get the wave equation (9) must be stressed.

We have obtained in Appendix A an $0(\eta^2)$ approximation of TE, TM fields in which the Green's function of the 1D-Helmholtz equation intervenes rather naturally. The object of this approximation was only to get a perception of the TE, TM behaviour, but it is clear that an important numerical analysis has to be performed when η is not very small.

Finally, it has ben assumed that 1D-nano doping may be described by a sequence of delta Dirac pulses, the nano dots being assimilated to points. This postulat could be generalized to 2D and 3D nano doping from the relations [5]

$$\delta(r)/\pi r = \delta(x)\,\delta(y)\; r = (x^2+y^2)^{1/2}$$

$$\delta(r)/2\pi r^2 = \delta(x)\,\delta(y)\,\delta(z)\; r = (x^2+y^2+z^2)^{1/2} \quad (28)$$

The following generalization of (5) could be used to describe nanodoped photonic crystals made of multilayer films [6]

$$\varepsilon_j(z) = = \varepsilon_0 + z_0\,\Sigma_{m=1}^{M}\,\eta_j\,\delta(z - mz_0)\;,\; j = 1,2\ldots Ï \quad (29)$$

in which j is the number of layers.

5. MAGNETIC NANO COMPOSITE FILMS

Magnetic nano composite films are used specially to enhance the film coercivity [7-11]. that is their resistance to becoming demagnetized. The non existence of magnetic monopoles prevents to imagine the doping of these films as made of nano dots and, we have instead to consider magnetic hollow nano spheres [12-17]. Then, the permeability $\mu(z)$ in $0 \le z \le d$ may be represented by the expansion

$$\mu(z) = \mu_0 + \textstyle\sum_{m=1}^{M} \nu(mz_0)\,\delta(z - mz_0) \quad (30)$$

mz_0 is the center of a hollow nano sphere and $\nu(mz_0)$ depends on its nature and on its radius [14, 15]. This permittivity satisfies also the condittion $\mu'(z) = 0$.

Proceeding as in (26a), we get from (30) for the mean value permittivity μ of this magnetic nano composite film giving the possibility to check its coercivity performance.

$$\mu = \mu_0 + 1/\omega d \textstyle\sum_m^M \nu(mz_0) \quad (30a)$$

Then, using(30) and assuming $\varepsilon = 1$, it is easily checked that the equations (8a,b) for TE,TM fields transform into

$$\partial_z E_y^{\dagger} = -\,i\omega\,\mu(z) H_x^{\dagger}\;,\; \partial_x E_y^{\dagger} = i\omega\,\mu(z)\,H_z^{\dagger}\;,$$

$$\partial_z H_x^{\dagger} - \partial_x H_z^{\dagger} + i\omega\, E_y^{\dagger} = 0 \quad (31a)$$

$$\partial_z H_y^{\dagger} = i\omega\, E_x^{\dagger}\;,\; \partial_x H_y^{\dagger} = -\,i\omega\, E_z^{\dagger}\;,$$

$$\partial_z E_x^{\dagger} - \partial_x E_z^{\dagger} - i\omega\mu(z)\, H_y^{\dagger} = 0 \quad (31b)$$

so that since $\mu'(z) = 0$, the components $E_y^{\dagger}$, $H_y^{\dagger}$ are still solutions of the wave equation (9) in which now $n^2(z) = \mu(z)$ and they take the form (13) with $\phi(z)$ satisfying the differential equation (12). Substituting (13) into (31a,b) gives the other two components of the TE, TM fields with according to (31a)

$$H_x^{\dagger}(x,z) = [i/\mu(z)]\,\beta^{\dagger}(x)\,[T_e^{1}\phi'(z) + T_e^{2}\,\phi'(\{d-z\})] \quad a)$$

$$H_z^{\dagger}(x,z) = [n_0/\mu(z)]\sin\theta^{\dagger}\,\beta^{\dagger}(x)$$

$$[T_e^{1}\phi(z) + T_e^{2}\,\phi(\{d-z\})] \quad b) \quad (32)$$

and from (31b)

$$E_x^{\dagger}(x,z) = in_0\,\beta^{\dagger}(x)\,[T_m^{1}\phi'(z) + T_m^{2}\,\phi'(\{d-z\})] \quad a)$$

$$E_z^{\dagger}(x,z) = n_0^2\,\sin\theta^{\dagger}\,\beta^{\dagger}(x)\,[T_m^{1}\phi(z) + T_m^{2}\,\phi(\{d-z\})] \quad b) \quad (33)$$

From there, we may proceed as in Sec.3, using the boundary conditions at z = 0 and z = d to get four equations to determine the four unknown amplitudes.

As previously stated, $\phi(z)$ is solution of Eq.(11) with $n^2(z) = \mu(z)$. Let $\nu = \mathrm{Max}_m \nu(mz_0)$ then assuming $\nu << 1$ and consequently $\nu(mz_0) << 1$ we have to the $0(\nu^2)$ order

$$\phi(z) = \phi_0(z) + \nu \textstyle\sum_{m=1}^{M} \phi_m(z) + 0(\nu^2) \quad (34)$$

with $\phi_0(z)$, $\phi_m(z)$given by (12a,b).

This analysis of magnetic nano composite films reposes on magnetic hollow nano sphere whose existence requires further works.

APPENDIX A

We discuss here the solutions of Eq.(11) rewritten for convenience

$$\partial_z^2\phi(z) + \omega^2[n^2(z) - n_0^2\,\sin^2\theta^{\dagger}]\phi(z) = 0 \quad (A.1)$$

in which, according to (5) :

$$n^2(z) = n_0^2 + \eta z_0\,\Sigma_{m=1}^{M}\,\delta(z - mz_0)\;,\; Mz_0 \le d \quad (A.2)$$

We start this analysis with the simple refractive index

$$n^2(z) = n_0^2 + \eta z_0\,\delta(\omega z - m\omega z_0) \quad (A.3)$$

so that the equation (A.1) becomes

$$\phi''(z) + \omega^2 n_0^2\,\cos^2\theta^{\dagger}\,\phi(\omega z) = -\omega^2\eta z_0\,\delta(z - mz_0)\,\phi(z) \quad (A.4)$$

We assume $\eta << 1$ very small and we look for the solutions of (A.4) to the $0(\eta^2)$ order in the

the form

$$\phi(z) = \phi_0(z) + \eta\,\phi_m(z) + 0(\eta^2) \quad (A.5)$$

Substituting (A.5) into (A.4) gives

$$\phi_0''(z) + \omega^2 n_0^2\,\cos^2\theta^{\dagger}\,\phi_0(z) + \eta\,[\phi_m''(z) + \omega^2 n_0^2$$

$$\cos^2\theta^{\dagger}\,\phi_m(z)] = -\omega^2\eta z_0\,\delta(z - mz_0)\,\phi_0(z) \quad (A.6)$$

supplying the two equations

$$\phi_0''(z) + \omega^2\, n_0^2\,\cos^2\theta^{\dagger}\,\phi_0(z) = 0 \quad a)$$

$$\phi_m''(z) + \omega^2 n_0^2\,\cos^2\theta^{\dagger}\,\phi_m(z) = -\,\omega^2 z_0\,\delta(z - mz_0)\,\phi_0(z) \quad b) \quad (A.7)$$

Taking as solution of (A.7a)

$$\phi_0(z) = \exp(in_0\cos\theta^{\dagger}\omega z) \quad (A.8)$$

the equation (A.7b) becomes

$$\phi_m''(z) + \omega^2 n_0^2\,\cos^2\theta^{\dagger}\,\phi_m(z) = -\omega^2\,z_0\,\exp(in_0\cos\theta^{\dagger}\omega z)\,\delta(z - mz_0) = -\omega^2\,z_0\,\exp(in_0\cos\theta^{\dagger}\omega mz_0)\,\delta(z - mz_0) \quad (A.9)$$

which is in fact the equation satisfied by the Green's function of the 1D-Helmholtz equation and this equation has the solution [3] for the infinite domain

$$\phi_m(z) = (i\omega z_0/2\pi\, n_0\cos\theta^{\dagger})\,\exp$$

$$[i\omega n_0\cos\theta^{\dagger}\,\{mz_0 + |z - mz_0|\}] \quad (A.10)$$

Now, with the refractive index (A.2), the equation (A.1) becomes

$$\phi''(z) + \omega^2\, n_0^2\,\cos^2\theta^{\dagger}\,\phi(z)$$

$$= -\omega^2\eta z_0\,\Sigma_{m=1}^{M}\,\delta(z - mz_0)\,\phi(z) \quad (A.11)$$

and we look for its solutions in the form

$$\phi(z) = \phi_0(z) + \eta\, \Sigma_{m=1}^{M}\, \phi_m(z) + 0(\eta^2) \quad (A.12)$$

Substituting (A.12) into (A.11) supplies the equations (A.7a) and (A.7b) for m = 1, 2…M with the solutions (A.10) which achieves to determine (A.12) to the $0(\eta^2)$ order.

APPENDIX B

To obtain the amplitudes R_e, T_e^1, T_e^2, $A_{t,e}$ for the TE field, we introduce the functions

$$\rho_e(z) = n_1 \cos\theta_i\, \phi(z) - i\, \phi'(z)\,,\ \sigma_e(z) = n_1 \cos\theta_i\, \phi(z) + i\, \phi'(z) \quad ((B.1)$$

Then, eliminating R_e from (19) and $A_{t,e}$ from (21) gives :

$$\rho_e(0)\, T_e^1 + \rho_e(d)\, T_e^2 = 2\, n_1 \cos\theta_i\, A_e\ ,\ \sigma_e(d)\, T_e^1 + \sigma_e(0)\, T_e^2 = 0 \quad (B.2)$$

from which T_e^1 and T_e^2 are obtained

$$T_e^1 = 2n_1\cos\theta_i\, \chi_e\sigma_e(0)\, A_e\ ,\ T_e^2 = -2n_1\cos\theta_i\, \chi_e\sigma_e(d)\, A_e \quad (B.3)$$

$$\chi_e = [\rho_e(0)\, \sigma_e(0) - \rho_e(d)\, \sigma_e(d)]^{-1} \quad (B.3a)$$

so that we get at once from the first relation (21)

$$A_{t,e} = 2n_1\cos\theta_i\, \chi_e\, [\sigma_e(0)\, \phi(d) - \sigma_e(d)\, \phi(0)]\, \gamma^{-1} A_e \quad (B.4)$$

while eliminating A_e from (19) and taking into account (B3) give

$$R_e = \chi_e\, [\sigma_e^2(0) - \sigma_e^2(d)]\, A_e \quad (B.5)$$

We proceed similarly for the TM field with the functions

$$\rho_m(z) = n_0^2 \cos\theta_i\, \phi(z) - i\, \phi'(z)\,,\ \sigma_m(z) = n_0^2 \cos\theta_i\, \phi(z) + i\, \phi'(\omega z) \quad (B.6)$$

Eliminating R_m from (23) and $A_{t,m}$ from (25) gives

$$\rho_m(0)\, T_m^1 + \rho_m(d)\, T_m^2 = 2\, n_0 n_1 \cos\theta_i\, A_m,$$

$$\sigma_m(d)\, T_m^1 + \sigma_m(0)\, T_m^2 = 0 \quad (B.7)$$

from which we get

$$T_m^1 = 2n_0n_1\cos\theta_i\, \chi_m\sigma_m(0)\, A_m\ ,$$

$$T_m^2 = -2n_0n_1\cos\theta_i\, \chi_m\sigma_m(\omega d) A_m \quad (B.8)$$

$$\chi_m = [\rho_m(0)\, \sigma_m(0) - \rho_m(d)\, \sigma_m(d)]^{-1} \quad (B.8a)$$

and, substituting (B.8) into the first relation (25), it comes

$$A_{t,m} = 2n_0^2\cos\theta_i\, \chi_m\, [\sigma_m(0)\, \phi(d) - \sigma_m(d)\, \phi(0)]\, \gamma^{-1} A_m \quad (B.9)$$

while eliminating A from (22) gives taking nto account (B.7)

$$R_m = \chi_m\, [\sigma_m^2(0) - \sigma_m^2(d)]\, A_m \quad (B.10)$$

To achieve to determine the TE and TM fields we have just to express $\phi(0)$ and $\phi(d)$ in terms of the solutions of Appendix A.

CONFLICT OF INTEREST

None declared.

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