# Hiding Charge in a Wormhole

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**Abstract:** Existence of wormholes can lead to a host of new effects like Misner-Wheeler "charge without charge" effect, where without being generated by any source an electric flux arrives from one "universe" and flows into the other "universe". Here we show the existence of an intriguing opposite possibility. Namely, a charged object (a charged lightlike brane in our case) sitting at the wormhole "throat" expels all the flux it produces into just one of the "universes", which turns out to be of compactified ("tube-like") nature. An outside observer in the non-compact "universe" detects, therefore, a neutral object. This charge-hiding effect takes place in a gravity/gauge-field system self-consistently interacting with a charged lightlike brane as a matter source, where the gauge field subsystem is of a special non-linear form containing a square-root of the Maxwell term and which previously has been shown to produce a QCD-like confining gauge field dynamics in flat space-time.

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## 1. INTRODUCTION

Misner-Wheeler "charge without charge" effect [1] stands out as one of the most interesting physical phenomena produced by wormholes. Misner and Wheeler realized that wormholes connecting two asymptotically flat space times provide the possibility of existence of electromagnetically non-trivial solutions, where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe. For a detailed account of the general theory of wormholes we refer to Visser's book [2] (see also [3, 4] and some more recent accounts [5-9]).

In the present paper we find the opposite effect in wormhole physics, namely, that a genuinely charged matter source of gravity and electromagnetism may appear *electrically neutral* to an external observer. Here we show this phenomenon to take place in a gravity/gauge-field system self-consistently coupled to a charged lightlike brane as a matter source, where the gauge field subsystem is of a special non-linear form containing a square-root of the Maxwell term. The latter has been previously shown [10-15] to produce a QCD-like confining ("Cornell" [16-18]) potential in flat space-time. In the present case the lightlike brane, which connects as a wormhole "throat" a non-compact "universe" with a compactified "universe", is electrically charged, however all of its flux flows into the

compactified ("tube-like") "universe" only. No Coulomb field is produced in the non-compact "universe", therefore, the wormhole hides the charge from an external observer in the latter "universe".

Let us recall that lightlike branes are singular null (lightlike) hypersurfaces in Riemannian space-time which provide dynamical description of various physically important phenomena in cosmology and astrophysics such as: (i) impulsive lightlike signals arising in cataclysmic astrophysical events (supernovae, neutron star collisions) [20]; (ii) dynamics of horizons in black hole physics -- the so called "membrane paradigm" [21]; (iii) the thin-wall approach to domain walls coupled to gravity [22-25]. The gravity/gauge-field system with a square-root of the Maxwell term was recently studied in [26] (see the brief review in Section 2 below) where the following interesting new features of the pertinent static spherically symmetric solutions have been found:

(i)     appearance of a constant radial electric field (in addition to the Coulomb one) in charged black holes within Reissner-Nordström-de-Sitter-type and/or Reissner-Nordström- *anti*-de-Sitter-type space-times, in particular, in electrically neutral black holes with Schwarzschild-de-Sitter and/or Schwarzschild- *anti*-de-Sitter geometry;

(ii)    novel mechanism of *dynamical generation* of cosmological constant through the nonlinear gauge field dynamics of the "square-root" Maxwell term;

(iii)   appearance of confining-type effective potential in charged test particle dynamics in the above black hole backgrounds.

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In Section 3 of the present paper we extend the analysis in [26] by finding new solutions of Levi-Civita-Bertotti-Robinson type [27-29], i.e., with space-time geometry of the form $\mathcal{M}_2 \times S^2$ with $\mathcal{M}_2$ being a two-dimensional anti-de Sitter, Rindler or de Sitter space depending on the relative strength of the electric field w.r.t. coupling of the square-root Maxwell term.

In our previous papers [30-40] we have provided an explicit reparametrization invariant world-volume Lagrangian formulation of lightlike $p$-branes (a brief review is given in Section 4) and we have used them to construct various types of wormhole, regular black hole and lightlike braneworld solutions in $D$=4 or higher-dimensional asymptotically flat or asymptotically anti-de Sitter bulk space-times. In particular, in refs. [38-40] we have shown that lightlike branes can trigger a series of spontaneous compactification-decompactification transitions of space-time regions, e.g., from ordinary compactified ("tube-like") Levi-Civita-Bertotti-Robinson space to non-compact Reissner-Nordström or Reissner-Nordström-de-Sitter region or *vice versa*. Let us note that wormholes with "tube-like" structure (and regular black holes with "tube-like" core) have been previously obtained within different contexts in refs. [41-49].

The essential role of the above mentioned proper world-volume Lagrangian formulation of lightlike branes manifests itself most clearly in the correct self-consistent construction [34, 37] of the simplest wormhole solution first proposed by Einstein and Rosen [50] -- the Einstein-Rosen "bridge" wormhole. Namely, in refs. [34, 37] it has been shown that the Einstein-Rosen "bridge" in its original formulation [50] naturally arises as the simplest particular case of *static* spherically symmetric wormhole solutions produced by lightlike branes as gravitational sources, where the two identical "universes" with Schwarzschild outer-region geometry are self-consistently glued together by a lightlike brane occupying their common horizon -- the wormhole "throat". An understanding of this picture within the framework of Kruskal-Szekeres manifold was subsequently provided in ref. [53], which involves Rindler's elliptic identification of the two antipodal future event horizons.

At this point let us strongly emphasize that the original notion of "Einstein-Rosen bridge" in ref. [50] is qualitatively different from the notion of "Einstein-Rosen bridge" defined in several popular textbooks (e.g., refs. [51, 52]) using the Kruskal-Szekeres manifold, where the "bridge" has *dynamic* space-time geometry. Namely, the two regions in Kruskal-Szekeres space-time corresponding to the two copies of outer Schwarzschild space-time region ($r > 2m$) (the building blocks of the original static Einstein-Rosen "bridge") and labeled ($I$) and ($III$) in ref. [51] are generally *disconnected* and share only a two-sphere (the angular part) as a common border ($U = 0, V = 0$ in Kruskal-Szekeres coordinates), whereas in the original Einstein-Rosen "bridge" construction [50] the boundary between the two identical copies of the outer Schwarzschild space-time region ($r > 2m$) is a three-dimensional lightlike hypersurface ($r = 2m$).

In Section 5 below we consider self-consistent coupling of gravity/gauge-field system with a square-root of the Maxwell term to a charged lightlike brane, which will serve as a matter source of gravity and (nonlinear) electromagnetism. In this Section we derive the main result of the present paper -- wormhole-like solutions joining a non-compact "universe" to a compactified ("tube-like") "universe" (of generalized Levi-Civita-Bertotti-Robinson type) *via* a wormhole "throat" realized by the charged lightlike brane, which completely hides its electric flux from an outside observer in the non-compact "universe". This new charge "confining" phenomena is entirely due to the presence of the "square-root" Maxwell term.

## 2. LAGRANGIAN FORMULATION. SPHERICALLY SYMMETRIC SOLUTIONS

We will consider the simplest coupling to gravity of the nonlinear gauge field system with a square-root of the Maxwell term known to produce QCD-like confinement in flat space-time [10-15]. The relevant action is given by (we use units with Newton constant $G_N = 1$):

$$S = \int d^4 x \sqrt{-G} \left[ \frac{R(G)}{16\pi} + L(F^2) \right] \quad , \quad L(F^2) = -\frac{1}{4} F^2 - \frac{f}{2} \sqrt{\varepsilon F^2} \quad , \quad (1)$$

$$F^2 \equiv F_{\kappa\lambda} F_{\mu\nu} G^{\kappa\mu} G^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \; .$$

Here $R(G)$ is the scalar curvature of the space-time metric $G_{\mu\nu}$ and $G \equiv \det \|G_{\mu\nu}\|$; the sign factor $\varepsilon = \pm 1$ in the square root term in (1) corresponds to "magnetic" or "electric" dominance; $f$ is a positive coupling constant. It is important to stress that we will *not* introduce any bare cosmological constant term.

Let us note that the Lagrangian $L(F^2)$ in (1) contains both the usual Maxwell term as well as a non-analytic function of $F^2$ and thus it is a *non-standard* form of nonlinear electrodynamics. In this way it is significantly different from the original purely "square root" Lagrangian $-\frac{f}{2} \sqrt{F^2}$ first proposed by Nielsen and Olesen [54] to describe string dynamics (see also refs. [55, 56]). The natural appearance of the "square-root" Maxwell term in effective gauge field actions was further motivated by 't Hooft [19] who has proposed that such gauge field actions are adequate for describing confinement (see especially Eq.(5.10) in [19]). He has in particular described a consistent quantum approach in which "square-root" gauge-field terms play the role of "infrared counterterms". Also, it has been shown in first three refs. [10-15] that the square root of the Maxwell term naturally arises as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell theory with $f$ appearing as an integration constant responsible for the latter spontaneous breakdown.

Let us also remark that one could start with the non-Abelian version of the gauge field action in (1). Since we will be interested in static spherically symmetric solutions, the non-Abelian gauge theory effectively reduces to an Abelian one as pointed out in the ref. [10].

The corresponding equations of motion read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi T_{\mu\nu}^{(F)} , \tag{2}$$

where

$$T_{\mu\nu}^{(F)} = L(F^2) G_{\mu\nu} - 4 L'(F^2) F_{\mu\kappa} F_{\nu\lambda} G^{\kappa\lambda} , \tag{3}$$

and

$$\partial_\nu \left( \sqrt{-G} L'(F^2) F_{\kappa\lambda} G^{\mu\kappa} G^{\nu\lambda} \right) = 0, \tag{4}$$

where $L'(F^2)$ denotes derivative w.r.t. $F^2$ of the function $L(F^2)$ in (1).

In our preceding paper [26] we have shown that the gravity-gauge-field system (1) possesses static spherically symmetric solutions with a radial electric field containing both Coulomb and *constant* vacuum pieces:

$$F_{0r} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi} r^2} \quad , \quad \varepsilon_F = \text{sign}(Q) , \tag{5}$$

and the space-time metric:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) , \tag{6}$$

$$A(r) = 1 - \sqrt{8\pi} |Q| f - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{2\pi f^2}{3} r^2 , \tag{7}$$

is Reissner-Nordström-de-Sitter-type with *dynamically generated* effective cosmological constant $\Lambda_{\text{eff}} = 2\pi f^2$ .

Appearance in (7) of a "leading" constant term different from 1 resembles the effect on gravity produced by a spherically symmetric "hedgehog" configuration of a nonlinear sigma-model scalar field with $SO(3)$ symmetry [57] (cf. also [58]).

## 3. GENERALIZED LEVI-CIVITA-BERTOTTI-ROBINSON SPACE-TIMES

Here we will look for static solutions of Levi-Civita-Bertotti-Robinson type [27-29] of the system (2) -- (4), namely, with space-time geometry of the form $\mathcal{M}_2 \times S^2$ where $\mathcal{M}_2$ is some two-dimensional manifold:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2(d\theta^2 + \sin^2\theta d\phi^2) , \tag{8}$$

$$-\infty < \eta < \infty \quad , \quad r_0 = \text{const} ,$$

and being:

- either purely electric type, where the sign factor $\varepsilon = -1$ in the gauge field Lagrangian $L(F^2)$ (1):

$$F_{\mu\nu} = 0 \text{ for } \mu, \nu \neq 0, \eta \quad , \quad F_{0\eta} = F_{0\eta}(\eta) ; \tag{9}$$

- or purely magnetic type, where $\varepsilon = +1$ in (1):

$$F_{\mu\nu} = 0 \text{ for } \mu, \nu \neq i, j \equiv \theta, \phi \quad , \quad \partial_0 F_{ij} = \partial_\phi F_{ij} = 0 . \tag{10}$$

In the purely electric case (9) the gauge field equations of motion become:

$$\partial_\eta \left( F_{0\eta} - \frac{\varepsilon_F f}{\sqrt{2}} \right) = 0 \quad , \quad \varepsilon_F \equiv \text{sign}(F_{0\eta}) , \tag{11}$$

yielding a constant vacuum electric field:

$$F_{0\eta} = c_F = \text{arbitrary const} . \tag{12}$$

The (mixed) components of energy-momentum tensor (3) read:

$$T^{(F)0}_{\phantom{(F)0}0} = T^{(F)\eta}_{\phantom{(F)\eta}\eta} = -\frac{1}{2} F_{0\eta}^2 \quad , \quad T_{ij}^{(F)} = g_{ij}(\frac{1}{2} F_{0\eta}^2 - \frac{f}{\sqrt{2}} | F_{0\eta} |) . \tag{13}$$

Taking into account (13), the Einstein eqs. (2) for $(ij)$, where $R_{ij} = \frac{1}{r_0^2} g_{ij}$ because of the $S^2$ factor in (8), yield:

$$\frac{1}{r_0^2} = 4\pi F_{0\eta}^2 \quad , \text{i.e. } r_0 = \frac{1}{2\sqrt{\pi} | c_F |} . \tag{14}$$

The (00) Einstein eq. (2) using the expression $R_0^0 = -\frac{1}{2} \partial_\eta^2 A$ (ref. [59]; see also [60]) becomes:

$$\partial_\eta^2 A = 8\pi \left( | c_F | | c_F | - \sqrt{2} f \right) . \tag{15}$$

Therefore, we arrive at the following three distinct types of Levi-Civita-Bertotti-Robinson solutions for gravity coupled to the non-Maxwell gauge field system (1):

(i) $AdS_2 \times S^2$ with strong constant vacuum electric field $| F_{0\eta} | = | c_F | > \sqrt{2} f$, where $AdS_2$ is two-dimensional anti-de Sitter space with:

$$A(\eta) = 4\pi | c_F | \left( | c_F | - \sqrt{2} f \right) \eta^2 \tag{16}$$

in the metric (8), $\eta$ being the Poincare patch space-like coordinate.

(ii) $Rind_2 \times S^2$ with constant vacuum electric field $| F_{0\eta} | = | c_F | = \sqrt{2} f$, where $Rind_2$ is the flat two-dimensional Rindler space with:

$$A(\eta) = \eta \text{ for } 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \text{ for } -\infty < \eta < 0 \tag{17}$$

in the metric (8).

(iii) $dS_2 \times S^2$ with weak constant vacuum electric field $| F_{0\eta} | = | c_F | < \sqrt{2} f$, where $dS_2$ is two-dimensional de Sitter space with:

$$A(\eta) = 1 - 4\pi | c_F | (\sqrt{2} f - | c_F |) \eta^2 \tag{18}$$

in the metric (8). For the special value $| c_F | = \frac{f}{\sqrt{2}}$ we recover the Nariai solution [61, 62] with $A(\eta) = 1 - 2\pi f^2 \eta^2$ and equality (up to signs) among energy density, radial and transverse pressures:

$$\rho = -p_r = -p_\perp = \frac{f^2}{4} \quad (T^{(F)\mu}_{\phantom{(F)\mu}\nu} = \text{diag}(-\rho, p_r, p_\perp, p_\perp)).$$

In all three cases above the size of the $S^2$ factor is given by (14). Solutions (17) and (18) are new ones and are specifically due to the presence of the non-Maxwell square-root term (with $\varepsilon = -1$) in the gauge field Lagrangian (1).

In the purely magnetic case (10) the gauge field equations of motion (4):

$$\partial_\nu \left[ \sin\theta (1 + \frac{f}{\sqrt{F^2}}) F^{\mu\nu} \right] = 0 \tag{19}$$

yield magnetic monopole solution $F_{ij} = B r_0^2 \sin\theta \varepsilon_{ij}$, where $B = \text{const}$, irrespective of the presence of the non-Maxwell square-root term. However, the latter does contribute to the energy-momentum tensor:

$$T^{(F)0}_{\phantom{(F)0}0} = T^{(F)\eta}_{\phantom{(F)\eta}\eta} = -\frac{1}{2} B^2 - f \mid B \mid \quad, \quad T^{(F)}_{ij} = \frac{1}{2} g_{ij} B^2 . \tag{20}$$

Taking into account (19), the Einstein eqs. (2) for (*ij*) yield (*cf.* (14)):

$$\frac{1}{r_0^2} = 4\pi B^2 + \sqrt{2} f \mid B \mid, \tag{21}$$

whereas the mixed-component (00) Einstein eq. (2) gives $\partial_\eta^2 A = 8\pi B^2$. Thus in the purely magnetic case we obtain only one solution -- $AdS_2 \times S^2$ space-time with magnetic monopole where:

$$A(\eta) = 4\pi B^2 \eta^2 \tag{22}$$

in the metric (8) and the size of the $S^2$ factor is determined by (22).

## 4. LAGRANGIAN FORMULATION OF LIGHTLIKE BRANE DYNAMICS

In what follows we will consider gravity/gauge-field system self-consistently interacting with a lightlike $p$-brane (*LL-brane* for short) of codimension one ( $D = (p+1)+1$ ). In a series of previous papers [30-40] we have proposed manifestly reparametrization invariant world-volume Lagrangian formulation in several dynamically equivalent forms of *LL-branes* coupled to bulk gravity $G_{\mu\nu}$ and bulk gauge fields, in particular, electromagnetic field $A_\mu$. Here we will use our Polyakov-type formulation given by the world-volume action:

$$S_{LL}q = -\frac{1}{2} \int d^{p+1}\sigma T b_0^{\frac{p-1}{2}} \sqrt{-\gamma} \left[ \gamma^{ab} \overline{g}_{ab} - b_0(p-1) \right], \tag{23}$$

$$\overline{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu} \partial_b X^\nu - \frac{1}{T^2}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) \quad, \quad \mathcal{A}_a \equiv \partial_a X^\mu A_\mu . \tag{24}$$

Here and below the following notations are used:

- $\gamma_{ab}$ is the *intrinsic* Riemannian metric on the world-volume with $\gamma = \det \| \gamma_{ab} \|$; $g_{ab}$ is the *induced* metric on the world-volume:

$$g_{ab} \equiv \partial_a X^\mu G_{\mu\nu}(X) \partial_b X^\nu , \tag{25}$$

which becomes *singular* on-shell (manifestation of the lightlike nature), *cf.* Eq. (29) below); $b_0$ is a positive constant measuring the world-volume "cosmological constant".

- $X^\mu(\sigma)$ are the $p$-brane embedding coordinates in the bulk $D$-dimensional space-time with Riemannian metric $G_{\mu\nu}(x)$ ( $\mu, \nu = 0, 1, \dots, D-1$ ); $(\sigma) \equiv \sigma^0 \equiv \tau, \sigma^i$ ) with i=1,...,p; $\partial_a \equiv \dfrac{\partial}{\partial \sigma^a}$.

- $u$ is auxiliary world-volume scalar field defining the lightlike direction of the induced metric (see Eq.(29) below) and it is a non-propagating degree of freedom (ref.[40]).

- T is *dynamical* (variable) brane tension (also a non-propagating degree of freedom).

- Coupling parameter $q$ is the surface charge density of the *LL-brane*.

The corresponding equations of motion w.r.t. $X^\mu$, $u$, $\gamma_{ab}$ and $T$ read accordingly (using short-hand notation (24)):

$$\partial_a (T \sqrt{|\overline{g}|} \overline{g}^{ab} \partial_b X^\mu) + + T \sqrt{|\overline{g}|} \overline{g}^{ab} \partial_b X^\lambda \partial_a X^\nu \Gamma^\mu_{\lambda\nu}$$

$$+ \frac{q}{T} \sqrt{|\overline{g}|} \overline{g}^{ab} \partial_a X^\nu (\partial_b u + q\mathcal{A}_b) \mathcal{F}_{\lambda\nu} G^{\mu\lambda} = 0. \tag{26}$$

$$\partial_a \left( \frac{1}{T} \sqrt{|\overline{g}|} \overline{g}^{ab} (\partial_b u + q\mathcal{A}_b) \right) = 0, \quad \gamma_{ab} = \frac{1}{b_0} g_{ab}. \tag{27}$$

$$T^2 + \varepsilon \overline{g}^{ab} (\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) = 0. \tag{28}$$

Here $\overline{g} = \det \| \overline{g}_{ab} \|$ and $\Gamma^\mu_{\lambda\nu}$ denotes the Christoffel connection for the bulk metric $G_{\mu\nu}$.

The on-shell singularity of the induced metric $g_{ab}$ (25), i.e., the lightlike property, directly follows Eq. (4) and the definition of $\overline{g}_{ab}$ (24):

$$g_{ab} \overline{g}^{bc} (\partial_c u + q\mathcal{A}_c) = 0. \tag{29}$$

Explicit world-volume reparametrization invariance of the *LL-brane* action (23) allows to introduce the standard synchronous gauge-fixing conditions for the intrinsic world-volume metric $\gamma_{00} = -1$, $\gamma_{0i} = 0$ ( $i = 1, \dots, p$ ). which reduces Eqs. (27) -- (28) to the following relations:

$$\frac{(\partial_0 u + q\mathcal{A}_0)^2}{T^2} = b_0 + g_{00} \quad, \quad \partial_i u + q\mathcal{A}_i = (\partial_0 u + q\mathcal{A}_0) g_{0i} (b_0 + g_{00})^{-1},$$

$$g_{00} = g^{ij} g_{0i} g_{0j}, \partial_0 \left( \sqrt{g^{(p)}} \right) + \partial_i \left( \sqrt{g^{(p)}} g^{ij} g_{0j} \right) = 0, g^{(p)} \equiv \det \| g_{ij} \|, \tag{30}$$

(recall that $g_{00}, g_{0i}, g_{ij}$ are the components of the induced metric (25); $g^{ij}$ is the inverse matrix of $g_{ij}$ ). Then, as shown in refs. [30-40], consistency of *LL-brane* dynamics in static "spherically-symmetric"-type backgrounds (in what follows we will use Eddington-Finkelstein coordinates, $dt = dv - \dfrac{d\eta}{A(\eta)}$ ):

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j \quad , \quad F_{v\eta} = F_{v\eta}(\eta), \text{rest} = 0 \quad (31)$$

with the standard embedding ansatz:

$$X^0 \equiv v = \tau \quad , \quad X^1 \equiv \eta = \eta(\tau) \quad , \quad X^i \equiv \theta^i = \sigma^i \; (i = 1, \ldots, p). \; (32)$$

requires the corresponding background (31) to possess a horizon at some $\eta = \eta_0$, which is automatically occupied by the *LL-brane*, i.e.:

$$\eta(\tau) = \eta_0 \quad , \quad A(\eta_0) = 0. \quad (33)$$

This property is called "horizon straddling" according to the terminology of Ref. [23]. Similar "horizon straddling" has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [36, 37].

## 5. SELF-CONSISTENT WORMHOLE-LIKE SOLUTIONS WITH LL-BRANES

Let us now consider a bulk gravity/gauge-field system in $D$=4 (1) self-consistently interacting with a $p = 2$ *LL-brane*:

$$S = \int d^4x \sqrt{-G} \left[ \frac{R(G)}{16\pi} - \frac{1}{4}F^2 - \frac{f}{2}\sqrt{-F^2} \right] + S_{\text{LL}}[q], \quad (34)$$

where $S_{\text{LL}}[q]$ is the *LL-brane* world-volume action (23) (with $p = 2$). It is now the *LL-brane* which will be the material and charge source for gravity and (nonlinear) electromagnetism.

The equations of motion resulting from (34) read:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi \left[ T_{\mu\nu}^{(F)} + T_{\mu\nu}^{(\text{brane})} \right], \quad (35)$$

$$\partial_\nu \left( \sqrt{-G}(1 - \frac{f}{\sqrt{-F^2}})F_{\kappa\lambda}G^{\mu\kappa}G^{\nu\lambda} \right) + j_{(\text{brane})}^\mu = 0, \quad (36)$$

together with the *LL-brane* equations (26) -- (28). $T_{\mu\nu}^{(F)}$ is the same as in (3). The energy-momentum tensor and the charge current density of the *LL-brane* are straightforwardly derived from the underlying world-volume action (23):

$$T_{(\text{brane})}^{\mu\nu} = -\int d^3\sigma \frac{\delta^{(4)}(x - X(\sigma))}{\sqrt{-G}} T\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\mu \partial_b X^\nu, \quad (37)$$

$$j_{(\text{brane})}^\mu = -q\int d^3\sigma \delta^{(4)}(x - X(\sigma))\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\mu (\partial_b u + q\mathcal{A}_b)T^{-1} \; (38)$$

Looking for solutions of static "spherically-symmetric"-type (31) for the coupled gravity-gauge-field-*LL-brane* system (34) amounts to the following simple steps:

(i)     Choose "vacuum" static "spherically-symmetric"-type solutions (31) of (35) -- (36) (i.e., without the delta-function terms due to the *LL-branes*) in each region $-\infty < \eta < \eta_0$ and $\eta_0 < \eta < \infty$ with a common horizon at $\eta = \eta_0$;

(ii)    The *LL-brane* automatically locates itself on the horizon according to "horizon straddling" property (33);

(iii)   Match the discontinuities of the derivatives of the metric and the gauge field strength (31) across the horizon at $\eta = \eta_0$ using the explicit expressions for the *LL-brane* stress-energy tensor charge current density (37) -- (38).

Using (30) -- (32) we find for the *LL-brane* energy-momentum tensor and charge current density:

$$T_{(\text{brane})}^{\mu\nu} = S^{\mu\nu}\delta(\eta - \eta_0) \quad , \quad j_{(\text{brane})}^\mu = \delta_0^\mu q\sqrt{\det\|G_{ij}\|}\delta(\eta - \eta_0), \; (39)$$

where $G_{ij} = C(\eta)h_{ij}(\theta)$ (*cf.* (31)). The non-zero components of the surface energy-momentum tensor $S_{\mu\nu}$ (with lower indices) and its trace are:

$$S_{\eta\eta} = \frac{T}{b_0^{1/2}} \quad , \quad S_{ij} = -Tb_0^{1/2}G_{ij} \quad , \quad S_\lambda^\lambda = -2Tb_0^{1/2}. \quad (40)$$

Taking into account (39) -- (40) together with (31) -- (33), the matching relations at the horizon $\eta = \eta_0$ become [38-40] (for a systematic introduction to the formalism of matching different bulk space-time geometries on codimension-one hypersurfaces ("thin shells") see the textbook [63]):

(A)     Matching relations from Einstein eqs. (35):

$$\left[\partial_\eta A\right]_{\eta_0} = -16\pi T\sqrt{b_0} \quad , \quad \left[\partial_\eta \ln C\right]_{\eta_0} = -\frac{8\pi}{\sqrt{b_0}}T \quad (41)$$

with notation $Y_{\eta_0} \equiv Y|_{\eta \to \eta_0 + 0} - Y|_{\eta \to \eta_0 - 0}$ for any quantity $Y$.

(B)     Matching relation from nonlinear gauge field eqs. (32):

$$\left[F_{v\eta}\right]_{\eta_0} = q \quad (42)$$

(C)     $X^0$-equation of motion of the *LL-brane* (the only non-trivial contribution of second-order *LL-brane* eqs. (26) in the case of embedding (32)):

$$\frac{T}{2}\left(\langle\partial_\eta A\rangle_{\eta_0} + 2b_0\langle\partial_\eta \ln C\rangle_{\eta_0}\right) - \sqrt{b_0}q\langle F_{v\eta}\rangle_{\eta_0} = 0 \quad (43)$$

with notation $\langle Y\rangle_{\eta_0} \equiv \frac{1}{2}\left(Y|_{\eta \to \eta_0 + 0} + Y|_{\eta \to \eta_0 - 0}\right)$.

We are looking for wormhole-type solutions to (34) with the charged *LL-brane* at the wormhole "throat" connecting a non-compact "universe" with Reissner-Nordström-de-Sitter-type geometry (5)–(7) (where the cosmological constant is *dynamically* generated) to a compactified ("tube-like") "universe" of (generalized) Levi-Civita-Bertotti-Robinson type (8)–(9). These wormholes possess the novel property of *hiding electric* charge from external observer in the non-compact "universe", i.e., the whole electric flux produced by the charged *LL-brane* at the wormhole "throat" is pushed into the "tubelike" "universe".

The first wormhole-type solution of the above kind we find is given by:

(a)     "left universe" of Levi-Civita-Bertotti-Robinson ("tube-like") type with geometry $Rind_2 \times S^2$ (17):

$$A(\eta) = -\eta \quad , \quad C(\eta) = r_0^2 \quad , \quad |F_{v\eta}| = \sqrt{2}f \quad \text{for } \eta < 0 ; \quad (44)$$

(b)   non-compact "right universe" comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle (Schwarzschild-type) horizon $r_0$ (*cf.* (5) -- (7)):

$$A(\eta) = 1 - \sqrt{8\pi} \, |Q| \, f - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} - \frac{2\pi f^2}{3}(r_0 + \eta)^2 \, ,$$

$$A(0) = 0 \, , \, \partial_\eta A(0) > 0 \, ,$$

$$C(\eta) = (r_0 + \eta)^2 \quad , \quad F_{v\eta} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}(r_0 + \eta)^2} \quad \text{for } \eta > 0 . \quad (45)$$

Substituting (44) -- (45) into the set of matching relations (41) -- (43) determines all parameters of the wormhole $(r_0, m, Q, b_0, q)$ in terms of the coupling constant $f$ in front of the square-root Maxwell term in (34):

$$Q = 0 \quad , \quad |q| = \frac{f}{\sqrt{2}} \quad , \quad \text{sign}(q) = -\text{sign}(F_{v\eta}) \, , \quad (46)$$

$$r_0^2 = \frac{1}{8\pi f^2} \quad , \quad m = \frac{11}{48\sqrt{2\pi} \, f} \quad , \quad b_0 = \frac{1}{8\sqrt{2\pi} \, f} + \frac{3}{16} . (47)$$

The second wormhole-type solution of the aforementioned kind reads:

(c)   "left universe" of Levi-Civita-Bertotti-Robinson ("tube-like") type with geometry $AdS_2 \times S^2$ (16):

$$A(\eta) = 4\pi \, |c_F| \left( |c_F| - \sqrt{2}f \right) \eta^2 \quad , \quad C(\eta) = r_0^2 \, ,$$
$$|F_{v\eta}| = |c_F| \quad > \quad \sqrt{2}f \quad \text{for} \quad \eta < 0; \quad (48)$$

(d)   non-compact Reissner-Nordström-de-Sitter-type "right universe" of the same kind as (45).

Substituting again (48), (45) into the matching relations (41) -- (43) we find for the wormhole parameters:

$$Q = 0 \quad , \quad |c_F| = |q| + \frac{f}{\sqrt{2}} \quad , \quad \text{sign}(q) = -\text{sign}(F_{v\eta}) \equiv -\text{sign}(c_F) , \quad (49)$$

$$r_0^2 = \frac{1}{4\pi c_F^2} \quad , \quad m = \frac{1}{2\sqrt{\pi} \, f} \left( 1 - \frac{f^2}{6c_F^2} \right) \, , \quad b_0 = \frac{|q| \left( |q| + \sqrt{2}f \right)}{4c_F^2} \quad (50)$$

The important observation here is that $Q = 0$ in both wormhole solutions (a)-(b) (Eqs. (44) -- (45), (46) -- (47) and (c)-(d) (Eqs. (48), (45), (49) -- (50). Therefore, the "right universe" in both cases turns out to be the exterior region of the electrically neutral *Schwarzschild-de-Sitter* black hole beyond the Schwarzschild horizon which carries a vacuum constant radial electric field $|F_{v\eta}| = \frac{f}{\sqrt{2}}$. On the other hand, according to (45), (46) and (45), (49) the whole flux produced by the *LL-brane* charge $q$ ($|F_{v\eta}| = \frac{f}{\sqrt{2}} + |q|$) flows only into the compactified "left universe" of Levi-

Civita-Bertotti-Robinson type ($Rind_2 \times S^2$ (17) or $AdS_2 \times S^2$ (16)).

The geometry of the above constructed charge-"hiding" wormhole solutions is illustrated in Fig. (**1**).
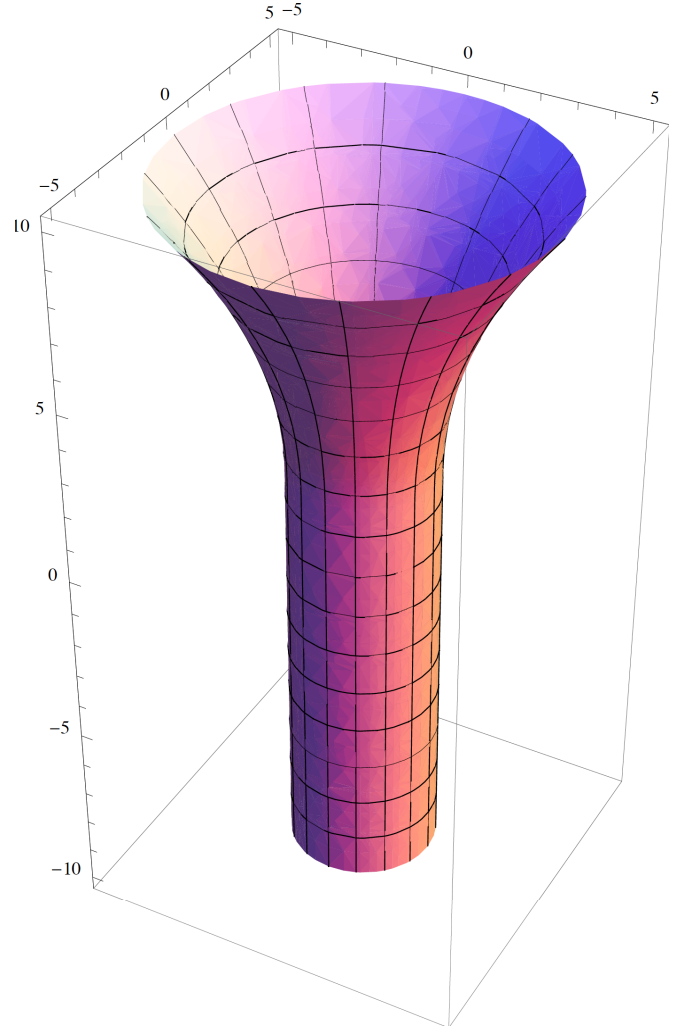


**Fig. (1).** Shape of $t = const$ and $\theta = \frac{\pi}{2}$ slice of charge-"hiding" wormhole geometry. The whole electric flux is expelled into the lower (infinitely long) cylindric tube.

## CONCLUSIONS

We have seen that a charged wormhole "throat" realized by a charged lightlike brane, when joining a compactified space-time with a non-compact space-time region, expels all of the electric flux it produces into the compactified ("tube-like") region when the gauge field dynamics is driven by an additional "square-root" Maxwell term known to produce QCD-like confining potential in flat space-time. Indeed, this effect can be understood from the point of view of an observer in the non-compact "universe" as an alternative way of achieving charge confinement in a fashion similar to the MIT bag model [64], where the role of the inside bag region is being played by the compactified Levi-Civita-Bertotti-Robinson space.

In an accompanying paper [65] we show that the above "charge-hiding" solution can be further generalized to a truly

*charge-confining* wormhole solution when we couple the bulk gravity/nonlinear-gauge-field system (1) self-consistently to *two* separate codimension-one charged *lightlike* branes with equal in magnitude but opposite charges. The latter system possesses a "two-throat" wormhole solution, where the "left-most" and the "right-most" "universes" are two identical copies of the exterior region of the neutral Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon, whereas the "middle" "universe" is of generalized Levi-Civita-Bertotti-Robinson "tube-like" form with geometry $dS_2 \times S^2$ (3). It comprises the finite-extent intermediate region of $dS_2$ between its two horizons. Both "throats" are occupied by the two oppositely charged lightlike branes and the whole electric flux produced by the latter is confined entirely within the middle finite-extent "tube-like" "universe" -- a property qualitatively resembling the quark confinement phenomenon in quantum chromodynamics.

## ACKNOWLEDGMENTS

## CONFLICT OF INTEREST

None declared.

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