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Saturation Properties and Density-dependent Interactions among Nuclear and Hyperon Matter

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Abstract: The density-dependent interrelations among properties of nuclear matter and hyperonic neutron stars are studied by applying the conserving nonlinear mean-field theory of hadrons. The nonlinear interactions that will be renormalized as effective coupling constants, effective masses and sources of equations of motion are constructed self-consistently by maintaining thermodynamic consistency (the Hugenholtz-Van Hove theorem, conditions of conserving approximations) to the nonlinear mean-field (Hartree) approximation. The characteristic density-dependent properties among nuclear matter and hyperonic neutron stars appear by way of effective coupling constants and masses of hadrons; they are mutually interdependent and self-consistently constrained *via* the bulk properties of infinite matter, such as incompressibility, K, symmetry energy, α_4, and maximum masses of neutron stars. Consequently, the density-dependence induced by nonlinear interactions of hadrons will determine and restrict the saturation properties (binding energy and density) of hyperons, hyperon-onset density and equation of state in high densities.

The nonlinear hadronic mean-field and quark-based hadronic models will predict essentially different density-dependent behavior for hadrons in terms of effective masses and coupling constants, and discrepancies between the models are shown and discussed, which would improve and compensate for both approaches to nuclear physics.

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1. INTRODUCTION

The relativistic linear σ-ω mean-field approximation of hadrons has been applied to finite and infinite nuclear matter system [1-3], and it is successful for simulations and descriptions of nuclear and high-density hadronic phenomena. As extensions of the linear σ-ω mean-field theory, nonlinear mean-field approximations and nonlinear chiral models [4-8] have been applied to examine nuclear and high-density phenomena quantitatively. The nonlinear mixing and self interactions of mesons, nonlinear vertex interactions can be understood as many-body effects and are renormalized as effective masses, effective coupling constants and density-sources in the renormalizable mean-field models, which is one of the important results obtained from the conserving nonlinear mean-field approximation [9, 10].

The theory of conserving approximations discusses self-consistent approximations that maintain *thermodynamic consistency* to microscopically constructed approximations, and it has been applied to diverse fields of many-body theories, relativistic field theoretical approach for finite nuclei and infinite nuclear matter [11-15]. The theory is based on the fundamental requirement of the Landau's quasiparticle theory [16, 17], which is expressed as:

$$\mu = \frac{\partial \varepsilon}{\partial \rho_B} = E(k_F)\,. \tag{1.1}$$

where μ is the chemical potential; ε and ρ_B are energy density and particle density; $E(k_F)$ is the single particle energy at the Fermi surface (Fermi energy). The requirement (1.1) together with Feynman diagrams that maintain certain symmetries, self-consistent relations between equations of motion and self-energies will determine self-consistent effective masses, effective coupling constants for nonlinear mean-field approximations [9]. The density functional theory is equivalent to the theory of conserving approximations [15], which is also based on the relation (1.1) [18, 19].

The nonlinear σ-ω-ρ mean-field lagrangian is renormalizable and has several parameters: coupling constants and masses of hadrons. The determination of coupling constants is essential for nonlinear mean-field lagrangians to extract physically meaningful results. Hence, it is imperative to have conditions to fix or confine parameters by way of theoretical and experimental

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requirements. The coupling constants of the current conserving nonlinear σ-ω-ρ mean-field approximation are confined with experimental data: the binding energy at saturation density (-15.75 MeV, $k_F = 1.30$ fm^{-1}), symmetry energy ($a_4 \simeq 30.0$ MeV) and the maximum mass of neutron stars ($M_{max} = 2.00\ M_{\odot}$). With these empirical data as constraints, the lower bound of incompressibility is simultaneously searched by adjusting nonlinear coupling constants. Since the nonlinear interactions are interrelated by conditions of thermodynamic consistency, the nonlinear coupling constants are not free to adjust. One can examine that nonlinear coupling constants are confined by searching the lower bound of incompressibility and maintaining the empirical constraints, which results in obtaining the upper bounds of nonlinear coupling constants.

The constraints will emerge as density-dependent correlations among physical quantities in nuclear matter, hyperonic matter and neutron stars, such as binding energy, effective masses of hadrons, incompressibility, symmetry energy and maximum mass of neutron stars. The self-consistent conserving approximation exhibits that the effective masses, effective coupling constants and other observables are strictly interrelated by way of density-dependent interactions. Although the admissible upper bound values of nonlinear coupling constants seem to be large, corrections to coupling constants and masses of hadrons become small as long as conditions of thermodynamic consistency are maintained; the nonlinear corrections seem to be properly truncated, which can be checked numerically. The properties of nonlinear corrections to effective masses and coupling constants would be an example of *naturalness* in the level of self-consistent mean-field approximations; naturalness and truncations of nonlinear corrections to physical quantities could be appropriately controlled and defined with *thermodynamic consistency*. This is an important result derived in the conserving nonlinear mean-field approximation [10].

As the binding energy of symmetric nuclear matter (fixed as -15.75 MeV at $k_F = 1.30$ fm^{-1} or, $\rho_0 = 0.148$ fm^{-3} in the current calculation) is important to study interactions of nucleons, the binding energy and density of hyperon matter are also essential to study interactions of hadrons. Since density-dependent interactions interconnect dynamical quantities of nucleons with those of hyperons, such as single particle energy, self-energy and effective masses of hyperons, the determination of physical quantities in symmetric nuclear matter simultaneously determine properties of binding energy and saturation, effective masses of hyperons. For example, neutron stars are expected to be composed of baryons, and the baryonic matter has been investigated by starting from symmetric nuclear matter through the process of general β-equilibrium phase transitions, such as (n,p)-(n,p,e)-(n,p,Λ,e) phase transitions [20]. The onset density of Λ in the phase transition, (n,p,e)-(n,p,Λ,e), depends on density-dependent effective masses and coupling constants, whose equation of state is delimited as $M_{\max} = 2.00\ M_{\odot}$ for the current calculation. Hence, nucleon-nucleon interactions simultaneously determine the onset density, effective masses and binding energy of hyperons.

Since the onset density of a hyperon depends on hadronic interactions and self-consistent single particle energies, it is important to investigate interactions of NY and YY, the order of onset of hyperons in symmetric nuclear matter and isospin asymmetric matter. For example, the determination of the order of the onset of Σ^- and Λ in isospin asymmetric (n,p,e) matter, either (n,p,e)-(n,p,Σ^-,e) or (n,p,e)-(n,p,Λ,e), has important information on interactions of nucleons as well as binding energy and saturation of hyperons, effective masses, coupling constants and the maximum mass of neutron stars. Therefore, it is imperative to determine the order of onset of hyperons, Σ^- and Λ, to check which hyperons could be energetically sensitive to be produced. This helps us understand the relation of self-consistency, charge neutrality and binding energy for nuclear and hyperonic matter.

The phase-transition conditions given by chemical potentials of hadrons and charge neutrality determine the onset-density of a hyperon, but the density will be altered when other hyperons are produced together. For example, Λ is produced at $k_F \simeq 1.7$ fm^{-1} when it is produced as the phase transition: (n,p,e)-(n,p,Λ,e). However, if Λ is produced along with Σ^- as (n,p,e)-(n,p,Σ^-,e)-(n,p,Σ^-,Λ,e), the onset-density of Λ is pushed up to a higher density: $k_F \simeq 2.4$ fm^{-1}. Similarly, the onset density of Σ^- appears at $k_F \simeq 1.6$ fm^{-1} when it is produced in the phase transition: (n,p,e)-(n,p,Σ^-,e). However, if the hyperonic matter changes through the phase transition (n,p,e)-(n,p,Λ,e)-(n,p,Λ,Σ^-,e), the onset-density of Σ^- is pushed up to a higher density: $k_F \simeq 2.4$ fm^{-1}. The same phenomena are observed with other hyperons, and generally the onset-density of a hyperon is pushed up to a higher density [20]. We denote the phenomenon as *the push-up of a hyperon onset-density in many-fold hyperon generations*.

The push-up of the hyperon onset-density can be understood from the concept of Fermi energy in the theory of Fermi-liquid [16, 17]. The phase transition (n,p,e)-(n,p,Λ,e)-(n,p,Λ,Σ^-,e) indicates the generation of the single particle energies, $E_n(k_F)$, $E_p(k_F)$, $E_\Lambda(k_F)$ and $E_{\Sigma^-}(k_F)$, respectively. The nucleon single-particle energies of (n,p,e), $E_n(k_F)$ and $E_p(k_F)$, are redistributed to (n,p,Λ) in the phase (n,p,Λ,e) to maintain phase conditions; hence, the respective single particle energies develop slowly, resulting in a softer equation of state and incompressibility. This fact exhibits that the phase transition, (n,p,e)-(n,p,Λ,e), requires a higher energy, which results in the phase-transition at a higher density. The phase transitions and variations of single particle energies are perceived as discontinuous changes of physical quantities, such as effective masses of baryons, incompressibility, K, and symmetry energy, a_4, in high densities. The redistribution of single particle energies at the phase transition can be examined numerically with chemical potentials (μ_n, μ_p, μ_Λ, μ_{Σ^-}). It is also checked with the fact that the equation of state (EOS) is discontinuously softened when hyperons are produced. Because of the push-up phenomena of the hyperon onset density, the hyperons relevant to determine the maximum mass of neutron stars are mainly Σ^- and Λ. The similar results are discussed in the nonrelativistic Brueckner-Hartree-Fock calculations [21, 22].

The single particle energies are important to study saturation and *self-boundedness* of binding energies of hyperons. It is found in the nonlinear σ-ω-ρ self-consistent approximation that binding energy of hyperons will exhibit essentially different properties according to coupling constants and density dependence. The binding energies of Λ produced by self-consistent effective masses, coupling constants of nucleons and maximum masses of neutron stars, $M_{max} = 2.50\ M_\odot$ and $M_{max} = 2.00\ M_\odot$, are compared with coupling constants required by the SU(6) quark model for the vector coupling constants [23, 24]. The coupling constants expected from the nonlinear mean-field approximation will produce the binding energy of Λ which is bounded and saturates at a high density (the potential of V_Λ is attractive). On the contrary, the coupling constants required by the SU(6) quark model generate positive binding energy in all densities, which shows that the potential of Λ is repulsive in all densities; the result contradicts with the experimental values, $V_\Lambda = -28 \sim -30$ MeV. This may be a discrepancy between the prediction of hadronic and quark-based models.

The many-body system of nuclear physics is the system of strong interaction of hadrons and composite particles of quarks and gluons. The hadronic models would be simple and consistent for nuclear phenomena in low-density region, whereas the effective quark models [25-29] should be consistent for high-density region. If calculations and predictions to nuclear physics from both models agree independently, we could obtain rigorous physical understanding for nuclear phenomena; both approaches could support and compensate each other to comprehend complex many-body physics in terms of each energy region of expertise. However, if there are certain discrepancies to nuclear physics from both approaches, it should be revealed as much as possible to understand properties of both models for nuclear physics. In the conserving mean-field approximation of hadrons, it is shown that the coupling constants of hyperons are expected to be $g_{\sigma H}/g_{\sigma N} \sim 1$ and $g_{\omega H}/g_{\omega N} \gtrsim 1$, whereas effective quark models require smaller values of coupling constant of hyperons, $g_{\omega H}/g_{\omega N} = 2/3$, ($H = \Sigma^-, \Lambda$). The values of coupling constant, $g_{\omega H}/g_{\omega N} \sim 1$, or $g_{\omega H}/g_{\omega N} = 2/3$ lead to essentially different results in terms of density-dependent interactions. The hadronic and effective quark models of hadrons seem to give distinct results for some physical quantities of nuclear matter [20]. The results will be discussed in the sec. 5 and remarks are in the sec. 6. The symmetric nuclear matter and hyperonic matter are very interesting to analyze theoretically and experimentally for both hadronic and effective quark-based approaches to nuclear physics.

2. THE CONSERVING NONLINEAR σ-ω-ρ MEAN-FIELD APPROXIMATION

The nonlinear σ-ω-ρ lagrangian with nonlinear vertex interactions is defined by

$$
\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B [i\gamma_\mu \partial^\mu - g^*_{\omega B}\gamma_\mu V^\mu - \frac{g^*_{\rho B}}{2}\gamma_\mu\, \boldsymbol{\tau}\cdot\mathbf{R}_\mu - (M_B - g^*_{\sigma B}\phi)]\psi_B \\
& + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2) - \frac{g_{\sigma 3}}{3!}\phi^3 - \frac{g_{\sigma 4}}{4!}\phi^4 \\
& - \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2) - \frac{g_{\sigma 3}}{3!}\phi^3 - \frac{g_{\sigma 4}}{4!}\phi^4 \\
& - \frac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{R}_\mu\cdot\mathbf{R}^\mu + \frac{g_{\rho 4}}{4!}(\mathbf{R}_\mu\cdot\mathbf{R}^\mu)^2 + \frac{g_{\sigma\rho}}{4}\phi^2\mathbf{R}_\mu\cdot\mathbf{R}^\mu + \frac{g_{\omega\rho}}{4}V_\nu V^\nu(\mathbf{R}_\mu\cdot\mathbf{R}^\mu) \\
& + \sum_l \bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l\ ,
\end{aligned} \tag{2.1}
$$

where $V_\mu V^\mu = V_0^2 - \mathbf{V}^2$ $(\mu = 0,1,2,3)$. The neutral and charged vector meson field strengths are given by $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $\mathbf{L}_{\mu\nu} = \partial_\mu\mathbf{R}_\nu - \partial_\nu\mathbf{R}_\mu - g_\rho\mathbf{R}_\mu\times\mathbf{R}_\nu$, respectively. The fields, ψ_B ($B = n, p, \Lambda, \Sigma, \cdots$) and ψ_l ($l = e^-, \mu^-$), denote the field of baryons and leptons. The nonlinear vertex interactions are introduced with the scalar field ϕ in order to maintain Lorentz-covariance, thermodynamic consistency in a simple form. The coupling constants for nonlinear interactions of mesons and vertex interactions will be confined by self-consistency, saturation properties of symmetric nuclear matter (binding energy and density, symmetry energy) and the maximum mass of neutron stars ($M_{max} = 2.00\ M_\odot$ in the present calculation).

The nonlinear σ-ω-ρ mean-field lagrangian, L_{NMF}, with density-dependent effective masses and coupling constants is defined by

$$
\begin{aligned}
\mathcal{L}_{NMF} = \quad & \sum_B \bar{\psi}_B [i\gamma_\mu \partial^\mu - g^*_{\omega B}\gamma_0 V_0 - \frac{g^*_{\rho B}}{2}\gamma_0 \tau_3 R_0 - (M_B - g^*_{\sigma B}\phi_0)]\psi_B \\
& -\frac{1}{2} m_\sigma^2 \phi_0^2, -\frac{g_{\sigma 3}}{3!}\phi_0^3 - \frac{g_{\sigma 4}}{4!}\phi_0^4 + \frac{1}{2} m_\omega^2 V_0^2 + \frac{g_{\omega 4}}{4!} V_0^4 + \frac{g_{\sigma\omega}}{4}\phi_0^2 V_0^2 \\
& + \frac{1}{2} m_\rho^2 R_0^2 + \frac{g_{\rho 4}}{4!} R_0^4 + \frac{g_{\sigma\rho}}{4}\phi_0^2 R_0^2 + \frac{g_{\omega\rho}}{4} V_0^2 R_0^2 + \sum_l \bar{\psi}_l (i\gamma_\mu \partial^\mu - m_l)\psi_l \ .
\end{aligned}
\tag{2.2}
$$

The lagrangian yields the nonlinear σ-ω-ρ Hartree approximation when direct interactions are properly renormalized, which is denoted as NHA [10]. The meson-fields operators are replaced by expectation values in the ground state: ϕ_0 for the σ-field, V_0 for the vector-isoscalar ω-meson. The neutral ρ-meson mean-field, R_0, is chosen for τ_3-direction in isospin space. The masses in (2.2) are: $M = 939$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV and $m_\rho = 770$ MeV, in order to compare the effects of nonlinear and density-dependent interactions with those of the linear σ-ω approximation discussed by Serot and Walecka [1].

The nonlinear model is motivated by preserving the structure of Serot and Walecka's linear σ-ω mean-field approximation [1], Lorentz-invariance and renormalizability, thermodynamic consistency, that is, Landau's hypothesis of quasiparticles [16, 17], the Hugenholtz-Van Hove theorem [30], the virial theorem [31], and conditions of conserving approximations [9-12]. The concepts of *effective masses* and *effective coupling constants* are naturally generated by nonlinear interactions of mesons and baryons. The conditions of conserving approximations will require self-consistent relations among single particle energy, effective masses and coupling constants, and then, empirical values of low-density nuclear matter and high-density neutron matter will be restricted with the effective masses and coupling constants [9]. The admissible values of effective coupling constants and masses are confined in certain values due to strong density-dependent correlations among physical quantities of nuclear matter and neutron stars. The purpose of the analysis is to study density-dependent correlations among properties of symmetric nuclear matter, hyperonic matter and neutron stars with the minimum constraints at nuclear matter saturation and the maximum masses of hyperonic neutron stars.

The density-dependent, effective coupling constants are induced by σ-field, preserving Lorentz-invariance and thermodynamic consistency as simple as possible, which is discussed in detail and listed here for convenience [9]. We have assumed that only nucleon-meson coupling constants are density-dependent in the current analysis since we are interested in the density-correlations produced by interactions of symmetric nuclear matter. The density-dependent nucleon-meson coupling constants that maintain thermodynamic consistency are defined by,

$$
\begin{aligned}
g^*_{\sigma N} &= g_{\sigma N} + (g_{\sigma\sigma N}/2m_\sigma)\phi_0 \ , \\
g^*_{\omega N} &= g_{\omega N} + g_{\sigma\omega N}\phi_0/m_\sigma \ , \\
g^*_{\rho N}/2 &= g_{\rho N}/2 + g_{\sigma\rho N}\phi_0/m_\sigma \ .
\end{aligned}
\tag{2.3}
$$

The effective masses of mesons compatible with the effective coupling constants (2.3) are required to be:

Since the effective masses of mesons and coupling constants depend on fields, it is clearly seen that they are density-dependent through the scalar field ϕ_0 and have to be determined self-consistently. Note that the effective mass depends on the (n, p) scalar source of nucleons, ρ_{sN}. The nonlinear mean-field approximation is thermodynamically consistent only if effective masses of mesons and coupling constants are renormalized as (2.3) and (2.4).

The introduction of nonlinear $\sigma\sigma N$-vertex interaction is equivalent to define the effective mass of nucleon as,

$$
\begin{aligned}
m_\sigma^{*2} &= m_\sigma^2 (1 + \frac{g_{\sigma 3}}{2m_\sigma^2}\phi_0 + \frac{g_{\sigma 4}}{3!m_\sigma^2}\phi_0^2 - \frac{g_{\sigma\omega}}{2m_\sigma^2} V_0^2 - \frac{g_{\sigma\rho}}{2m_\sigma^2} R_0^2 - \frac{g_{\sigma\sigma N}}{2m_\sigma^3}\rho_{sN}) \ , \\
m_\omega^{*2} &= m_\omega^2 (1 + \frac{g_{\omega 4}}{3!m_\omega^2} V_0^2 + \frac{g_{\sigma\omega}}{2m_\omega^2}\phi_0^2 + \frac{g_{\omega\rho}}{2m_\omega^2} R_0^2) \ , \\
m_\rho^{*2} &= m_\rho^2 (1 + \frac{g_{\rho 4}}{3!m_\rho^2} R_0^2 + \frac{g_{\sigma\rho}}{2m_\rho^2}\phi_0^2 + \frac{g_{\omega\rho}}{2m_\rho^2} V_0^2) \ .
\end{aligned}
\tag{2.4}
$$

$$M_N^* = M_N - g_{\sigma N}^*\phi_0 = M_N - g_{\sigma N}\phi_0 - (g_{\sigma\sigma N} / 2m_\sigma)\phi_0^2 \,. \tag{2.5}$$

Since the effective mass of hyperon H is defined by,

$$M_H^* = M_H - g_{\sigma H}\phi_0 \,, \tag{2.6}$$

the effective masses of nucleons and hyperons are obtained from (2.5) and (2.6):

$$M_H - M_H^* = \frac{g_{\sigma H}}{g_{\sigma N}^*}(M_N - M_N^*) \,. \tag{2.7}$$

The total scalar source is obtained by the requirement of self-consistency:

$$\Sigma^s = \Sigma_N^s + \Sigma_H^s = -\frac{g_{\sigma N}^{*2}}{m_\sigma^{*2}}(\rho_{sN}^* + \rho_{sH}) \,, \tag{2.8}$$

and the scalar sources of nucleons (N) and hyperons (H) are respectively given by [19, 20]

$$\Sigma_N^s = i\frac{g_{\sigma N}^*}{m_\sigma^{*2}}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}$$
$$\left\{(g_{\sigma N}^* - g_{\sigma\omega N}V_0\gamma^0 / m_\sigma - g_{\sigma\rho N}R_0\gamma^0\tau_3 / m_\sigma)G_D(q)\right\} = -\frac{g_{\sigma N}^{*2}}{m_\sigma^{*2}}\rho_{sN}^* \,, \tag{2.9}$$

where ρ_{sN}^* is the modified scalar density defined by $g_{\sigma N}^*\rho_{sN}^* = g_{\sigma N}^*\rho_{sN} - g_{\sigma\omega N}V_0\rho_B/m_\sigma - g_{\sigma\rho N}R_0\rho_3/m_\sigma$; $G_D(q)$ is Green's function of baryons [1]. The hyperon sources are

$$\Sigma_H^s = -\frac{g_{\sigma N}^{*2}}{m_\sigma^{*2}}\sum_H\frac{g_{\sigma H}/g_{\sigma N}^*}{\pi^2}\int_0^{k_{F_H}}dqq^2\frac{M_H^*}{E_H^*(q)} = -\frac{g_{\sigma N}^{*2}}{m_\sigma^{*2}}\rho_{sH} \,. \tag{2.10}$$

where k_{F_H} is the Fermi-momentum of the hyperon H, and $E_H^*(k) = (k^2 + M_H^{*2})^{1/2}$. The sum is performed to baryons, and N is used to denote proton and neutron: $N = (p,n)$; the hyperons are denoted as, $H = \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \cdots$. The hyperon coupling constants are not density-dependent in the current investigation; however, the contributions of hyperon coupling constants are effectively modified as $g_{\sigma H}/g_{\sigma N}^*$.

The ω-meson and ρ-meson contributions to the self-energy are given by

$$\Sigma_\omega^\mu = -\frac{g_{\omega N}^{*2}}{m_\omega^{*2}}\rho_\omega\delta_{\mu,0} \quad \text{and} \quad \Sigma_{\rho\binom{p}{n}}^\mu = \mp\frac{g_{\rho N}^{*2}}{4m_\rho^{*2}}\rho_3\delta_{\mu,0} \,, \tag{2.11}$$

where the isoscalar density, ρ_ω, is given by

$$\rho_\omega = \rho_p + \rho_n + \sum_H r_{HN}^\omega\rho_H \,, \tag{2.12}$$

and the density-dependent ratios of hyperon-nucleon coupling constants on ω-meson, r_{HN}^ω, are defined self-consistently which will be explained in the next section. The self-energies, $\Sigma_{\rho p}^\mu$ and $\Sigma_{\rho n}^\mu$, are briefly denoted as $\Sigma_{\rho\binom{p}{n}}^\mu$; the isovector density is denoted as $\rho_3 = (k_{F_p}^3 - k_{F_n}^3)/3\pi^2$ where the Fermi momentum k_{F_p} is for proton and k_{F_n} for neutron. The baryon-isovector density, ρ_{3B}, and the ratios of sigma-nucleon coupling constants on ρ-meson are also defined; for example, $\rho_{3B} = \rho_3 + r_{\Sigma N}^\rho\rho_{3\Sigma}$, where $r_{\Sigma N}^\rho = g_{\rho\Sigma}/g_{\rho N}^*$ and $\rho_{3\Sigma} = \rho_{\Sigma^+} - \rho_{\Sigma^-}$.

The energy density, pressure of isospin-asymmetric and charge-neutral nuclear matter are calculated by way of the energy-momentum tensor as:

$$\varepsilon_{NHA} = \sum_B\frac{1}{\pi^2}\int_0^{k_{F_B}}dkk^2E_B(k) + \frac{m_\sigma^2}{2}\phi_0^2 + \frac{g_{\sigma 3}}{3!}\phi_0^3 + \frac{g_{\sigma 4}}{4!}\phi_0^4 - \frac{m_\omega^2}{2}V_0^2 - \frac{g_{\omega 4}}{4!}V_0^4 - \frac{g_{\sigma\omega}}{4}\phi_0^2V_0^2$$
$$-(\frac{m_\rho^2}{2} + \frac{g_{\rho 4}}{4!}R_0^2 + \frac{g_{\sigma\rho}}{4}\phi_0^2 + \frac{g_{\omega\rho}}{4}V_0^2)R_0^2 + \sum_{l=e^-,\mu^-}\frac{1}{\pi^2}\int_0^{k_{F_l}}dkk^2E_l(k) \,, \tag{2.13}$$

$$p_{NHA} = \frac{1}{3\pi^2}\sum_B\int_0^{k_{F_B}}dk\frac{k^4}{E_B^*(k)} - \frac{m_\sigma^2}{2}\phi_0^2 - \frac{g_{\sigma 3}}{3!}\phi_0^3 - \frac{g_{\sigma 4}}{4!}\phi_0^4 + \frac{m_\omega^2}{2}V_0^2 + \frac{g_{\omega 4}}{4!}V_0^4 + \frac{g_{\sigma\omega}}{4}\phi_0^2V_0^2$$
$$+(\frac{m_\rho^2}{2} + \frac{g_{\rho 4}}{4!}R_0^2 + \frac{g_{\sigma\rho}}{4}\phi_0^2 + \frac{g_{\omega\rho}}{4}V_0^2)R_0^2 + \sum_{l=e^-,\mu^-}\frac{1}{3\pi^2}\int_0^{k_{F_l}}dk\frac{k^4}{E_l^*(k)} \,, \tag{2.14}$$

where k_{F_B} is the Fermi-momentum for baryons. One can check that the thermodynamic relations, such as $\varepsilon_{NHA} + p_{NHA} = \rho_B E_n(k_{F_n})$ and the chemical potential, $\mu = \partial\varepsilon_{NHA} / \partial\rho_B = E_n(k_{F_n}) = E^*(k_{F_n}) - \Sigma^0(k_{F_n})$, are exactly satisfied for a given baryon density, $\rho_B = 2k_F^3/3\pi^2$. One should note that the chemical potentials and self-energies depend on effective masses and coupling constants when thermodynamic consistency is to be checked.

The conditions of thermodynamic consistency of propagators, self-energies and energy density with effective masses of hadrons (M_N^*, m_σ^*, m_ω^*, m_ρ^*) and effective coupling constants (g_σ^*, g_ω^*, g_ρ^*) can be directly proved [9-12]. The functional derivative of energy density, $\varepsilon_{NHA}(\phi_0, V_0, R_0, n_i)$, with respect to the baryon number distribution, n_i, is given by:

$$\frac{\delta\varepsilon_{NHA}}{\delta n_i} = E(k_i) + \sum_i \left(\frac{\delta\varepsilon_{NHA}}{\delta\phi_0}\frac{\delta\phi_0}{\delta n_i} + \frac{\delta\varepsilon_{NHA}}{\delta V_0}\frac{\delta V_0}{\delta n_i} + \frac{\delta\varepsilon_{NHA}}{\delta R_0}\frac{\delta R_0}{\delta n_i}\right). \tag{2.15}$$

Thermodynamic consistency requires: $\frac{\delta\varepsilon_{NHA}}{\delta\phi_0} = 0$, $\frac{\delta\varepsilon_{NHA}}{\delta V_0} = 0$ and $\frac{\delta\varepsilon_{NHA}}{\delta R_0} = 0$ [9]. The conditions independently generate meson equations of motion and determine self-energies of the approximation. The self-energies calculated by propagators and the condition of conserving approximations become equivalent, only if the effective masses and effective coupling constants of mesons and coupling constants are given by (2.3) and (2.4). The nonlinear mean-field approximation becomes thermodynamically consistent, relativistic, field-theoretical approximation with the effective masses of mesons and coupling constants.

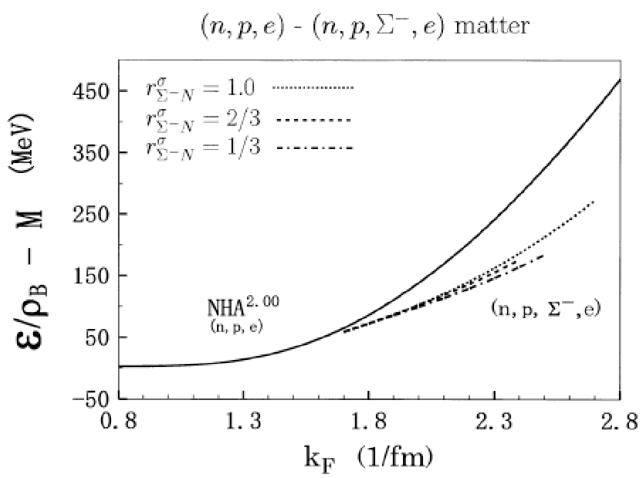

Fig. (1). The binding energies of (n,p,e) and (n,p,Σ^-,e). The onset density of Σ^- is about $k_F = 1.6$ fm^{-1}. The ratios of Σ^--coupling constants, $r^{\sigma}_{\Sigma^- N} = g_{\sigma\Sigma^-} / g_{\sigma N} = 1.0$ (dotted line), $r^{\sigma}_{\Sigma^- N} = 2/3$ (dashed line) and $r^{\sigma}_{\Sigma^- N} = 1/3$ (dash-dotted line), are used respectively. The other coupling constants are fixed as in the Table **1**.

3. THE PHASE TRANSITIONS FROM (n,p,e) TO (n,p,H,e)

The current nonlinear σ-ω-ρ mean-field approximation has several coupling constants whose values are not determined at the outset; however, with given experimental values of nuclear matter at saturation and the maximum mass of neutron stars at high density, adjusting coupling constants for searching the lower bound of incompressibility can delimit the values of coupling constants. One should be careful that if a parameter is changed, it affects saturation density and energy, incompressibility, symmetry energy and maximum mass of neutron stars.

The numerical procedure to determine nonlinear parameters is as follows. First, all nonlinear parameters have to be adjusted to produce constraints at saturation searching for the minimum value of incompressibility. Second, one has to produce EOS at high-density hyperonic matter in order to calculate the maximum mass of neutron stars. Note the (n, p, e)-(n, p, H ,e) phase transition (the first-order phase transition is assumed) when EOS is linked to TOV equation to calculate the mass of neutron stars. If the EOS does not produce the required maximum mass of neutron stars ($M_{max} = 2.00$ $M_{\odot}$), one has to adjust parameters to produce the maximum mass, and then, go back to nuclear matter to adjust constraints at saturation searching for the minimum value of incompressibility. Again, one has to produce EOS and calculate the maximum mass of neutron stars until both constraints should be satisfied. The upper bound of nonlinear parameters and the minimum value of incompressibility are found in the iterative process. One should be careful that thermodynamic consistency to one's approximation is essential to obtain the convergence of the numerical procedure.

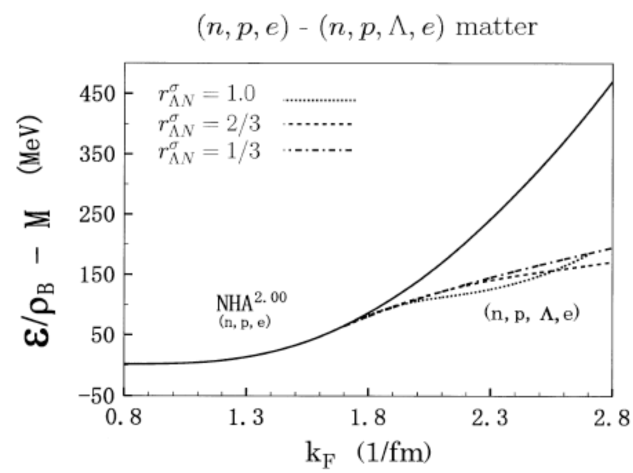

Fig. (2). The binding energies of (n,p,e) and (n,p,Λ,e). The onset density of Λ is about $k_F = 1.7$ fm^{-1}. The ratios of Λ-coupling constants $r^{\sigma}_{\Lambda N} = g_{\sigma\Lambda} / g_{\sigma N} = 1.0$ (dotted line), $r^{\sigma}_{\Lambda N} = 2/3$ (dashed line) and $r^{\sigma}_{\Lambda N} = 1/3$ (dash-dotted line), are used respectively.

In Figs. (**1**) and (**2**), the binding energies of (n,p,e)-(n,p,Σ^-,e) and (n,p,e)-(n,p,Λ,e) matter are shown. By comparing binding energies of phase-transitions from (n,p,e) to (n,p,H,e) matter, it is clearly examined that the equation of state (EOS) becomes softer when a hyperon, H, is produced. Note that *the hyperon-coupling ratios* are defined by ($r^{\sigma}_{\Sigma^- N} = g_{\sigma\Sigma^-}/g^*_{\sigma N}$, $r^{\omega}_{\Sigma^- N} = g_{\omega\Sigma^-}/g^*_{\omega N}$) and ($r^{\sigma}_{\Lambda N} = g_{\sigma\Lambda}/g^*_{\sigma N}$, $r^{\omega}_{\Lambda N} = g_{\omega\Lambda}/g^*_{\omega N}$), respectively. The binding energies of hyperons with different coupling ratios, $r^{\sigma}_{\Sigma^- N} = 1.0, 2/3, 1/3$, exhibit almost the same results and produce similar maximum masses of neutron stars. The ratios, r^{σ}_{HN} and r^{ω}_{HN}, are related to each other and will be explained in detail in sec. 4.

The phase transition begins at $k_{F_{\Sigma^-}} \simeq 1.6$ fm^{-1} and $k_{F_\Lambda} \simeq 1.7$ fm^{-1} respectively; the onset densities are almost fixed, even if the given ratios of coupling constants are changed as $r^\sigma_{HN} = 1, 2/3, 1/3$, and the results are similar to those in the ref. [20]. The properties of nuclear matter and EOS of neutron stars are sensitive to density-dependent interactions, but the hyperon-onset densities of Σ^- and Λ are not so sensitive, which should be experimentally checked if the onset densities of Σ^- and Λ are almost fixed in symmetric nuclear matter.

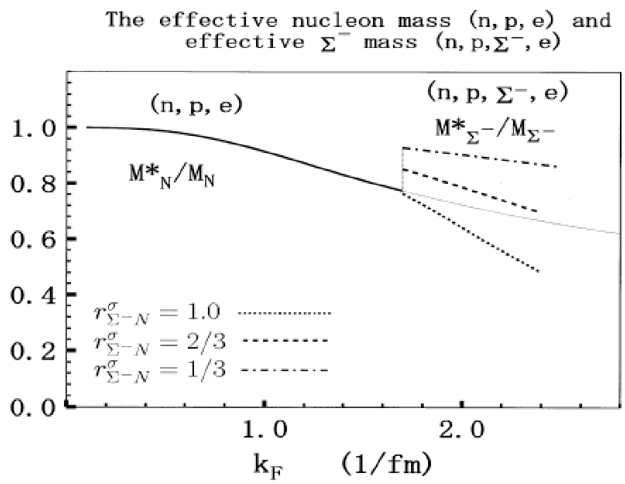

Fig. (3). The effective masses of N and Σ^-. Note that the effective mass of hyperon shows $M^*_{\Sigma^-}/M_{\Sigma^-} \sim 1$ when $r^\sigma_{\Sigma^- N} = 2/3, r^\sigma_{\Sigma^- N} = 1/3$. The smaller coupling ratios mean less density-dependent interactions for the hyperon.

The nonlinear coupling constants in the Table **1**, $g_{\sigma 4}$, $g_{\omega 4}$, $g_{\sigma\omega}$, $|g_{\sigma\rho}|$, $|g_{\omega\rho}|$, $|g_{\sigma\sigma N}|$, $g_{\sigma\omega N}$ and $g_{\sigma\rho N}$, should be understood as the upper limit to be consistent with binding energy at saturation (-15.75 MeV at $k_F = 1.30$ fm^{-1}), $a_4 = 30.0$ MeV and $M_{max} = 2.00$ M_{e}. The effective masses of hadrons are $M^*_N/M < 0.84$, $m^*_\sigma/m_\sigma < 1.06$, $m^*_\omega/m_\omega < 1.02$ at saturation, but incompressibility is the lower bound, $K > 256$ MeV, as explained in the section 2. One can vary the combinations of the values of nonlinear coupling constants so that constraints are satisfied, but the results will be $M^*_N/M < 0.84$, $m^*_\sigma/m_\sigma < 1.06$, $m^*_\omega/m_\omega < 1.02$, and $K > 256$ MeV, at nuclear matter saturation density. Note that $g_{\rho 4}$ is an exception and exhibits almost no effect in the numerical calculations.

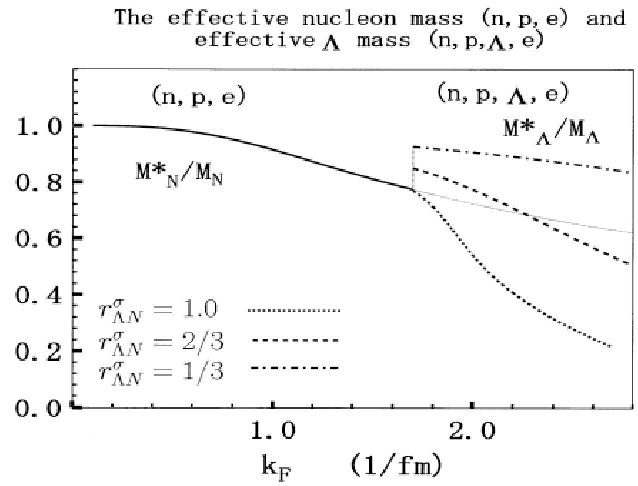

Fig. (4). The effective masses of N and Λ. Note that the effective mass of hyperon shows $M^*_\Lambda/M_\Lambda \sim 1$ when $r^\sigma_{\Lambda N} = 2/3$, $r^\sigma_{\Lambda N} = 1/3$. The smaller coupling ratios indicate less density-dependent interactions for the hyperon.

The equations of motion, self-energies (2.9) ~ (2.11) enable one to obtain the effective coupling constants and masses, (2.3) and (2.4). In Figs. (**3**). and (**4**), the effective masses of nucleons and hyperons (Σ^-, Λ) after hyperon-onset densities are shown respectively. The hyperon effective masses, $M^*_{\Sigma^-}$ and M^*_Λ, depend on the values of coupling ratios and change discontinuously when $r^\sigma_{HN} < 1$. The effective masses become $M^*_{\Sigma^-}/M_{\Sigma^-} \sim 1$ and $M^*_\Lambda/M_\Lambda \sim 1$ for $r^\sigma_{HN} = 1/3, 2/3$, which show that density-dependent interactions of hyperons are weak in high densities and generate softer EOS, resulting in the smaller

Table 1. Coupling Constants and Properties of Nuclear Matter

g_σ 7.629	g_ω 6.675	g_ρ 5.810	$g_{\sigma 3}$ (MeV) 20.0	$g_{\sigma 4}$ 80.0	$g_{\omega 4}$ 80.0	$g_{\rho 4}$ 4.00	$g_{\sigma\omega}$ 72.0	$g_{\sigma\rho}$ -42.0	$g_{\omega\rho}$ -42.0
$g_{\sigma\sigma N}$ -38.50	$g_{\sigma\omega N}$ 11.00	$g_{\sigma\rho N}$ 31.35	g^*_σ 6.879	g^*_ω 7.103	g^*_ρ 8.252				
M^*_N/M 0.84	m^*_σ/m_σ 1.06	m^*_ω/m_ω 1.02	K (MeV) 256	a_4 (MeV) 30.0					

The properties of symmetric nuclear matter connected with isospin-asymmetric, beta-equilibrium matter (n, p, e) whose EOS produces $M_{max}(n, p, e) = 2.00$ $M_\odot$ are listed. The coupling constants are chosen from the data NHA$^{2.00}$ in the paper [9]. The results with the beta-equilibrium matter (n, p, e) whose EOS produces $M_{max}(n, p, e) = 2.50$ $M_\odot$ are discussed in the Ref. [20]. (Note that the current coupling constants, ($g_{\sigma\sigma N}$, $g_{\sigma\omega N}$, $g_{\sigma\rho N}$), correspond to $m_\sigma \times (g_{\sigma\sigma N}, g_{\sigma\omega N}, g_{\sigma\rho N})$ in the paper [16]).

maximum masses of neutron stars (see, the Table **1**). On the contrary, the effective mass of hyperons for $r^{\sigma}_{HN} = 1$ show that density-dependent interactions of hyperons become relatively strong, but the EOS will not become harder than that of (n, p, e), resulting in the smaller maximum masses of neutron stars. As the EOS becomes soft when hyperons are produced as (n, p, e)-(n, p, Σ^-, e) and (n, p, e)-(n, p, Λ, e) matter, the two-fold hyperon production such as ($n, p, \Sigma^-, \Lambda, e$) makes the EOS softer. Hence, the EOS of many-hyperon matter ($n, p, \Sigma^-, \Lambda, H_1, H_2, \cdots, e$) with the ratio, $r^{\sigma}_{HN} < 1$, would generate much softer EOS and be unable to support observed masses of neutron stars.

Hadronic models for nuclear physics model-independently indicate strong density-dependent interactions and correlations among properties of nuclear matter and neutron stars, which is indicated by effective masses, $M^*_N/M_N < 1$ in high densities. On the contrary, if the coupling ratios, $r^{\omega}_{HN} = 1/3$ and 2/3, required by SU(6) effective quark model are employed in the nonlinear mean-field approximation of hadrons, the results suggest that density-dependent interactions of hadrons appear to be weak in effective masses, equations of state and incompressibilities. The similar results are obtained and discussed in [20], and this is a prominent discrepancy between hadronic and effective quark models. The hadronic mean-field model demands the strong density-dependent interactions ($r^{\omega}_{\Sigma^- N} \geq 1.0$ and $r^{\omega}_{\Lambda N} \geq 1.0$) which are consistent with properties of nuclear and neutron stars. The effective masses of hyperons predicted from the hadronic mean-field and SU(6) effective quark model are intrinsically different. This should be investigated further to examine consistency and restriction for both hadronic and effective quark models.

4. THE COUPLING CONSTANTS, BINDING ENERGY AND ONSET-DENSITY OF AN HYPERON

Suppose that (n, p, H, e)-phase is generated after (n, p, e)-phase. The phase transition condition is given by chemical potentials as,

$$\mu_H = \mu_n - q_H \mu_e \,, \tag{4.1}$$

where μ_H, μ_n and μ_e are the hyperon, neutron and electron chemical potentials, and q_H is the hyperon charge in the unit of e. The phase transition conditions (4.1) are generally obtained by minimizing the energy density $\varepsilon(n, p, H, e)$, and the baryons are restricted by the baryon-number conservation and charge-neutrality. The leptons are produced to maintain charge-neutrality and the lepton densities slowly increase for a low density region, but they decrease rapidly and vanish in high densities since the energies of leptons are absorbed and used to produce higher energy hyperons; these phenomena are also observed in the current numerical calculations. The muon can be generated but restricted in a region narrower than that of an electron with the phase-equilibrium condition, $\mu_{\mu^-} = \mu_{e^-}$, and so, the effect of the muon chemical potential is smaller than that of an electron.

The hyperon coupling constants, r^{σ}_{HN} and r^{ω}_{HN}, are related to each other, since the coupling constants are required to produce the minimum value of binding energy (saturation energy) at the hyperon onset density. The relation of hyperon coupling constants can be calculated in terms of the effective masses, coupling constant and binding energy of a hyperon in the current conserving mean-field approximation. The binding energy at the onset-density, α_H, would be expected as the lowest energy level of the hyperon H (the hyperon single particle energy at saturation). The Hugenholtz-Van Hove theorem of a self-bound system at the onset density ($\rho_H = 0$) leads to,

$$\begin{aligned}\alpha_H = ((\varepsilon/\rho_B)_H - M_H)_{\rho_H = 0} &= E_H(0) - M_H = E^*_H(0) - \Sigma^0_{\omega H} - M_H \\ &= g_{\omega H} V_0 + M^*_H - M_H \,.\end{aligned} \tag{4.2}$$

By employing the effective masses of baryons (2.7) and the self-energy of ω-meson (2.11) with $\Sigma^0_{\omega} = -g^*_{\omega N} V_0$, one can obtain,

$$r^{\omega}_{HN} = \frac{m^{*2}_{\omega}}{g_{\omega N} g^*_{\omega N} \rho_{\omega}} \left(\frac{g_{\sigma H}}{g^*_{\sigma N}}(M_N - M^*_N) + \alpha_H\right) = \frac{m^{*2}_{\omega}}{g_{\omega N} g^*_{\omega N} \rho_{\omega}} (M_H - M^*_H + \alpha_H)\,, \tag{4.3}$$

where $\rho_{\omega} = \rho_p + \rho_n$; since $\rho_H = 0$, α_H is the lowest binding energy of a hyperon. The hyperon-coupling constants and the lowest binding energies of hyperons are expressed with effective masses and coupling constants of hadrons related to nonlinear interactions, nuclear observables and masses of neutron stars. The hyperon-onset density and hyperon EOS are intimately related to properties of nuclear matter by way of nonlinear and density-dependent interactions. The binding energies of Λ and Σ^- are chosen as $\alpha_{\Lambda} = -28$ MeV and $\alpha_{\Sigma^-} = 20$ MeV. Since the value of α_{Σ^-} has not been settled yet, we have varied the binding energy as $-20 \leq \alpha_{\Sigma^-} \leq 20$ MeV, and evaluated the EOS and neutron stars. If α_{Σ^-} is negative (attractive), it softens the EOS compared to that of EOS when α_{Σ^-} is positive (repulsive), but the onset density, EOS, maximum mass of neutron stars are qualitatively similar. It may be different for certain properties of finite nuclei and hypernuclei, which should be investigated for quantitative analyses.

The incompressibility, K, and nucleon symmetry energy, a_4, are respectively calculated in the conserving mean-field approximation as [32-34],

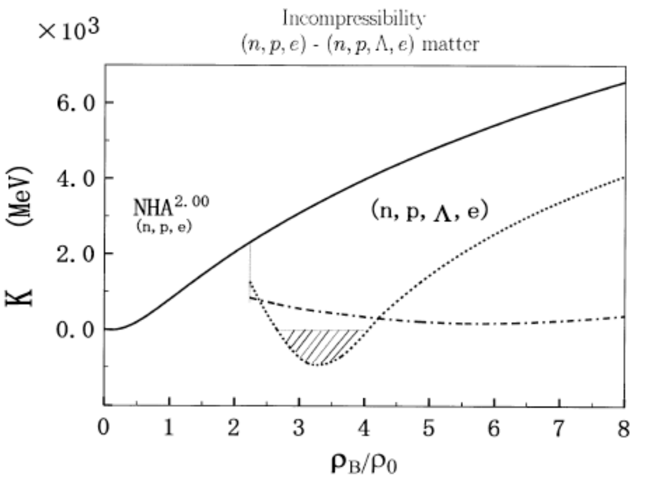

Fig. (5). Incompressibilities of (n,p,e)-(n,p,Λ,e) matter are compared with isospin asymmetric (n,p,e) matter (solid line). The coupling ratios for dotted line are $r^{\sigma}_{\Lambda N}=1.00$ and $r^{\omega}_{\Lambda N}=1.24$, and $r^{\sigma}_{\Lambda N}=0.595$ and $r^{\omega}_{\Lambda N}=2/3$ for dash-dotted line. Note that the shaded area of incompressibility (dotted line) shows the density region unstable against density fluctuations ($2.65\le\rho_B/\rho_0\le 4.21$) (see, for example, [35]).

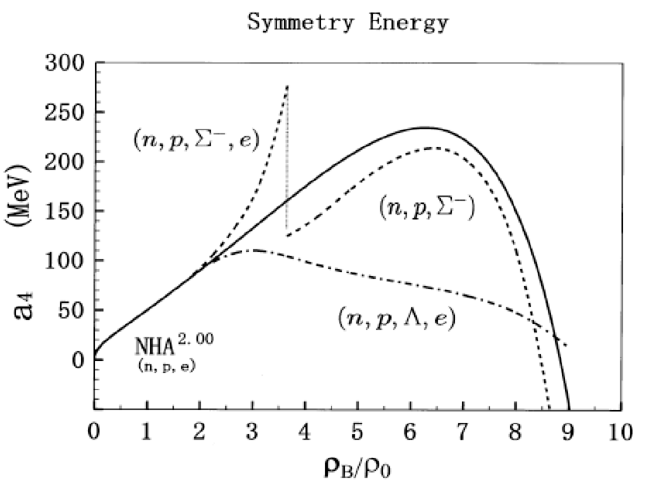

Fig. (6). Symmetry energies of (n,p,e), (n,p,Λ,e) and (n,p,Σ^-,e)-(n,p,Σ^-) are compared. The coupling ratios for dashed line are $r^{\sigma}_{\Sigma^-N}=1.00$ and $r^{\omega}_{\Sigma^-N}=1.31$, and $r^{\sigma}_{\Lambda N}=1.00$ and $r^{\omega}_{\Lambda N}=1.24$ for dash-dotted line. The ratios required by SU(6) quark model, $r^{\omega}_{\Sigma^-N}=2/3$ and $r^{\omega}_{\Lambda N}=2/3$, give similar results, respectively.

$$K = 9\rho_B \frac{\partial^2 \mathcal{E}}{\partial \rho_B^2}\ , \qquad a_4 = \frac{1}{2}\rho_N \left[\left[\frac{\partial^2 \mathcal{E}}{\partial \rho_3^2} \right]_{\rho_N} \right]_{\rho_3=0} \tag{4.4}$$

The computation of nucleon symmetry energy must be performed by maintaining phase equilibrium conditions, which will fix mean-fields, ϕ_0, V_0, R_0 and the ground state energy, $\mathcal{E}(\rho_p,\rho_n)$; then, the derivative of the energy density $\mathcal{E}(\rho_p,\rho_n)$ can be calculated by changing ρ_p and ρ_n with fixed $\rho_N=\rho_p+\rho_n$ and mean-fields. Incompressibility and symmetry energy are important not only at nuclear matter saturation but also in high densities as probes for heavy-ion collisions and density-dependent correlations between properties of nuclear matter and neutron stars [35-41].

In Fig. (**5**), incompressibilities of Λ matter with coupling ratios, ($r^{\sigma}_{\Lambda N}=1.0$, $r^{\omega}_{\Lambda N}=1.24$) and ($r^{\sigma}_{\Lambda N}=0.595$, $r^{\sigma}_{\Lambda N}=2/3$), are compared. The hyperon-onset and softening of EOS are perceived as the discontinuity and abrupt reduction of incompressibility as shown in the Fig. (**5**). The coupling constants of Λ are expected to be unity, $g^{\sigma}_{\Lambda N}, g^{\omega}_{\Lambda N}\approx 1.0$, in order to be consistent with properties of nuclear and neutron matter. The shaded area of incompressibility (dotted line) is the density region unstable against density fluctuations ($2.65\le\rho_B/\rho_0\le 4.21$), which is numerically found in the current conserving mean-field approximation. The similar phenomenon for nuclear matter in low densities is discussed in the Ref. [32]. The dash-dotted line ($r^{\sigma}_{\Lambda N}=0.595$, $r^{\omega}_{\Lambda N}=2/3$) shows that incompressibility is small for high densities. In Fig. (**6**), symmetry energies of (n,p,Σ^-,e) with ratios $r^{\sigma}_{\Sigma^-N}=1.0$, $r^{\omega}_{\Sigma^-N}=1.31$) and (n,p,Λ,e) matter with ratios ($r^{\sigma}_{\Lambda N}=1.0$, $r^{\omega}_{\Lambda N}=1.24$) are compared with (n,p,e) matter. The symmetry energies will increase monotonically about saturation of nuclear matter, but they reach the maximum values in a high density and decrease respectively, which is consistently examined in the conserving nonlinear mean-field approximations [10, 20]. The ratios required by SU(6) quark model ($r^{\sigma}_{\Sigma^-N}=0.473$, $r^{\omega}_{\Sigma^-N}=2/3$) and ($r^{\sigma}_{\Lambda N}=0.595$, $r^{\omega}_{\Lambda N}=2/3$) exhibit qualitatively similar results for the nuclear symmetry energy.

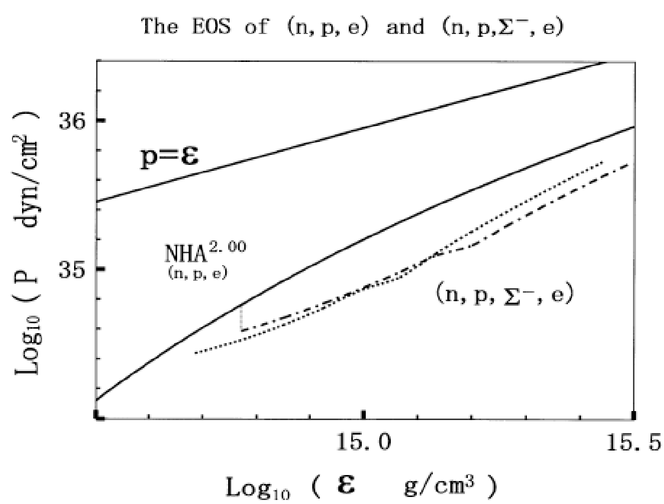

Fig. (7). The equations of state (EOS) for (n,p,e) and (n,p,e)-(n,p,Σ^-,e) matter. The EOS of NHA$^{2.00}_{(n,p,e)}$ yields $M_{max}(n,p,e)=2.00\ M_{\odot}$. The EOS with $r^{\sigma}_{\Sigma^-N}=0.473$ and $r^{\omega}_{\Sigma^-N}=2/3$ (dash-dotted line) produces $M_{max}(n,p,e)=1.25\ M_{\odot}$, which is smaller than the expected lower bound mass of neutron stars (see, the Table **1**).

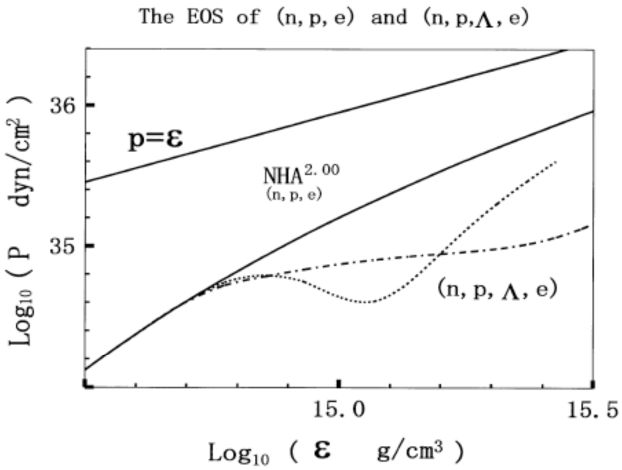

Fig. (8). The equations of state for (n, p, e)-(n, p, Λ, e) matter. The EOS with $r^{\sigma}_{\Lambda N} = 0.595$ and $r^{\omega}_{\Lambda N} = 2/3$ (dash-dotted line) produces $M_{max}(n, p, e) = 1.42\ M_{\odot}$. The results in the case of $r^{\sigma}_{\Lambda N} = 1.00$ and $r^{\omega}_{\Lambda N} = 1.24$ (dotted line) depend intimately on the Λ-onset and phase transition densities (compare with the unstable density region for incompressibility in Fig. **5**).

In Figs. (**7**) and (**8**), the equations of state for (n, p, e)-(n, p, Σ^-, e) and (n, p, e)-(n, p, Λ, e) matter are shown; $p = E$ is the relativistic limit of EOS. The equations of state for (n, p, e)-(n, p, Σ^-, e) discontinuously change with the coupling ratios, whose discontinuities originate from charge-neutrality and phase-equilibrium conditions constrained by self-consistent single particle energies [20]. As expected from energy densities in Fig. (**1**), the equations of state produce similar maximum masses of neutron stars. However, the coupling constant less than unity, such as $r^{\omega}_{\Sigma^- N} \sim 1/3$ and 2/3, are not appropriate, since they produce much softer equations of state which produce the maximum mass of neutron stars, $M_{max} \lesssim 1.30\ M_{\odot}$, close to the observed minimum mass of neutron stars ~ 1.30 $M_{\odot}$ (see, the Table **2**). The equations of state for Λ in Fig. (**8**) clearly exhibit softer equations of state and a phase transition (dotted line), or an unstable density region with respect to density fluctuations [32]. The unstable density region is consistent with the negative incompressibility ($K < 0$) shown as the shaded density region in Fig. (**5**), which indicates that (n, p, Λ, e) matter may go through a phase transition; thermodynamic quantities, such as pressure, energy density and chemical potentials, have to be carefully evaluated by Maxwell construction [42].

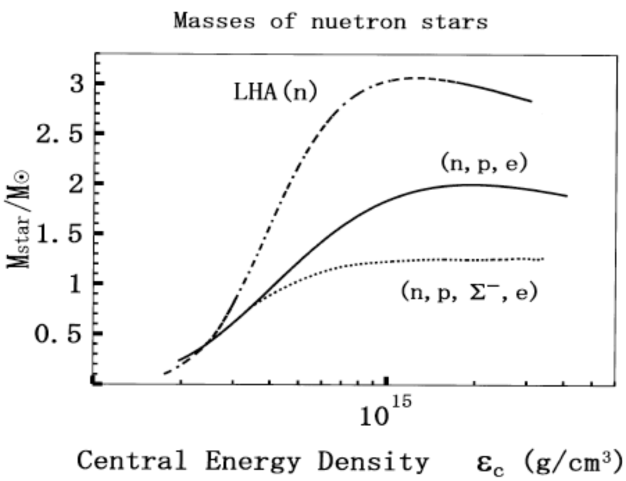

Fig. (9). Masses of neutron stars. LHA(n) is the pure neutron star calculated by employing the linear σ-ω mean-field approximation. The solid line exhibits isospin asymmetric (n, p, e) neutron stars, and the dotted line is for (n, p, e)-(n, p, Σ^-, e) neutron stars ($M_{max} = 1.25\ M_{\odot}$) with the ratios, $r^{\sigma}_{\Sigma^- N} = 0.473$, $r^{\omega}_{\Sigma^- N} = 2/3$.

The masses of neutron stars are shown in Fig. (**9**) using the equations of state for (n, p, e)-(n, p, Σ^-, e) with the coupling ratios, $r^{\sigma}_{\Sigma^- N} = 0.473$, $r^{\omega}_{\Sigma^- N} = 2/3$. The masses of (n, p, e)-(n, p, Λ, e) neutron stars with coupling ratios, $r^{\sigma}_{\Lambda N} = 0.595$, $r^{\omega}_{\Lambda N} = 2/3$, are shown in the Fig. (**10**). The results are compared with the pure neutron [42] and isospin asymmetric (n, p, e) matter [10]. The equation of state for Λ ($r^{\sigma}_{\Lambda N} = 1.00$) is too complicated to use for the maximum mass calculation of neutron stars, because the phase transition of (n, p, e)-(n, p, Λ, e) and density fluctuations (n, p, Λ, e) occur in a density region where the maximum mass of neutron stars apparently depend on; we tentatively

Table 2. Hyperon Coupling Ratios and Properties of (n, p, Σ^-, e), (n, p, Λ, e) Neutron Stars

(i) (n, p, e)-(n, p, Σ^-, e)

$r^{\sigma}_{\Sigma^- N}$	$r^{\omega}_{\Sigma^- N}$	M_{max}	E_c
1.00	1.31	1.37	2.75
0.473	2/3	1.25	1.57
0.200	1/3	1.25	1.58

(ii) (n, p, e)-(n, p, Λ, e)

$r^{\sigma}_{\Lambda N}$	$r^{\omega}_{\Lambda N}$	M_{max}	E_c
1.00	1.24	—	—
0.595	2/3	1.42	1.16
0.359	1/3	1.41	1.07

The maximum masses, M_{max}, and central energy densities, ε_C (10^{15} g / cm^3), of neutron stars produced by way of (n, p, e)-(n, p, Σ^-, e) and (n, p, e)-(n, p, Λ, e) are listed respectively. The EOS of the hyperon-phase (n, p, Σ^-, e) and (n, p, Λ, e) are calculated with the ratios, $r^{\omega}_{HN} = g_{\omega H}/g_{\omega N} = 2/3$ [25-27] and other coupling ratios for comparison. The calculation of (n, p, Λ, e) matter with the coupling ratio, $r^{\sigma}_{\Lambda N} = g_{\sigma\Lambda} / g_{\sigma N} = 1.0$, is explained in the sec. 4 (see, the Fig. **5**).

linked the equations of state smoothly in several ways and evaluated the maximum mass numerically, resulting in 1.10 ~ $M_{\max}$ ~ 1.40. We will evaluate the maximum mass more accurately by including vacuum fluctuation corrections and hadron-quark phase transitions in the near future.

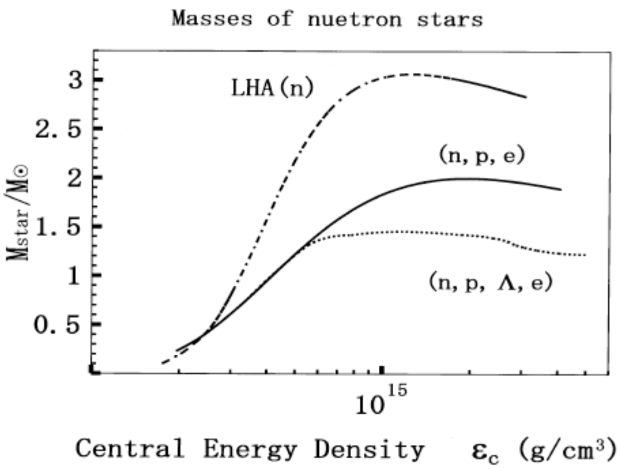

Fig. (10). Masses of neutron stars. The dotted line shows (n,p,e)-(n,p,Λ,e) neutron stars ($M_{max} = 1.42$ $M_{\odot}$) with the ratios, $r^{\sigma}_{\Lambda N} = 0.595$, $r^{\omega}_{\Lambda N} = 2/3$.

As a summary of this section, we conclude that the (n,p,e)-(n,p,H,e) phase transition and hyperon onset-density are important for the equation of state and calculations of neutron stars. The effective masses and coupling constants of isospin symmetric (n,p) nuclear matter and onset-density are related with the binding energy and saturation properties of hyperons. Hence, properties of nucleons and order of hyperon-onset in symmetric nuclear matter are important for the analysis of binding energies, NN, NY interactions and determination of maximum mass of neutron stars. It is also confirmed in the current calculation that hyperons relevant to determine properties of neutron stars are Σ^- and Λ; the similar results are also derived in a nonrelativistic Brueckner-Hartree-Fock calculation for β-stable neutron star matter [21, 22], stating that the only strange baryons entering in β-stable matter up to baryonic densities of 1.2 fm^{-3} are Σ^- and Λ. By comparing the results of $M^{(n,p,e)}_{\max} = 2.50$ $M_{\odot}$ and the current calculation, $M^{(n,p,e)}_{max} = 2.00$ $M_{\odot}$, the results of $M^{(n,p,e)}_{\max} = 2.50$ $M_{\odot}$ [20] are more likely to explain experimental data of nuclear and neutron stars. The maximum mass of neutron stars would be explained appropriately with isospin asymmetric neutron stars with $M^{(n,p,e)}_{\max} \leq 2.50$, ($n,p,\Sigma^-,e$) and ($n,p,\Sigma^-,\Lambda,e$) matter [21]. The nonlinear σ-ω-ρ mean-field approximation has revealed interesting properties of nuclear and hyperonic matter, which should be extended to extract more rigorous conclusions.

5. BINDING ENERGIES OF PURE Λ MATTER

Thermodynamic consistency with constraints of nuclear and neutron matter will induce density-dependent relations among physical quantities, and nonlinear coupling constants of a mean-field approximation are confined, resulting in a self-consistent approximation. When the effective masses and coupling constants of nucleons are self-consistently determined in each density, binding energies and density-dependent interactions of hyperons are self-consistently determined with the coupling constants, $r^{\omega}_{\Lambda N}$ and $r^{\sigma}_{\Lambda N}$, fixed by the eq. (4.3), which is an important result in the self-consistent calculation. Therefore, the determination of binding energy, effective masses and coupling constants will simultaneously determine hyperon binding energies, saturation properties and effective masses of pure Λ-matter. Although information on hyperon-hyperon (YY) interactions is not readily obtained from experiments in free space, it is crucial to study interactions for finite nuclei, symmetric nuclear matter and hyperonic matter [43].

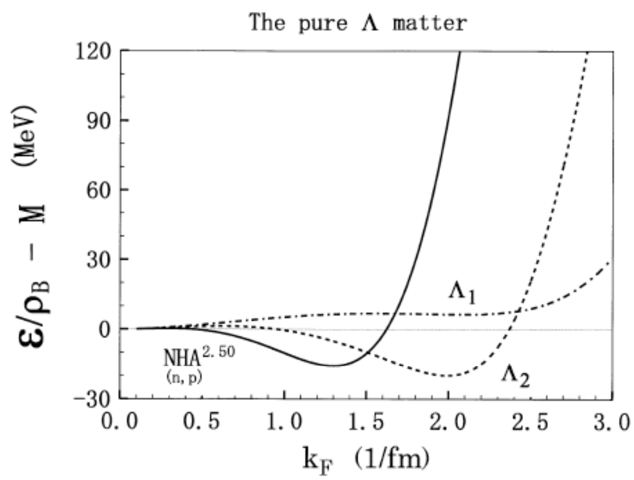

Fig. (11). Binding energies of pure Λ matter. The β-equilibrium (n,p,e)-matter generated by NHA$^{2.50}_{(n,p)}$ will produce $M_{max} = 2.50$ M$_{\odot}$ [20]. The pure Λ-matter with coupling ratios, ($r^{\sigma}_{\Lambda N} = 0.650, r^{\omega}_{\Lambda N} = 2/3$) is denoted as Λ_1 (dash-dotted line). Note that Λ_1 is positive in all densities. The coupling ratios, ($r^{\sigma}_{\Lambda N} = 1.00, r^{\omega}_{\Lambda N} = 1.06$) is denoted as Λ_2 (dashed line).

In Fig. (**11**), the binding energy of symmetric nuclear matter (-15.75 MeV, at $k_F = 1.30$ fm^{-1}) defined by the conserving nonlinear mean-field approximation is shown with a solid line (NHA$^{2.50}_{(n,p)}$); the equation of state for β-equilibrium (n,p,e)-matter will produce $M_{max} = 2.50$ M$_{\odot}$. The pure Λ-matter with coupling ratios, ($r^{\sigma}_{\Lambda N} = 0.650, r^{\omega}_{\Lambda N} = 2/3$) is shown with dash-dotted line, denoted as Λ_1. The pure Λ-matter with coupling ratios, ($r^{\sigma}_{\Lambda N} = 1.00, r^{\omega}_{\Lambda N} = 1.06$) is shown with the dotted line, Λ_2. The coupling ratio, $g_{\omega\Lambda}/g_{\omega N} = r^{\omega}_{\Lambda N} = 2/3$, is employed from the SU(6) quark model for the vector coupling constants [23, 24], and $r^{\sigma}_{\Lambda N} = 0.65$ is obtained from the eq. (4.3). The binding energy of $\Lambda_1(r^{\omega}_{\Lambda N} = 2/3)$ shows that it is positive in all densities, which indicates that YY interaction be repulsive. On the contrary, the binding energy of Λ_2 (a dashed line) with $r^{\sigma}_{\Lambda N} = 1.00, r^{\omega}_{\Lambda N} = 1.06$ shows that the binding energy exhibits saturation at a high density. The YY interaction is attractive at $k_F = 1.30$ fm^{-1} and the binding energy is -19.95 MeV, at $k_F \sim 2.00$ fm^{-1}. The results of $r^{\omega}_{\Lambda N} = 2/3$ and $r^{\omega}_{\Lambda N} = 1.06$ give essentially different results for the properties of hyperonic matter.

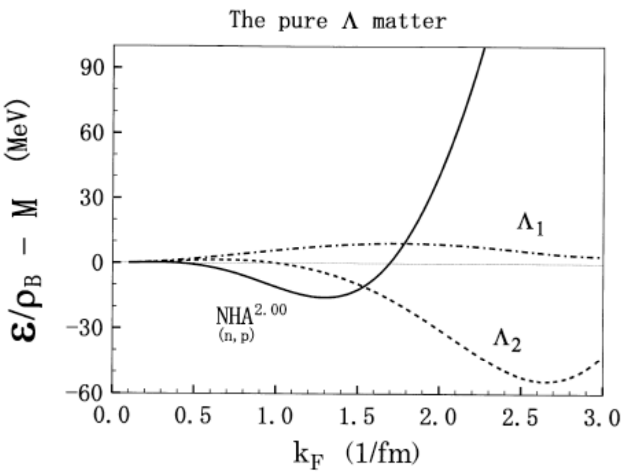

Fig. (12). Binding energies of pure Λ matter. The β-equilibrium (n, p, e)-matter generated by $\mathrm{NHA}^{2.00}_{(n,p)}$ will produce $M_{max} = 2.00$ $M_{\odot}$. The pure Λ-matter (Λ_1) with coupling ratios, ($r^{\sigma}_{\Lambda N} = 0.595, r^{\omega}_{\Lambda N} = 2/3$), is positive in all densities and unbound, whereas the (Λ_2) matter with the coupling ratios, ($r^{\sigma}_{\Lambda N} = 1.00, r^{\omega}_{\Lambda N} = 1.24$) is deeply bound.

In Fig. (**12**), the binding energy of symmetric nuclear matter, which will produce the EOS of β-equilibrium (n, p, e)-matter with the maximum mass of neutron stars, $M_{max} = 2.00\ M_{\odot}$, is compared with Λ_1, ($r^{\sigma}_{\Lambda N} = 0.595, r^{\omega}_{\Lambda N} = 2/3$; dash-dotted line), and Λ_2 ($r^{\sigma}_{\Lambda N} = 1.00, r^{\omega}_{\Lambda N} = 1.24$; dashed line). The coupling constants required by SU(6) quark model will produce positive binding energy in all densities, whereas the coupling constants required by hadronic model will produce self-bound matter. Therefore, the coupling strength, $g_{\omega\Lambda}/g_{\omega N} = r^{\omega}_{\Lambda N} = 2/3$, required by quark model clearly generates different results for binding energies of hyperons.

The hyperon coupling constants required by the hadronic mean-field model ($r^{\sigma}_{\Lambda N} \sim 1.00$ and $r^{\omega}_{\Lambda N} \sim 1.00$) and the SU(6) quark model for the vector coupling constants ($r^{\omega}_{\Lambda N} = 2/3$) exhibit essentially different results on the problem of density-dependent interactions, binding energies and saturation of hyperonic matter. This may indicate another important discrepancy between hadronic model and effective quark model for hadrons, which should be investigated in other many-body approximations and hadronic models.

6. REMARKS

The conserving nonlinear mean-field approximation has exhibited consistent properties for symmetric nuclear and hyperonic matter. The effective masses of hadrons, incompressibility and symmetry energy have shown strong density-dependent behavior consistently in the hadronic nonlinear mean-field approximation. The effective masses and coupling constants are important to examine self-consistent, density-dependent interactions; therefore, conditions of thermodynamic consistency [9-15] are essential to extract consistent results from approximations. The conserving nonlinear mean-field approximation for nuclear matter has shown consistent density-dependent phenomena when it is connected with β-equilibrium, hyperonic matter and neutron stars.

The analysis in the paper [20] and the present calculation lead us to the conclusion that the expected values of effective masses of hadrons and incompressibility should be, $M^*_N/M_N \sim 0.70$, $m^*_{\sigma}/m_{\sigma} \sim 1.02$, $m^*_{\omega}/m_{\omega} \sim 1.01$, $K \sim 320$ MeV and $a_4 = 30$ MeV at saturation density of symmetric nuclear matter, in order to appropriately explain empirical values of nuclear matter and neutron stars. The effective mass of the scalar-isovector ω meson, m^*_{ω}/m_{ω}, will slightly increase at saturation; on the contrary, the effective quark-model for hadrons [28, 29] predicts the decreasing effective mass of m^*_{ω}/m_{ω}. This is also another discrepancy between hadronic and quark-based hadronic models. The physical reason of the increase of effective mass of omega meson, m^*_{ω}/m_{ω}, can be clearly shown in the nonlinear mean-field approximation of hadrons in terms of thermodynamic consistency [10]; it should be actively investigated in other hadronic mean-field approximations. Although the characteristic behavior of effective mass of omega meson, m^*_{ω}/m_{ω}, has not been determined yet [44], it will certainly have great impact on understanding hadronic interactions; this problem will be discussed in the conserving nonlinear Hartree-Fock approximation (σ, ω, π, ρ) in the near future. The results and discrepancies shown in the present calculations should be checked by extending the current approximation to conserving, nonlinear relativistic Hartree-Fock, Ring, Brueckner-Hartree-Fock approximations so as to extract more quantitative results.

The hyperon-onset densities in phase transitions, such as (n, p, e)-(n, p, Σ^-, e) and (n, p, e)-(n, p, Λ, e), are observed fairly fixed numerically, and the push-up phenomena of hyperon-onset densities, abrupt softening of the hyperon EOS, discontinuous variations of incompressibility and symmetry energy are consistently examined in the present calculations; this is consistent with the results obtained in the ref. [20]. It is also confirmed in the current nonlinear mean-field approximation that the baryons entering in β-stable matter up to baryonic densities relevant to neutron star calculations are Σ^- and Λ. The similar results are also derived in a nonrelativistic Brueckner-Hartree-Fock calculation [21, 22], which suggests that nonlinear terms in mean-field approximations be a manifestation of many-body interactions.

The determinations of properties of symmetric nuclear matter, such as binding energy at saturation, effective masses and coupling constants, incompressibility and symmetry energy, will simultaneously decide binding energy and saturation properties of hyperonic matter; the self-consistent relations are important to examine density-dependent correlations among nuclear and hyperonic matter. This fact is clearly shown in the results of the effective masses of hadrons and binding energies of Λ-matter. The conserving nonlinear mean-field approximation and effective quark models require different coupling constants for hyperons. Since the hyperon coupling ratios, $r^{\omega}_{\Lambda N} = 2/3$, required by SU(6) quark model produce weak density-dependent interactions for hadrons at saturation and high densities, it is not compatible with the coupling ratio, $r^{\sigma}_{\Lambda N} \sim 1.0$, demanded by the current nonlinear mean-field approximation. One should note that the conserving nonlinear mean-field approximation includes foregoing linear and nonlinear mean-field approximations and reproduces those results examined so far by adjusting coupling constants. The discrepancy between the nonlinear mean-field approximation and the effective quark models for hadrons may not be a simple matter which is corrected by adjusting coupling constants; however, the investigations of discrepancy between the hadronic and quark-based hadronic models would be constructive for both theoretical approaches. The discrepancy will be very interesting to investigate by extending the current nonlinear mean-field model to include chiral symmetry.

The results obtained in the nonlinear mean-field approximation are important to understand hadronic models of nuclear physics and should be applied to investigate properties of hypernuclei [45], heavy-ion, high-energy scattering experiments and compared to those of other hadronic models and quark-based effective hadronic models. The conserving nonlinear mean-field approximation has revealed interesting properties of hadronic matter; it is imperative to investigate binding energies of hypernuclei, isospin asymmetric matter and magic nuclei [46] in terms of hadronic and quark degrees of freedom. The applications to high-energy, high-density photoproduction reactions [47, 48] may be interesting to examine limitations and applicability of the hadronic mean-field models.

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