Stability Control in a Supply Chain: Total Costs and Bullwhip Effect Reduction

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Abstract: The bullwhip effect refers to the phenomenon of demand distortion in a supply chain. By eliminating or controlling this effect, it is possible to increase product profitability. The main focus of this work is to apply a control technique, based on the divergence of system, to reduce the bullwhip effect in a single-product one echelon supply chain, in which an Order-Up-To (OUT) order policy is applied. First the relationships between bullwhip, stability of the supply chain and the total costs are analyzed. Second the divergence-based control strategy is applied to stabilize the supply chain dynamics with a considerable reduction of the total costs (> 30%) and, in relevant cases, of the bullwhip effect.

Keywords: Bullwhip effect, order-up-to policy, supply chain.

1. INTRODUCTION

The main objective of an optimal ordering policy is that of keeping close production and demand, while inventory levels and capacity requirements are kept at minimum levels [1]. However, a typical effect that arises in supply chains is the bullwhip effect. This effect refers to the phenomenon that occurs when orders to the supplier have larger variance than the ones from the customers, i.e. variance amplification [2-5].

The first academic description of the bullwhip phenomenon is usually ascribed to Forrester [6], who explained it as a lack of information looping between the components of the supply chain and by the non-linear interactions existing, which are difficult to deal with using managerial intuition. Sterman [7] simulated the bullwhip effect in the Beer Game model and confirmed these explanations. Other causes of bullwhip effect are [8-9]: overreaction to backlogs, errors in demand forecasting, neglecting to order in an attempt to reduce inventory, lack of communication and coordination up and down the supply chain, delay times for information and material flow and fluctuating prices. In [8, 9] several countermeasures were also presented.

The economic impact of bullwhip effect was studied in [10], who showed that by eliminating it, one could decrease the stock expenses around 15-30%. For this reason, different bullwhip reduction techniques have appeared in literature. These techniques are based on improving demand forecast, improving the communication in the supply chain, applying a proportional controller to the order policy [1], filtering the demand in order to reduce the variability [11], replacing crisp orders by fuzzy numbers [3], applying Petri nets to improve the coordination of the supply chain [12], using genetic algorithm (GA) to optimize the order policy [13, 14]. In [15] the impact of the centralization of the information was studied and it was shown that, even if an improvement of the coordination can decrease the bullwhip effect, it cannot eliminate it.

The distortion of the demand along a supply chain is in relationship with its parametric sensitivity. Local stability analysis for different types of complex supply chains was studied in [16], whereas Kleijnen [17] reviewed the role of sensitivity analysis in supply chain simulations. Non-linear behaviour, the relationships between global stability and damped oscillations, and the extraordinary complexity and the large variety of bifurcations have been studied in the Beer Game model [18-19]. Techniques to reduce the bullwhip effect based on considering the supply chain as a dynamical system and the application of control techniques have been recently summarized by Sarimveis *et al.* [20]. These control methodologies span from the application of a proportional controller [1, 21] to highly sophisticated techniques such as model predictive control [22].

In this work we will try to control the local stability of a simple supply chain in order to reduce the costs as well as the bullwhip effect. However, since the considered system is linear, local and global stabilities are the same and therefore, we will simply speak about stability. The supply chain considered is single-product, one echelon with a zero replenishment lead time and only a single, order-of-events, review period in which an Order-Up-To (OUT) order policy is applied and an auto regressive and moving average (ARMA) model is used to forecast the demand. Using this model the relationship between bullwhip and stability will be analysed, as well as the relationships between the divergence criterion and stability conditions. By stabilising the supply chain by controlling the order policy with the divergence of the system, we will show how it is possible to reduce the total costs and, in several cases, the bullwhip. The novelty of this control technique with respect to others techniques that have

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studied the relationship between total cost, bullwhip and stability in supply chains, is that this strategy can easily be extended on-line to provide an order policy without the knowledge of the system investigated. Using delayed vectors of a state space variable and applying non-linear dynamical systems techniques for the reconstruction of the divergence as in [23, 24], we would be able of detecting the stability properties of the chain and acting with a right order policy.

In this paper, we have developed the control strategy and assessed its performance using the analytical values of the divergence obtained from the model of the supply chain.

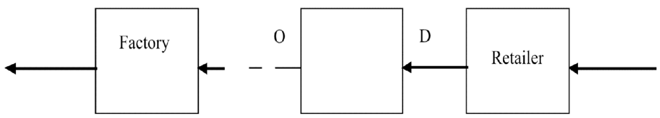

Fig. (1). Block diagram of a one echelon supply chain. The arrows indicate the direction of the Orders.

2. THE SUPPLY CHAIN AND ORDER POLICY MODEL

Let us consider a supply chain as represented in Fig. (**1**). The inventory is used to improve the service to the customers and to protect the production system from fluctuations in the demand. Different replenishment policies exist [25]. Two basic types of inventory replenishment rules are: fixed order and periodic re-ordering systems [26]. Fixed order systems result in the same quantity of product ordered at varying time intervals. In periodic systems a variable amount of products are ordered at fixed time intervals and the decision maker has to determine an Order-Up-To (OUT) level for each period. In this work OUT policy will be considered. Furthermore, following [5], we have chosen to model demand pattern as an ARMA stochastic process of order one. Gilbert [27] has shown that, when an independently and identically distributed ARMA demand pattern has passed through an OUT policy, it does not change the pattern. This implies that the results obtained studying the response of an ARMA demand pattern is relevant for any echelon of a supply chain.

Two of the most important processes, which are in a certain way entangled, are the ordering and the delivering of purchased items. The delays and the non-linearities in the supply chain model give rise to the bullwhip effect, i.e. the fluctuations in the orders that increase as the orders move up in the chain. The bullwhip is quantified as follows:

$$Bullwhip = \frac{\mathrm{var}(O)}{\mathrm{var}(D)} \tag{1}$$

where O is the order to the supplier or to the production and D is the customer demand.

Costs Calculation

Following [1], when the orders are higher than the capacity limit K, then ordering cost c_0 is charged instead of c, see Eq. (2). This means that when the orders in each period are above the normal capacity limit, it is assumed that there is another source of supply. Examples of this alternative source of supply may include overtime working, purchasing or subcontracting. Holding and Shortage costs are calculated in accordance with Eqs. (3) and (4), where and s and h are the stock out costs and the inventory holding costs per unit per period, respectively. The sum of these costs yields the total cost per period, Eq. (5).

-Ordering costs (OC_t):

$$OC_t = \begin{cases} c \cdot O_t & \text{if } O_t < K \\ c \cdot K + c_0 \cdot (O_t \text{ - } K) & \text{if } O_t \geq K \end{cases} \tag{2}$$

- Holding costs (HC_t):

$$HC_t = \begin{cases} h \cdot NS_t & \text{if } NS_t > 0 \\ 0 & \text{if } NS_t \leq 0 \end{cases} \tag{3}$$

- Shortage costs (SC_t):

$$SC_t = \begin{cases} s \cdot |NS_t| & \text{if } NS_t < 0 \\ 0 & \text{if } NS_t \geq 0 \end{cases} \tag{4}$$

Finally, the total costs (TC_t) are calculated as:

$$TC_t = OC_t + HC_t + SC_t \tag{5}$$

Order Policy: OUT Policy

The Order Up To (OUT) policy is frequently used as a standard ordering algorithm, in many material requirement planning (MRP) systems [24], to achieve customer service, inventory and capacity trade-off. At each time step, one reviews the inventory position and then places an "order" to bring it "-up-to" a defined level. In this case, the inventory level is reviewed at the beginning of the period and ordering decision is made. At the end of the period, the customer order is received and demand is formulated and fulfilled. Therefore, it takes one period to receive the order placed. Unmet demand in a period is backordered [1, 5]. This OUT policy is able, in some cases, to minimize the inventory costs and the bullwhip [28].

The Order Up to Level, S_t, is updated every period according to the OUT policy:

$$S_t = \overline{D_t} + k\sigma_{D_t} \tag{6}$$

where $\overline{D_t}$ is the estimated demand at the end of period t, σ_{D_t} is the standard deviation of the demand and k, the safety factor, is defined as:

$$k = G^{-1}\left(\frac{s}{s+h}\right) \tag{7}$$

where G is the standard normal cumulative distribution,

$$G(x) = G(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(t-\mu)^2}{2\sigma^2}} dt \tag{8}$$

The classical Order-Up-To policy definition is completed when net stock NS_t is subtracted from inventory position:

$$O_t = S_t - NS_t = \overline{D_t} + k\sigma_{D_t} - NS_t \tag{9}$$

In accordance with [1, 5], a modification to the classical OUT policy was introduced to provide more freedom in shaping its dynamic response. They proposed to use a proportional controller by introducing a constant $1/T_i$ in the inventory position feedback loop as follows:

$$O_t = \overline{D_t} + \frac{1}{T_i}(k\sigma_{D_t} - NS_t) \tag{10}$$

Chen and Disney [1, 5] called this the modified OUT policy. The following equation completes the definition of NS_t:

$$NS_t = NS_{t-1} + O_{t-1} - D_{t-1} \tag{11}$$

where D_{t-1} is the real demand at time t-1.

Demand Forecast

In this work we will consider the demand to be a stochastic variable:

$$D_t = \eta_t + \mu \tag{12}$$

where η_t is a random normal variable, with mean zero and unity variance, and μ is the mean of the demand.

Let us consider an ARMA process that can be written as:

$$\overline{D}_t = \mu + \rho(\overline{D}_{t-1} - \mu) + \varepsilon_t + \theta \cdot \left(D_{t-1} - \overline{D}_{t-1}\right) \tag{13}$$

where ρ and θ are the model parameters with the initial condition $\overline{D}_0 = \mu + \varepsilon_0$. The forecast error ε_t is supposed to be a white noise process. In addition, we consider that $\varepsilon_{t-1} = D_{t-1} - \overline{D}_{t-1}$ has been actualized with the last known values and therefore is not a stochastic value.

Finally, the following numerical scenario will be considered. The average demand is μ=4 units per period, the cost to produce a unit in normal production is c = 100 € per unit per period, and in overtime production is c_0 = 200 € per unit per period. The inventory holding cost is h=10 € per unit per period, the shortage cost is s = 50 € per unit per period and the capacity limit K = 12 units per period. The inventory safety factor is set to $k\sigma_D = 0.2\mu$ [1, 5].

3. STABILITY AND BULLWHIP

Let us consider, the dynamical system given by the Eqs. (10), (11) and (13), which represents one level of the supply chain of Fig. (**1**). These equations can be written in a matricial form as follows:

$$\begin{bmatrix} O_t \\ NS_t \\ \overline{D}_t \end{bmatrix} = \begin{bmatrix} -1/T_i & -1/T_i & \rho-\theta \\ 1 & 1 & 0 \\ 0 & 0 & \rho-\theta \end{bmatrix} \begin{bmatrix} O_{t-1} \\ NS_{t-1} \\ \overline{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{T_i}(k\cdot\sigma + D_{t-1}) - \mu\cdot\rho + \theta\cdot D_{t-1} + \varepsilon_t + \mu \\ -D_{t-1} \\ -\mu\cdot\rho + \theta\cdot D_{t-1} + \varepsilon_t + \mu \end{bmatrix} \tag{14}$$

The matrix (3x3) on the right side is the Jacobian, J, of the system i.e. the matrix of all first-order partial derivatives. The stability of the dynamics is given by the eigenvalues λ_1, λ_2, λ_3 of this matrix:

$$J = \begin{bmatrix} -1/T_i & -1/T_i & \rho-\theta \\ 1 & 1 & 0 \\ 0 & 0 & \rho-\theta \end{bmatrix} \tag{15}$$

i.e. the solutions of the equation

$$\det(J - \lambda I) = (\rho - \theta - \lambda)\left[\left(-\frac{1}{T_i} - \lambda\right)(1-\lambda) + \frac{1}{T_i}\right] = 0 \tag{16}$$

which are

$$\lambda_1 = 0,\ \lambda_2 = \rho - \theta,\ \lambda_3 = 1 - \frac{1}{T_i} \tag{17}$$

The stability is ensured if all the eigenvalues are smaller than one in absolute value then $|\rho\text{-}\theta| < 1$ which implies $-1 < \rho\text{-}\theta < 1$, and $|1\text{-}1/T_i| < 1$ and therefore $T_i > 1/2$.

Stability means that a small change in the demand forecast ($\overline{D_t}$) will produce a small change on the Order (O_t) or on the Net Stock (NS_t) and the bullwhip cannot be too high.

Bullwhip measure, Eq. (1) considers only Order and Demand forecast and, hence, the variation of Net Stock is not taken into account. For this reason, it is not a global measure of the stability of the supply chain. In the specific case when T_i=*1*, and stability is given by the value of λ_2, the generated bullwhip surface when the parameters ρ and θ are varying between -1 and 1 is showed in Fig. (**2**). The stability limits are indicated by the planes 1 and -1, whereas λ_2 is represented by the inclined plane ρ-θ. The stability region in the parameter space (ρ, θ) is the one inside the projection of the intersection of the surface ρ-θ with the surfaces planes *1* and *-1*. As we can see, this region contains even the parameter values for which bullwhip equals to 1 (intersections of bullwhip surface with surface plane *1*) that means no bullwhip. Nevertheless it may occur that regions with bullwhip relatively small (for example for high values of ρ and small values of θ), are in the instability region, i.e. ρ-θ>*1*.

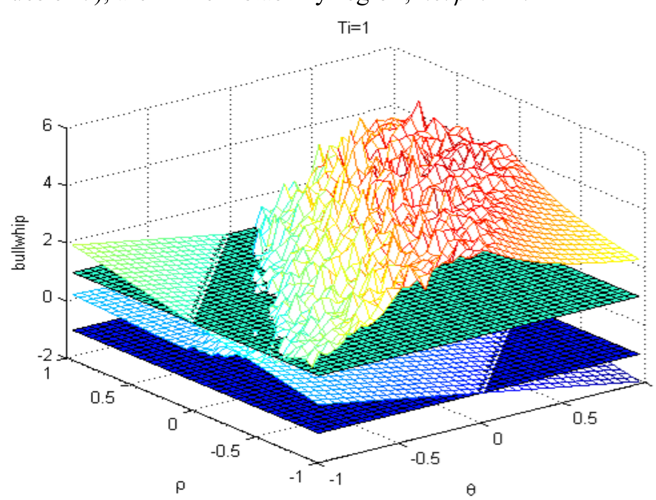

Fig. (2). Bullwhip surface (with noise) for T_i=1, Surfaces planes 1 (dark green) and -1 (dark blue) and the inclined plane ρ-θ.

4. STABILITY ANALYSIS USING DIVERGENCE

In this Section we will calculate the divergence of the supply chain given by Eq. (14) to develop a new type of control that is able of reducing the cumulative total costs and bullwhip. The divergence of a dynamical system is a local property defined as the trace of the Jacobian and then is given by:

$$div(J) = \rho - \theta - \frac{1}{T_i} + 1 \qquad (18)$$

The advantage in using the divergence to control the order policy resides in the fact that, in principle, it can be calculated using only the time series of the state variables: Orders, Net Stock and realized Demand as in [23-24], without the knowledge of the supply chain model. In addition, sometimes only one state variable is enough to calculate this value if it represents the dynamics of the system. The approach is based on the application of state space reconstruction techniques introduced in [29].

Here we have calculated the divergence using the model of the supply chain. The calculation of *div* using only temporal series of one measured variable of the system will be part of our future work but we believe that it is important to underline this aspect in order to understand the final objective of the present work.

Divergence and Stability

The divergence of the supply chain is a measure of the stability in the sense that it gives the rate of expansion or contraction of infinitesimal volume in the state space i.e. the volume that the states of the system can occupy, after one time step, starting from a given set of initial conditions.

When $T_i = 1$ the divergence becomes the same as λ_2. Then asking for $|div| < 1$ the stability condition is satisfied because all the eigenvalues are smaller than 1 in absolute value.

If $T_i \neq 1$ and $\lambda_1, \lambda_2, \lambda_3$ have the same sign, then $|div| < 1 \Rightarrow |\lambda_1| < 1,\ |\lambda_2| < 1, |\lambda_3| < 1$, i.e. the stability condition is satisfied. Our objective is to check if $|div| < 1$ is a good strategy to control the bullwhip and reduce the costs in general.

5. COSTS REDUCTION USING DIVERGENCE-BASED CONTROL

To test the divergence control techniques we will concentrate in two particular values of T_i : $T_i = 0.7$ and $T_i = 2$. We have chosen these two values because the divergence plane cross the surface -1 and 1 for $T_i = 0.7$ and $T_i = 2$, respectively, as it can be seen in Fig. (**3**). In addition, it can be also observed that there is an increment of the total costs with the increase of the absolute value of divergence, $div = \rho - \theta + 1 - 1/T_i$, beyond the absolute value of one. In this figure we have represented the costs only for $-0.5 < \rho, \theta < 0.5$ instead for $-1 < \rho, \theta < 1$ because of their exponential increase beyond this region.

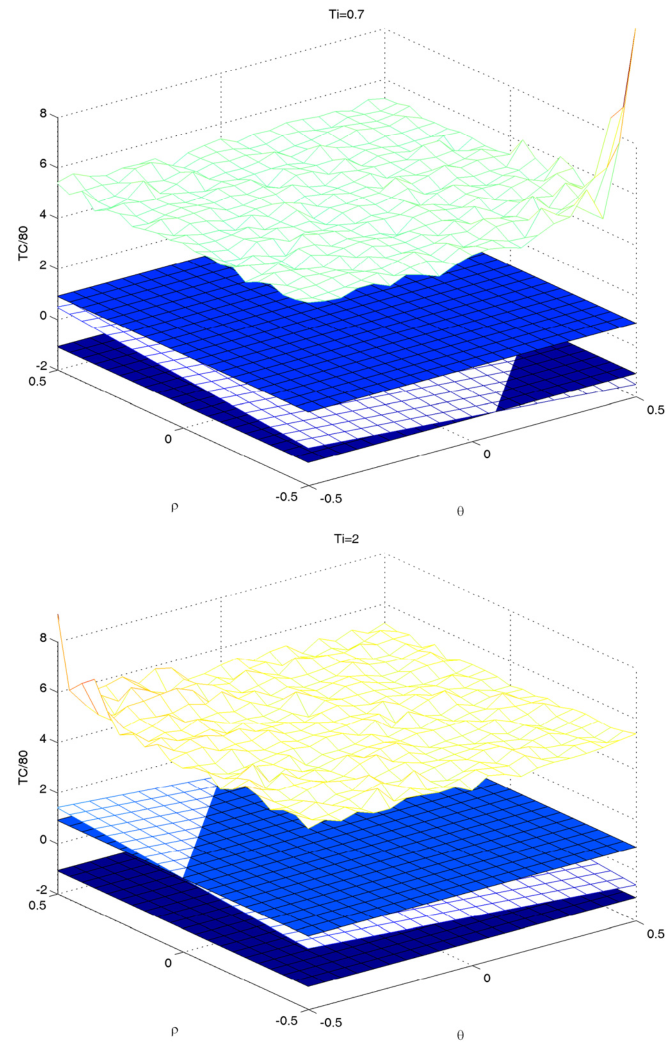

Fig. (3). Total costs (surface with noise) after 100 time units, surfaces planes 1 (light blue), -1 (dark blue), and *ρ-θ*, respectively for T_i=0.7, T_i=2.

Now we are interested in checking if by acting on the parameter T_i, and maintaining divergence values in the range [-1,1], the total costs will be reduced. The T_i parameter is then modified only when divergence becomes bigger than 1 in absolute value in order to change the divergence value as follows:

$$if \ |\mathrm{div}_{old}| \geq 1,$$

$$div_{new} = div_{old} - sign(div_{old}) \cdot (|div_{old}| - 1) \qquad (19)$$

In practice, in order to obtain $|div_{new}| < 1$ we subtract $sign(div_{old}) \cdot (|div_{old}| - 1)$ from $-1/T_i$ as follows:

$$(-1/T_i)_{new} = (-1/T_i)_{old} - sign(div_{old}) \cdot (|div_{old}| - 1)$$

and then

$$(T_i)_{mew} = -\left[(-1/T_i)_{old} - sign(div_{old})(|div_{old}| - 1)\right]^{-1}$$

With this calculation, we obtain a T_i value that allows having a divergence smaller than 1 in absolute value.

Graphically, the evolution of the controlled divergence with respect to the not controlled one is represented in Fig. (**4**).

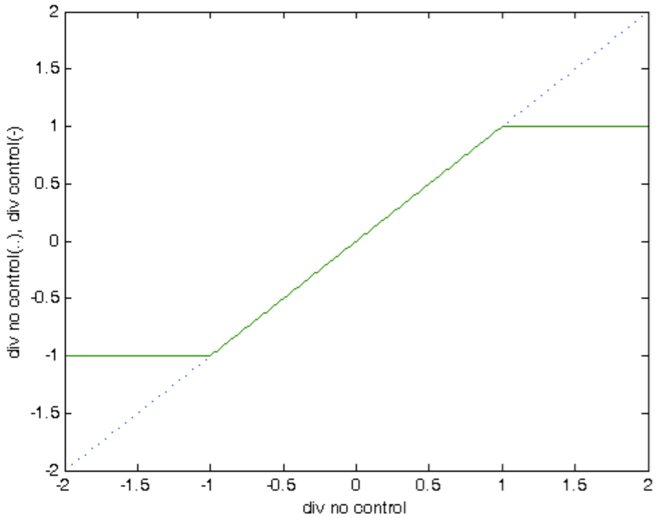

Fig. (4). Controlled (continuous) and non controlled divergence (dotted line) in respect to non controlled one in the interval [-2,2].

Applying this kind of control divergence-based, it is possible to observe (Fig. **5**) the reduction of the total costs surfaces for T_i=0.7 and T_i=2. This reduction occurs in correspondence with the divergence values that now stay inside the [-1,1] interval.

To analyse how this type of control strategy affects the cumulative total costs over time, two particular cases are considered for which the divergence in absolute value is bigger than one and then the control will intervene modifying the results:

a) $T_i = 2$, $\rho = 0.5$, $\theta = -0.5$

b) $T_i=0.7$, $\rho = -0.5$, $\theta = 0.5$

In both cases (Figs. **6**, **7**), the advantage in applying divergence control is clear the cumulative total costs during the simulation period, defined as

$$TC_{cum} = \sum_{t=1}^{n} TC_t \tag{20}$$

are reduced. In order to quantify the reduction we may calculate the relative rate of increase of cumulative total costs, $R=[TC_{cum}(n)-TC_{cum}(1)]/TC_{cum}(1)$, with and without control. Without control R is 0.48 for case a) and 1.5 in case b). Applying control we obtain R = 0.15 in case a) and R=0.41 in case b) respectively (see Figs. **6**, **7**).

When the control is activated, the improvement in the costs occurs for all the parameters values, ρ and θ, (see Figs. **3**, **5**), and not only for the special case represented.

We have analyzed the effects of changing the initial value of the control parameter T_i. The results are presented in Table **1**. In the first column there are the T_i value considered, in the second and third the logarithm of the mean total costs when the parameters ρ and θ are varying between -1 and 1 with a step of 0.05; each case has been run during twenty time steps and twenty times in order to average the values obtained. In the last column, we have presented the percentage of the increment in the cumulative total costs (TC_{cum}) that is obtained if we do not apply the divergence control strategy. By considering all cases we can observe that in average they are reduced approximately by 63% applying our control strategy. In particular, for low T_i values the reduction in TC_{cum} is lower than for high T_i values, and after a fast increase, until T_i =2.5, the values tend to oscillate around 65% approximately. It should be noticed that when $|div| < 1$ no control action is performed and therefore, as T_i increases the actuation of the control algorithm are more frequent, see Eq. (18).

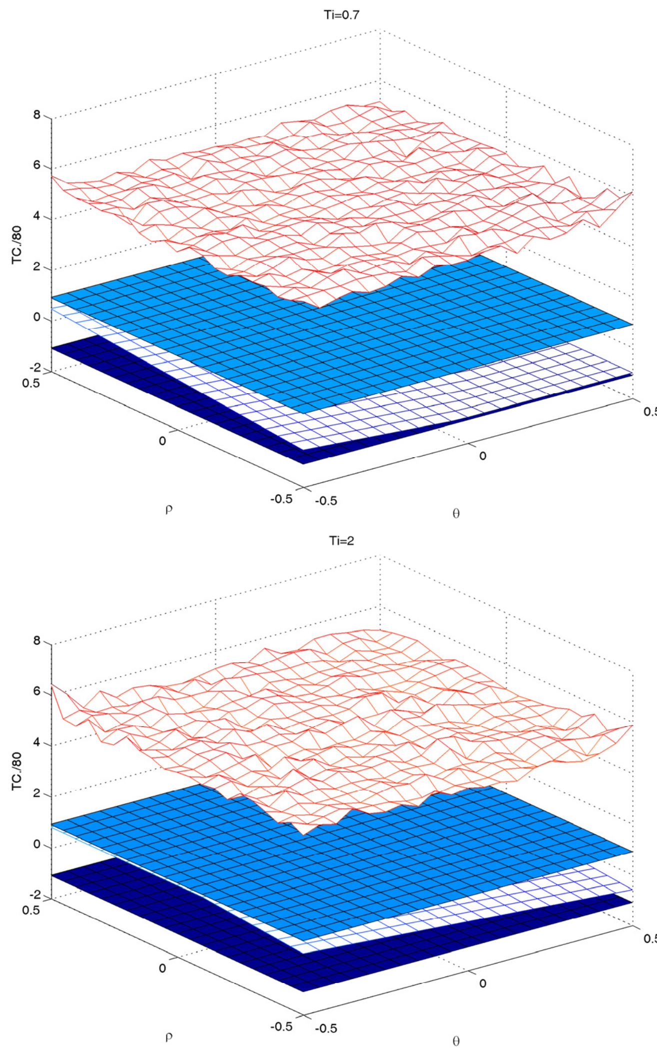

Fig. (5). Controlled divergence total costs (surface with noise) after 100 time units, surfaces planes 1 (light blue), -1 (dark blue), and ρ-θ, respectively for T_i=0.7, T_i=2.

6. BULLWHIP REDUCTION USING DIVERGENCE-BASED CONTROL

As it has been discussed previously, the control strategy based on maintaining the absolute value of the divergence of the supply chain between [-1,1], is able to reduce the costs. Let us now analyse what are the effects on the bullwhip values. In Fig. (**8**), we can see the bullwhip surface for T_i=0.7 without and with divergence control. If one observes the part of the parameter space, in correspondence with high θ values (near to 1) and small ρ values (near to -1), it is possible to see how the activation of the control implies a reduction of the bullwhip effect. These parameters values correspond to an order policy that give more importance to the error between real and forecast demand than to the deviation of forecasted demand from the mean value. For the other parameter values the bullwhip effect reduction does not occur. In the middle of parameters space where $|div| < 1$ the divergence control is not activated. In the other part of parameter's space where $|div| > 1$, small θ values and high ρ values, the control is also activated but the bullwhip is not reduced. Nevertheless, in this case, as it can be seen from Fig. (**8**), the bullwhip is very small even without control.

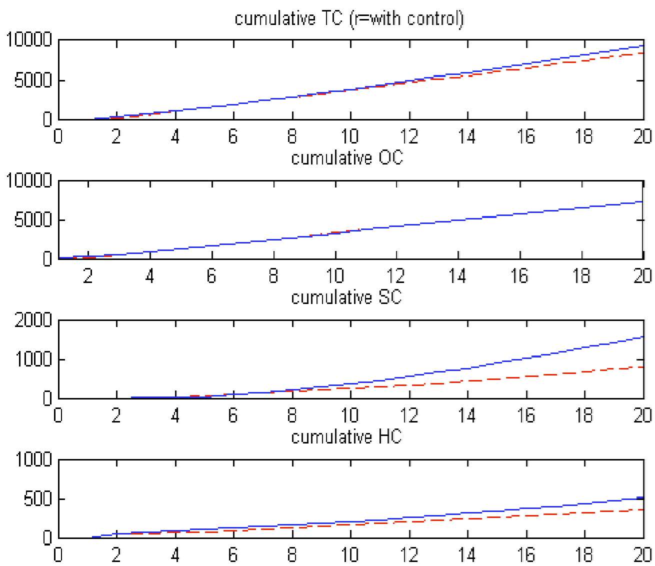

Fig. (6). Costs without (continuous line) and with (dot line) divergence control in time, case a) $T_i = 2$, $\rho = 0.5$, $\theta = -0.5$. Costs reduction: R without control - R with control= 0.33.

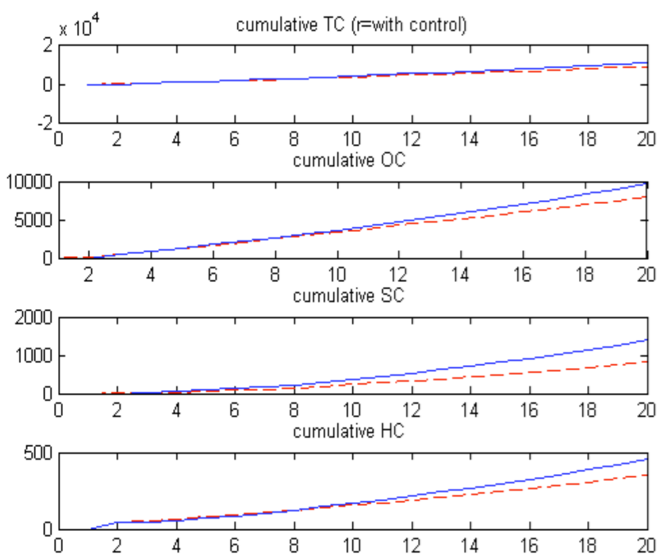

Fig. (7). Costs without (continuous line) and with (dot line) divergence control in time, case b): T_i=0.7 ρ=-0.5, θ=0.5. Costs reduction: R without control - R with control= 1.09.

Table 1. Logarithm of the Mean Value of Total Costs for Different T_i Values, when ρ and θ are Varying Between -1 and 1 with an Step of 0.05, without (TC^{nc}_{cum}) and with (TC^{c}_{cum}) Divergence Control for 20 Simulation Runs each Combination of Parameters

T_i	TC^{nc}_{cum}	TC^{c}_{cum}	Gain (%)
0.5	10.7734	10.7083	6.3026
1.0	11.5877	10.9883	45.0859
1.5	11.5993	10.8698	51.7850
2.0	11.7926	11.0314	53.2894
2.5	11.9066	10.8179	66.3346
3.0	11.9005	10.9408	61.6992
3.5	11.8798	11.0365	56.9712
4.0	12.1444	11.3136	56.4299
4.5	12.1076	11.2705	56.7036
5.0	12.1493	11.0166	67.7838
5.5	12.1466	11.2143	60.6353
6.0	11.9636	11.2891	49.0589
6.5	12.1481	10.9644	69.3856
7.0	11.9673	10.9451	64.0197
7.5	12.0793	11.1492	60.5486
8.0	12.1676	10.9176	71.3495
8.5	12.0981	10.9855	67.1297
9.0	12.1813	10.7704	75.6076
9.5	12.3316	11.3031	64.2457
10.0	11.8306	10.6557	69.1150
10.5	11.9813	11.0265	61.5111
11.0	12.0294	11.1342	59.1474
11.5	12.1204	10.8850	70.9282
12.0	12.2081	11.2591	61.2872
12.5	12.4134	11.0014	75.6345
13.0	11.9585	11.0297	60.4973
13.5	12.2391	11.3215	60.0523
14.0	12.2050	10.9544	71.3667
14.5	11.5667	10.8630	50.5249
15.0	12.4562	11.0025	76.6296
15.5	12.1701	10.9530	70.3912
16.0	11.9659	10.9846	62.5176
16.5	12.3291	11.0453	72.3017
17.0	12.2331	10.8312	75.3871
17.5	12.2027	10.9775	70.6301
18.0	12.1794	11.1155	65.4893
18.5	12.2126	11.1991	63.7054
19.0	12.1499	11.0093	68.0373
19.5	12.3574	11.2053	68.4027
20.0	12.1506	11.0727	65.9691

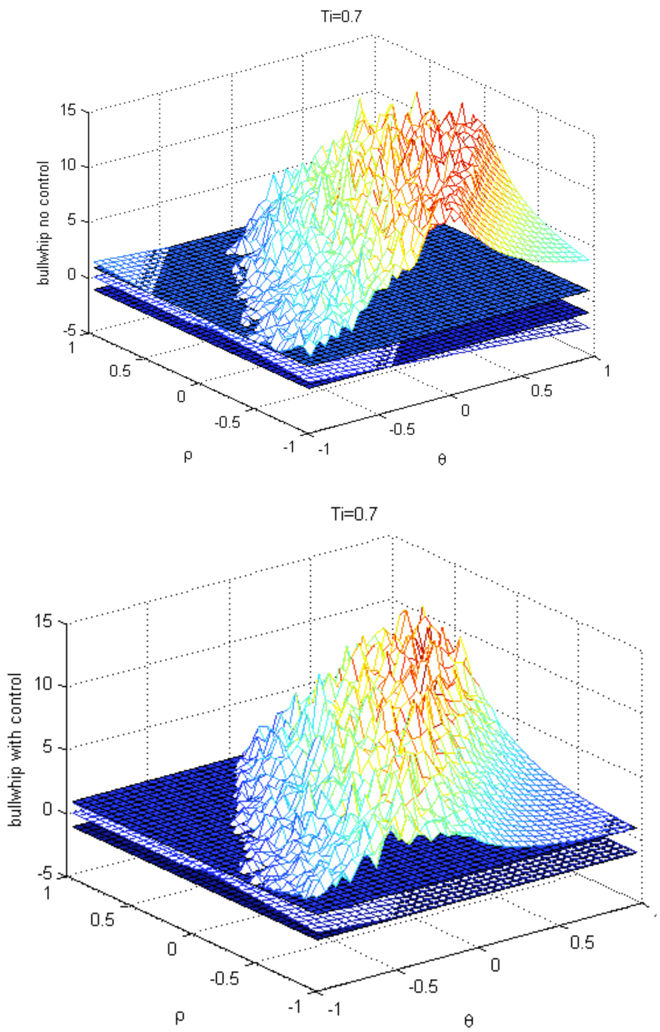

Fig. (8). Bullwhip surface without and with divergence control for T_i=0.7.

If we consider the other value of the control parameter, T_i=2, see Fig. (**9**), we can observe a similar behaviour. In this case, perhaps because the control acts in a region where the bullwhip surface is more smooth (high θ values and small ρ values), it is even more clear the action of the control on the reduction of the bullwhip. In fact a clear discontinuity in the bullwhip slope is observed in Fig. (**9**). The results for different T_i values as a function of ρ and θ are presented in Table 2. Similarly to Table **1**, in Table **2** the results correspond to 20 time steps and each combination of parameters was run 20 times and averaged. In the second and third column there are mean bullwhip values without and with control. In the third column the percentage of reduction applying control indicate that we can have a strong reduction only for certain values of T_i, whereas for the others we obtain similar values. Applying the control strategy there is a strong reduction of bullwhip for T_i values from 0.5 until 2 then the difference in bullwhip with or without control becomes nearly zero.

Comparing Tables **1** and **2**, we can observe that in the interval of T_i values between 0.5 and 2, by applying the control strategy based on divergence we have both bullwhip reduction and a decreasing of cumulative total costs. For T_i higher than 2 the gain applying the control strategy remains high but there is no a bullwhip reduction. High T_i values imply that the order policy without control does not consider the difference between the net stock and the real demand variance, see Eq. (10).

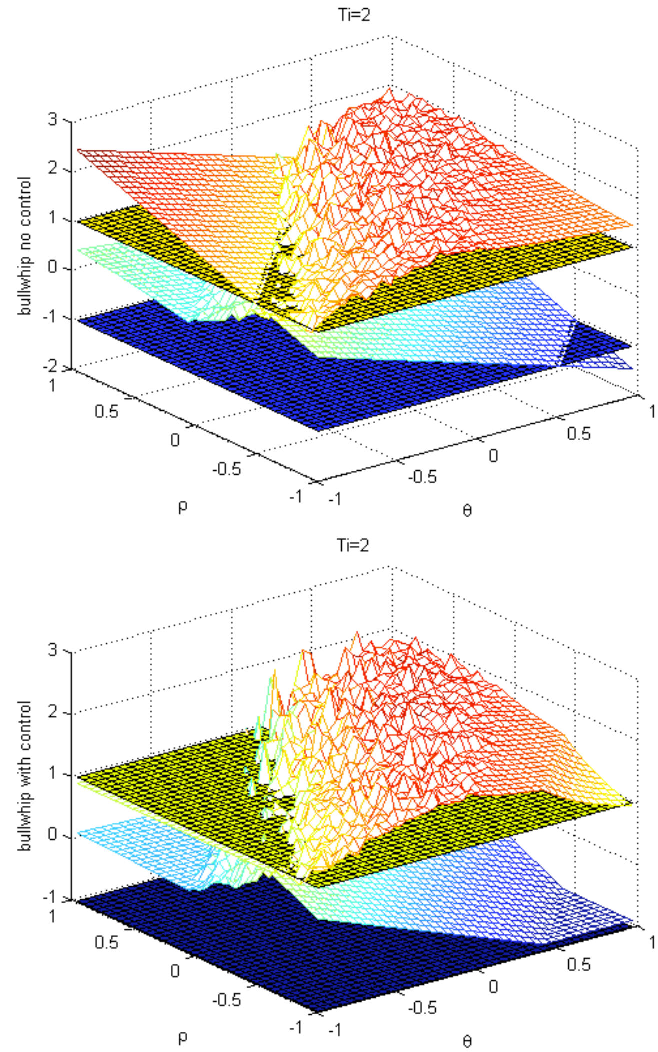

Fig. (9). Bullwhip surface without and with divergence control for T_i=2.0.

7. CONCLUSIONS AND FUTURE WORK

In this work we have considered, following [1, 5] a single-product one echelon supply chain with a zero replenishment lead time and only a single, order-of events review period (see Fig. **1**) in which an Order-Up-To (OUT) order policy is applied and the demand is forecasted using an ARMA model, Eq. (13). Using this model the relationships between bullwhip and stability have been analysed, as well as the relationships between the divergence based criterion and stability conditions.

By comparing bullwhip and stability surface (see Fig. **2**) in the (ρ, θ) plane we have found that the stability region contains the parameter values for which the bullwhip surface is equal to one, but, on the other side, small bullwhip values can be obtained even in the instability region, i.e. when the surface ρ-θ has values higher than 1 or smaller than -1.

Table 2. Mean Values of Bullwhip for Different T_i Values - when ρ and θ are Varying Between -0.5 and -1 and 0.5 and 1, Respectively- without (B_{nc}) and with (B_c) Divergence Control and its Differences ($(B_{nc} - B_c)\cdot 100/B_{nc}$) in Percentage for 20 Simulation Runs

T_i	B_{nc}	B_c	Diff. (%)
0.5	2.8237	1.2959	54.1063
1.0	1.5449	1.2574	18.6096
1.5	1.3292	1.2341	7.1547
2.0	1.1685	1.1634	0.4365
2.5	1.1378	1.1230	1.3008
3.0	1.0886	1.1028	-1.3044
3.5	1.0799	1.0722	0.7130
4.0	1.0533	1.0656	-1.1678
4.5	1.0343	1.0432	-0.8605
5.0	1.0308	1.0391	-0.8052
5.5	1.0270	1.0295	-0.2434
6.0	1.0245	1.0235	0.0976
6.5	1.0142	1.0205	-0.6212
7.0	1.0118	1.0200	-0.8104
7.5	1.0135	1.0149	-0.1381
8.0	1.0031	1.0129	-0.9770
8.5	1.0096	1.0105	-0.0891
9.0	1.0061	1.0074	-0.1292
9.5	1.0076	1.0100	-0.2382
10.0	1.0021	1.0047	-0.2595
10.5	1.0004	1.0027	-0.2299
11.0	1.0023	1.0067	-0.4390
11.5	0.9998	1.0021	-0.2300
12.0	1.0006	1.0058	-0.5197
12.5	1.0003	1.0011	-0.0800
13.0	0.9989	1.0014	-0.2503
13.5	0.9999	0.9997	0.0200
14.0	1.0003	1.0044	-0.4099
14.5	1.0005	1.0009	-0.0400
15.0	0.9995	0.9993	0.0200
15.5	0.9986	1.0010	-0.2403
16.0	1.0002	1.0001	0.0100
16.5	0.9978	0.9995	-0.1704
17.0	0.9972	1.0001	-0.2908
17.5	1.0005	1.0001	0.0400
18.0	0.9988	0.9998	-0.1001
18.5	0.9962	0.9995	-0.3313
19.0	0.9996	0.9976	0.2001
19.5	0.9996	0.9994	0.0200
20.0	0.9972	0.9978	-0.0602

The divergence criterion, $|div| < 1$, is introduced in order to reduce bullwhip and costs by decreasing the instability of the supply chain. For some values of control parameters, T_i=1, it was showed that checking $|div| < 1$ is identical to check the stability of the system using its eigenvalues. It was found that (Fig. **3**) in correspondence of the values of ρ and θ for which the divergence is bigger than one in absolute value, the total costs increase. If the divergence values are controlled by changing the control parameter T_i in order to maintain $|div| < 1$ we obtain a substantial reduction of the total costs (Fig. **5**).

In the two particular cases analyzed, the advantages of the application of divergence control are checked by measuring the cumulative total costs in correspondence of some parameters values for which we are sure that the control is applied, see Figs. (**6**) and (**7**). In both cases, a reduction of the cumulative total costs of more than 30% is obtained. In addition, as it can be seen in Figs. (**8**) and (**9**), the control using divergence allows a bullwhip reduction too for some of the T_i values considered. This is clear in the region of $\rho<0$ and $\theta>0$. When $\rho>0$ and $\theta<0$, even though there is a reduction of costs, this is not accompanied by a reduction in the bullwhip, but the bullwhip is already small in this region. The extension of these previous results to a general set of parameters can be seen in Tables **1** and **2**. In these cases we can observe that this control strategy always allows a decreasing of cumulative Total Costs but not of bullwhip effect. Anyway, as it is well known, the bullwhip along cannot be considered in general an indication of the performance of a supply chain.

As it was pointed out in [21], the functioning of the proportional controller depends on the demand model and its parameter values and care must be taken before implementing it on real-life situations. On the contrary, using the divergence the control parameter, T_i, adapts continuously, avoiding $|div| \geq 1$. In addition, the major advantage in using divergence is that, in principle, it can be calculated using only the time series of one of the variables without the model equations [23-24] and, therefore, it should be possible to extent it to real-life situations. Our future work will aim at developing an efficient algorithm for on-line control of supply chains without the need of forecasting the demand, based on state space reconstruction and divergence estimation.

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