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A Non-Random Optimization Approach to a Disposal Mechanism Under Flexibility and Reliability Criteria

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Abstract: This paper focuses on an optimal disposal mechanism by extending the work of Tripathy *et al.* [11] to consider the system cost as fuzzy under flexibility and reliability criteria. It has been observed that by using a non-random optimization technique the derived disposal plan may result in no substantial difference in the system cost between the crisp and fuzzy model. Various numerical analyses demonstrate the effectiveness of proposed method. The results of the best order size obtained by the robust study are in general, very satisfactory. Finally through numerical examples sensitivity analysis show the influence of key model parameters. Based on consistency study, fuzzy strategy is better off than that of fixed cost strategy. Through sensitivity and robust analysis the results indicate that the performance of the proposed approach is superior to that of its crisp model.

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INTRODUCTION

The EOQ model is a simple mathematical model to deal with inventory management issues in a production inventory system. It is considered to be one of the most popular inventory models used in the industry, see Silver [1], Silver *et al.* [2], Urgelleitti [3], Wagner [4], and Whitin [5]. However, these models are developed under many restrictive assumptions. There are inexhaustible number of studies that focus on imperfect production processes in inventory management, see Cheng [6, 7], Hadley *et al.* [8], Porteus [9], Rosenblatt *et al.* [10], Tapiero *et al.* [11] and Tripathy *et al.* [12] in which the items produced are of perfect quality but in reality product quality is never perfect with the production process and quality assurance established to monitor the product quality. Many studies approach this problem in a fuzzy framework, see Lee *et al.* [13], Vujosevic *et al.* [14] and Zimmermann [15]. Although there is some literature focussing attention on analytical approaches, Tripathy *et al.* [16] however, concentrated on using fuzzy optimization techniques for making in a disposal mechanism under flexibility and reliability criteria. Through very representative data our analyses show how such an approach can significantly improve fuzzy optimization in disposal mechanism. It will also show how a decision maker can dynamically access the various strategies as more information will become available during the optimization process. It is believed that the approach will be appealing because of its case of implementation. In real life situation uncertainty arises in decision making problem due to either lack of knowledge or inherent vagueness. In this paper economic order quantity in the fuzzy sense is tied with process flexibility and reliability considerations. The approach presented here is different from Tripathy *et al.* [16] in which the total cost function is expressed as:

$$Z(r,\lambda,q)=\frac{Sr}{\lambda q}+ar(1-r)^{c}\lambda^{-(b+1)}+\frac{Hq}{2r}$$
$$=F_1+F_2S+F_3H \quad (1)$$

where, $F_1=ar(1-r)^{c}\lambda^{-(b+1)}$, $F_2=\frac{r}{\lambda q}$ & $F_3=\frac{q}{2r}$

The EOQ which minimizes $Z(r,\lambda,q)$ in (1) is

$$q^{*}=\sqrt{\frac{2S}{\lambda H}}\times r \quad (2)$$

where, $Z(r,\lambda,q)$ is the total relevant cost per unit time.

The method is then relaxed and two important costs affecting the output of inventory model, namely setup cost and inventory holding cost are assumed to be fuzzy. The motivation for the study lies in the author's observation that the setup and inventory holding cost are not precisely known in practice but subjectively estimated. This puts an observational bias incorporating errors to the total cost. The computational error can be greatly reduced by the procedure of defuzzifying the fuzzy variables to compute the total cost. There are two important aspects of working with this paper, the first one being related to the development of theoretical basis for the fuzzy procedure using the work of Mahata *et al*. [17] and the other one being the numerical evaluation of this fuzzy model as applied to crisp model of Tripathy *et al.* [16] exposing its relative efficiency.

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Table 1. Major Characteristics of Inventory Models on Selected Researches

Author(s) and Published Year	Structure of the Model	Deterioration	Production Cost	Simulated Sample Studies	Demand Linked to Quality	Robust Analysis
Cheng (1991)	Crisp	No	Directly related to process reliability and demand is constant.	No	No	No
Vujosevic *et al*. (1996)	Fuzzy	No	No	No	No	No
Lee and Yao (1998)	Fuzzy	No	No	No	No	No
Tripathy *et al*. (2003)	Crisp	No	Directly related to process reliability and inversely related to demand rate.	No	Yes	No
Mahata and Goswami (2006)	Fuzzy	Yes	No	No	No	No
Tripathy *et al*. (2009)	Crisp	No	Inversely related to both process reliability and to demand rate	No	Yes	No
Present Paper (2009)	Fuzzy	No	Inversely related to both process reliability and to demand rate	Yes	Yes	Yes

In this paper a generalized mathematical model is developed in which the shortcomings of all the previous models are taken care of. The primary difference of this paper as compared to previous studies is that, we introduce a generalized inventory model by relaxing the traditional inventory model in different ways which are described in Table **1**. We then establish several theoretical results to characterize the optimal solution under various situations and prescribe five numerical examples to illustrate the given model. The adoption of this method ultimately depends on its cost savings as well as sensitivity to values of various cost parameters.

ASSUMPTIONS AND NOTATIONS

Here in the model Tripathy *et al.* [16], demand for the product exceeds supply, all items are subjected to inspection and all defective items are discarded. The unit cost of production is inversely related to both process reliability and demand according to the general power function. $P(r,\lambda) = a(1-r)^c \lambda^{-b}$ where, *a, b* and *c* are nonnegative real numbers so as to be chosen to provide the best fit of the estimated cost function. S and H be the setup cost per production run and inventory carrying cost per unit time respectively. q and λ indicate production quantity per production run and demand respectively. r and $p(r,\lambda)$ be the production process reliability and unit cost of production respectively. Similarly $Z(r,\lambda,q)$ and $M_{\tilde{TC}}(r,\lambda,q)$ be the total cost of setup, production and inventory holding per unit time in crisp and fuzzy strategy respectively.

DESCRIPTION OF THE PROPOSED MODEL

The proposed fuzzy model is the simplest and the most efficient, but the uncertainty itself is almost neglected. Since it calculates the fuzzy cost values and then determines the minimum of their defuzzifyied values. It requires less computational efforts for comparison of fuzzy numbers. The triangular fuzzy models are simple and there are no computational problems at all. However it is not easy to say which of the described models is the best or most appropriate. There is a general recommendation that it is always better to keep uncertainty in the model as long as possible. From that point of view the triangular fuzzy method is the best one. In any case one may opine that an analysis like the one presented in this paper provides the decision maker with a differ insight into the problem.

We replace the set up cost and inventory holding cost by fuzzy numbers $\tilde{S}$ and $\tilde{H}$ respectively. By expressing $\tilde{S}$ and $\tilde{H}$ as the normal triangular fuzzy numbers (S_1,S_0,S_2) and (H_1,H_0,H_2), where, $S_1=S-\Delta_1$, $S_0=S$, $S_2=S+\Delta_2$, $H_1=H-\Delta_3$, $H_0=H$, $H_2=H+\Delta_4$ such that $0<\Delta_1<S$, $0<\Delta_2$, $0<\Delta_3<H$, $0<\Delta_4$. $\Delta_1,\Delta_2,,\Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy set-up cost and fuzzy inventory holding cost are considered as:

$$\mu_{\tilde{S}}(S) = \begin{cases} \dfrac{S-S_1}{S_0-S_1} & ; \quad S_1 \le S \le S_0 \\ \dfrac{S_2-S}{S_2-S_0} & ; \quad S_0 \le S \le S_2 \\ 0 & ; \quad other\ wise \end{cases} \tag{3}$$

$$\mu_{\tilde{H}}(H)=\begin{cases}\dfrac{H-H_1}{H_0-H_1} & ; \quad H_1 \le H \le H_0 \\ \dfrac{H_2-H}{H_2-H_0} & ; \quad H_0 \le H \le H_2 \\ 0 & ; \quad other\ wise\end{cases} \tag{4}$$

Then the centroid for $\tilde{S}$ and $\tilde{H}$ are given by

$M_{\tilde{S}} = \dfrac{S_1+S_0+S_2}{3} = S + \dfrac{\Delta_2-\Delta_1}{3}$ and

$M_{\tilde{H}} = \dfrac{H_1+H_0+H_2}{3} = H + \dfrac{\Delta_4-\Delta_3}{3}$ respectively.

For fixed values of r, λ and q, let

$Z(S,H)=F_1(r,\lambda,q)+F_2(r,\lambda,q)S+F_3(r,\lambda,q)H=y$.

Let $S=\dfrac{y-F_1-F_2H}{F_2}$, $\dfrac{\Delta_2-\Delta_1}{3}=\phi_1$ and $\dfrac{\Delta_4-\Delta_3}{3}=\phi_2$

By extension principle the membership of the fuzzy cost function is given by,

$$\begin{aligned}\mu_{\tilde{Z}(\tilde{S},\tilde{H})}(y) &= \underset{(S,H)\in Z^{-1}(y)}{Sup}\left\{\mu_{\tilde{S}}(S)\wedge\mu_{\tilde{H}}(H)\right\} \\ &= \underset{H_1\le H\le H_2}{Sup}\left\{\mu_{\tilde{S}}\left(\frac{y-F_1-F_3H}{F_2}\right)V\mu_{\tilde{H}}(H)\right\}\end{aligned} \tag{5}$$

Now,

$$\mu_{\tilde{S}}\left(\frac{y-F_1-F_3H}{F_2}\right)=\begin{cases}\dfrac{y-F_1-F_2S_1-F_3H}{F_2(S_0-S_1)} & ; \quad u_2\le H\le u_1 \\ \dfrac{F_1+F_2S_2+F_3H-y}{F_2(S_2-S_0)} & ; \quad u_3\le H\le u_2 \\ 0 & ; \quad other\ wise\end{cases} \tag{6}$$

where, ... $u_1=\dfrac{y-F_1-F_2S_1}{F_3}$, $u_2=\dfrac{y-F_1-F_2S_0}{F_3}$ and

$u_3=\dfrac{y-F_1-F_2S_2}{F_3}$

When $u_2 \le H$ and $H \le u_1$ then $y \le F_1+F_2S_0+F_3H_0$ and $y \ge F_1+F_2S_1+F_3H_1$. It is clear that for every $y\in\left[F_1+F_2S_1+F_3H_1, F_1+F_2S_0+F_3H_0\right]$, $\mu_y(y)=PQ$. From the equations 3 and 6, the value of PQ may be found by solving the following equation:

$$\frac{H-H_1}{H_0-H_1}=\frac{y-F_1-F_2S_1-F_3H}{F_2(S_0-S_1)}$$

or $H=\dfrac{(y-F_1-F_2S_1)(H_0-H_1)+F_2H_1(S_0-S_1)}{F_2(S_0-S_1)+F_3(H_0-H_1)}$

Therefore,

$$PQ=\frac{H-H_1}{H_0-H_1}=\frac{y-F_1-F_2S_1-F_3H_1}{F_2(S_0-S_1)+F_3(H_0-H_1)}=\mu_1(y), \quad \text{(say)} \tag{7}$$

When $u_3 \le H$ and $H \le u_2$ then $y \le F_1+F_2S_2+F_3H_2$ and $y \ge F_1+F_2S_0+F_3H_0$. It is evident that for every $y\in\left[F_1+F_2S_0+F_3H_0, F_1+F_2S_2+F_3H_2\right]$, $\mu_{\tilde{y}}(y)=P'Q'$. From the equations 3 and 6, the value $P'Q'$ may be found by solving the following equation.

$$\frac{H_2-H}{H_2-H_0}=\frac{F_1+F_2S_2+F_3H-y}{F_2(S_2-S_0)}$$

or $H=\dfrac{F_2H_2(S_2-S_0)-(F_1+F_2S_2-y)(H_2-H_0)}{F_2(S_2-S_0)+F_3(H_2-H_0)}$

Therefore,

$$P'Q'=\frac{H_2-H}{H_2-H_0}=\frac{F_1+F_2S_2+F_3H_2-y}{(S_2-S_0)F_2+(H_2-H_0)F_3}=\mu_2(y),$$

(say) (8)

Thus the membership function for fuzzy total cost is given by

$$\mu_{\tilde{Z}(\tilde{S},\tilde{H})}(y)=\begin{cases}\mu_1(y) & ; \quad F_1+F_2S_1+F_3H_1\le y\le F_1+F_2S_0+F_3H_0 \\ \mu_2(y) & ; \quad F_1+F_2S_0+F_3H_0\le y\le F_1+F_2S_2+F_3H_2 \\ 0 & ; \quad other\ wise\end{cases} \tag{9}$$

Now let $P_1=\int_{-\infty}^{\infty}\mu_{\tilde{Z}(\tilde{S},\tilde{H})}(y)\,dy$ and $R_1=\int_{-\infty}^{\infty}y\,\mu_{\tilde{Z}(\tilde{S},\tilde{H})}(y)\,dy$.

Hence the centroid for fuzzy total cost is given by

$$\begin{aligned}M_{\tilde{TC}}(r,\lambda,q)&=\frac{R_1}{P_1}=F_1(r,\lambda,q)+F_2(r,\lambda,q)S+F_3(r,\lambda,q)H \\ &+\phi_1F_2(r,\lambda,q)+\phi_2F_3(r,\lambda,q)\end{aligned} \tag{10}$$

$M_{\tilde{TC}}(r,\lambda,q)=F_1+\left(S+\phi_1\right)F_2+\left(H+\phi_2\right)F_3$ subject to

$$0\le r\le 1 \tag{11}$$

Now for minimizing total average cost per unit time, the first partial derivative of $M_{\tilde{TC}}(r,\lambda,q)$ with respect to r, λ and q are set to zero to obtain the necessary optimality conditions:

$$a\lambda^{-(b+1)}\left(\frac{(1-r)^c-rc(1-r)^{c-1}}{r^2}\right)+\frac{1}{\lambda q}\left(S+\phi_1\right)-\frac{q}{2r^2}\left(H+\phi_2\right)=0 \tag{12}$$

$$-a(b+1)(1-r)^c\lambda^{-(b+2)}-\frac{r}{\lambda^2q}\left(S+\phi_1\right)=0 \quad \text{and}$$

$$-\frac{r}{\lambda q^2}\left(S+\phi_1\right)+\frac{1}{2r}\left(H+\phi_2\right)=0 \qquad (13)\ \&\ (14)$$

from which $r=\dfrac{1}{c+1}$ (15)

$$\lambda=\left[\frac{(S+\phi_1)(H+\phi_2)}{2a^2(b+1)^2(1-r)^{2c}r^2}\right]^{-\frac{1}{2b+1}} \text{ and } \quad q=\sqrt{\frac{2(S+\phi_1)}{\lambda(H+\phi_2)}}\times r, \quad (16) \& (17)$$

Hence,

$$M_{\tilde{TC}}(r,\lambda,q)=\frac{ac^c}{(c+1)^{c+1}}\times L^{\frac{b+1}{2b+1}}+\sqrt{2(S+\phi_1)(H+\phi_2)}\times L^{\frac{1}{2(2b+1)}}$$

where, $L=\dfrac{(S+\phi_1)(H+\phi_2)}{2a^2(b+1)^2r^2(1-r)^{2c}}$ (18)

SPECIAL CASE

After substituting the stationary points

$$r^*=\frac{1}{c+1} \quad (19)$$

$$\lambda^*=\left[\frac{(S+\phi_1)(H+\phi_2)(1+c)^{2(c+1)}}{2a^2(b+1)^2\times c^{2c}}\right]^{-\frac{1}{2b+1}} \text{ and}$$

$$q^*=\frac{1}{(1+c)}\sqrt{\frac{2(S+\phi_1)}{\lambda^*(H+\phi_2)}} \quad (20) \& (21)$$

As shown in the Appendix-A, we derive the sufficient condition for stationary points *(r*,λ*, q*)* as

$$\begin{vmatrix} f_{r^*r^*} & f_{r^*\lambda^*} & f_{r^*q^*} \\ f_{\lambda^*r^*} & f_{\lambda^*\lambda^*} & f_{\lambda^*q^*} \\ f_{q^*r^*} & f_{q^*\lambda^*} & f_{q^*q^*} \end{vmatrix}>0 \Rightarrow \begin{vmatrix} A & D & E \\ G & B & F \\ I & J & C \end{vmatrix}>0$$

OBSERVATION AND PROPOSITION

Our objectives in this section are to analyze the structure of our model in order to gain insights into its behaviour corresponding to reliability, demand rate and production lot size. For the purpose of brevity, all the proofs are omitted for the following propositions.

Proposition 1

If $r<.5$, f_r turns to be positive; which means that the curve of total average cost $M_{\tilde{TC}}(r,\lambda,q)$ is a rising one from left to right.

Proposition 2

The curve of total average cost $M_{\tilde{TC}}(r,\lambda,q)$ is concave upward in *r*, λ and *q*.

Proposition 3

The total average cost $M_{\tilde{TC}}(r,\lambda,q)$ is an increasing function with respect to H and decreasing function with respect to both λ and *q*.

Numerical Analysis

Here the following numerical examples are considered to illustrate the effectiveness of the proposed model.

Example – 1

a=10, b=1, c=1, H=5, S=100, Δ_1=0.1, Δ_2=0.2, Δ_3=0.01and Δ_4=0.02.

Table 2. Crisp and Fuzzy Values with Relative Error

H	5	10	15	20	25
r_1	0.5	0.5	0.5	0.5	0.5
λ_1	0.464158883	0.368403149	0.321829794	0.292401773	0.271441761
q_1	4.641588835	3.684031503	3.218297953	2.924017738	2.714417617
$Z_1(r,\lambda,q)$	58.01986047	92.10078765	120.6861733	146.2008872	169.6511013
$\begin{vmatrix} A & D & E \\ G & B & F \\ I & J & C \end{vmatrix}>0$	742324.5439	3803972.985	9527967.299	18238743.29	30248694.97
r^*	0.5	0.5	0.5	0.5	0.5
λ^*	0.464004269	0.368321673	0.321770211	0.292353051	0.271399846
q^*	4.64158883	3.684438951	3.218774674	2.92450502	2.71490013
$M^*_{\tilde{TC}}(r,\lambda,q)$	58.0585379	92.14164968	120.7308732	146.2496219	169.7038824
$\begin{vmatrix} A & D & E \\ G & B & F \\ I & J & C \end{vmatrix}>0$	744153.8235	3810034.092	9540793.474	18261096.16	30283571.66
R. E.	6.67×10^{-2}	4.44×10^{-2}	3.70×10^{-2}	3.33×10^{-2}	3.11×10^{-2}

Example – 2

a=10, b=1, c=1, H=10, S=100, Δ_1=0.1, Δ_2=0.2, Δ_3= 0.01and Δ_4=0.02.

Example – 3

a=10, b=1, c=1, H=15, S=100, Δ_1=0.1, Δ_2=0.2, Δ_3= 0.01and Δ_4=0.02.

Example – 4

a=10, b=1, c=1, H=120 S=100, Δ_1=0.1, Δ_2=0.2, Δ_3= 0.01and Δ_4=0.02.

Example – 5

a=10, b=1, c=1, H=25, S=100, Δ_1=0.1, Δ_2=0.2, Δ_3= 0.01and Δ_4=0.02.

Table **2** presents' five sets of solutions and obtains the computing results for each case as follows:

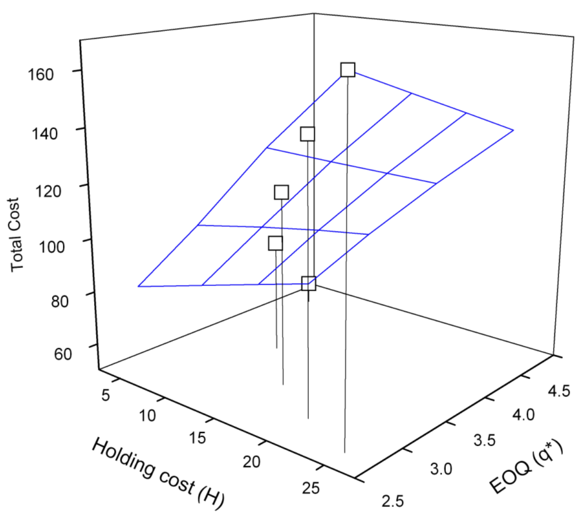

Fig. (1). The Behaviour of Total Cost to the Holding Cost and Changing Lot Size.

SENSITIVITY ANALYSIS

In Tables **3.1** to **3.5**, it can be seen that the demand λ, the optimal order quantity q and the optimal total average cost $M_{\tilde{TC}}$ are insensitive to the change of the parameters Δ_1, Δ_2, Δ_3 and Δ_4. The demand λ, the optimum order quantity q and the optimum total average cost $M_{\tilde{TC}}$ are very sensitive to the changes of the parameters *a, b* and *c*. This is a very practical aspect of the proposed fuzzy model.

The above mentioned tables illustrate the relative changes of λ, order quantity q and total cost $M_{\tilde{TC}}$, when each parameter is being changed from -60% to +60%. For all cases it also shows that the 60% decrease in parameter *b* gives a minimum optimal total average cost.

In the Tables **3.1** to **3.5**,

Relative error $\dfrac{(\lambda_* - \lambda^*)}{\lambda^*} \times 100\% = \overline{\lambda}_* \%$;

$\dfrac{(q_* - q^*)}{q^*} \times 100\% = \overline{q}_* \%$ and $\dfrac{M_{\tilde{TC}_*} - M_{\tilde{TC}^*}}{M_{\tilde{TC}^*}} \times 100\% = \overline{M}_{\tilde{TC}_*} \%$

EFFECTIVENESS OF THE TOTAL AVERAGE COST

We focus on the analysis of effectiveness of cost estimate in Table **4** which are studied in comparison. This estimates which type of relationship on decision parameters play the superior role in respect to the relative performance.

The sensitivity analysis $M_{\tilde{TC}_i}$ (*i=1,2,3,4*) is the minimum total cost of the decision parameter Δ_i and $M^*_{\tilde{TC}_i}$ is the optimum total cost for (*i=1,2,3,4*).

ROBUST

In the previous section it is observed that, the best order size q^* comes out to be the solution of the equation (18). In practice, however, the exact form of the decision parameters characterized by the numerical values of the parameters Δ_1, Δ_2, Δ_3 and Δ_4, that suitably represent variations in the proportion of the units in a particular situation may not be known in advance. The possible non availability of such records and the errors of estimates must however, be kept in mind. Hence it will be worthwhile to investigate how the best order size q^* changes with the change in any or more of the four parameters Δ_1, Δ_2, Δ_3 and Δ_4 of the given functional values of the best order size q^* for different values of Δ_1, Δ_2, Δ_3 and Δ_4 that are shown in Tables **5.1** to **5.9**. Tables **5.1(a)** to **5.9(a)** summarise the values of the ratio q'^* for several values of decision parameters, q' is the changing value of q^*.

INTERPRETATION OF THE ANALYSES

Tables **5.1** and **5.2** summarize the results of an analysis where for a fixed value of Δ_2' or Δ_1', the best order size is a decreasing function of Δ_4', while it is an increasing or decreasing function of Δ_2' or Δ_1' respectively for a fixed Δ_4'.

Combining the results of Tables **5.3** and **5.4**, for a fixed value of Δ_2' or Δ_1', the best order size is an increasing function of Δ_3', and for alternate case it is an increasing or decreasing function of Δ_2' or Δ_1' respectively for a fixed Δ_3'. Of course, the sensitivity analysis focused on changes in one decision parameter at a time.

But when several parameters are favourable to this strategy, the gains offered on this strategy remain modest. Tables **5.5** and **5.6** show for fixed values of Δ_1' and Δ_2', the best order size is an increasing function of Δ_3' or Δ_4', while it is a decreasing function of Δ_1' and Δ_2' for a fixed Δ_3' or Δ_4'. Tables **5.7** and **5.8** indicate that, for a fixed value of Δ_2'

Table 3.1. Sensitivity Analysis for H=5

Parameter	% Change	λ_*	q_*	$M_{\tilde{TC}_*}$	$\bar{\lambda}_*\%$	$\bar{q}_*\%$	$\bar{M}_{\tilde{TC}_*}\%$
a	+60	0.330082713	5.503212257	68.83601303	-28.86213877	18.56311402	18.5631184
	+40	0.251900252	6.299605492	78.79756527	-45.71165206	35.72088616	35.72089157
	+20	0.158687215	7.937005563	99.27871122	-30.5317054	70.99760133	70.99760829
	-20	0.158687215	7.937005561	59.5672266	-30.5317054	70.9976133	2.59856475
	-40	0.251900252	6.299605509	47.27853893	-45.71165206	35.72088653	-18.56746546
	-60	0.330082713	5.503212267	41.30160809	-28.86213877	18.56311424	-28.86212849
b	+60	0.286521478	5.906756443	77.57786199	-38.25024959	27.25720996	33.62007518
	+40	0.187106962	7.309416011	99.26535006	-59.67559471	57.4765943	70.97459504
	+20	0.092998806	10.36785629	146.9758825	-37.1005463	123.3686927	153.151195
	-20	0.00101424532	99.27871537	1614.354639	-99.78141466	2038.895085	2680.536716
	-40	5.87545×10^{-11}	412483.9939	7567248.203	-99.99999999	8886598.262	13033724.96
	-60	$9.81456 \times 10^{+11}$	3.1914808×10^{-6}	$7.185618956 \times 10^{-5}$	$2.115187436 \times 10^{14}$	-99.99993124	-99.99987623
c	+60	0.558208708	4.231841088	52.93327875	20.30249403	-8.827747502	-8.827744093
	+40	0.612257256	4.040737928	50.5428967	31.95078082	-12.94494028	-12.94493708
	+20	0.671539053	3.858264682	48.26046056	44.72691263	-16.87620422	-16.87620408
	-20	0.807878292	3.51766667	43.99789423	74.11009897	-24.21416892	-24.21804644
	-40	0.886101093	3.358814497	42.01317113	90.96830616	-27.63653525	-27.63653263
	-60	0.971897811	3.207135816	40.11592379	109.458808	-30.90435337	-30.90435061
Δ_1	+60	0.463983618	4.642001448	58.0637014	-0.004450605	0.008889585	0.00889361
	+40	0.63973313	4.642207674	58.06628073	-0.00667149	0.013332589	0.013336246
	+20	0.463963008	4.64241387	58.06886009	-0.008892375	0.017774947	0.017778935
	-20	0.463942402	4.642826285	58.0740185	-0.013333282	0.026660159	0.026663778
	-40	0.4639321	4.643032466	58.07659773	-0.01555352	0.031765329	0.031106243
	-60	0.463921799	4.64323867	58.07917678	-0.017773543	0.035544725	0.035548397
Δ_2	+60	0.464045471	4.640764087	58.04822388	0.008879659	-0.017768549	-0.017764863
	+40	0.464066095	4.640351597	58.04306429	0.013324446	-0.026655376	-0.026651739
	+20	0.464086724	4.639939067	58.03790443	0.01777031	-0.035543066	-0.035539079
	-20	0.464127991	4.639113991	58.02758415	0.026663978	-0.053318789	-0.053314725
	-40	0.464148631	4.638701421	58.02242341	0.031112213	-0.062207341	-0.062203581
	-60	0.464045471	4.63890752	58.02964595	0.008879659	-0.057767072	-0.049763481
Δ_3	+60	0.463963022	4.641176769	58.06885665	-0.008889357	-0.008866166	0.01777301
	+40	0.463942423	4.640970703	58.07401338	-0.013328756	-0.013317142	0.026654959
	+20	0.463921827	4.64076476	58.07916986	-0.017767508	-0.017755859	0.035536478
	-20	0.463880646	4.640352726	58.08948223	-0.026642642	-0.026631053	0.053298500
	-40	0.463860061	4.63976025	58.09463828	-0.031079024	-0.039395561	0.062179278
	-60	0.46383948	4.639554385	58.09979384	-0.035514543	-0.043830788	0.071059212
Δ_4	+60	0.464086697	4.642027162	58.03791138	0.017764491	0.009443576	-0.035527108
	+40	0.46412795	4.64243981	58.02759457	0.026655142	0.018333808	-0.053296777
	+20	0.464169219	4.642852598	58.0172767	0.035549241	0.027227056	-0.071068272
	-20	0.4642518	4.64367862	57.99663829	0.053346707	0.04502316	-0.10661586
	-40	0.464293113	4.644091849	57.9821887	0.062250289	0.053925909	-0.131503828
	-60	0.46433444	4.644505229	57.97599616	0.071156888	0.062831911	-0.142169856

Table 3.2. Sensitivity Analysis for H=10

Parameter	% Change	λ_*	q_*	$M_{\tilde{TC}_*}$	$\bar{\lambda}_*\%$	$\bar{q}_*\%$	$\bar{M}_{\tilde{TC}_*}\%$
a	+60	0.262015917	4.368387591	109.246093	-28.86220491	18.56316929	18.563205
	+40	0.199955568	5.000555492	125.0555585	-45.71170185	35.72094852	36.26859464
	+20	0.125964114	6.300304713	157.5601311	-65.80051536	70.99766876	70.99773191
	-20	0.125964114	6.300304713	94.53607831	-65.80051536	70.99766876	2.598638768
	-40	0.199955568	5.000555492	75.03332907	-45.71170185	35.72094852	-18.56741297
	-60	0.262015917	4.368387591	65.54765473	-28.86220491	18.56346929	-28.86207816
b	+60	0.209117865	4.889781292	128.3995441	-43.22412165	32.71440664	39.35016851
	+40	0.127330207	6.266416825	170.1451528	-65.42961864	70.07791168	84.65607398
	+20	0.56696898	9.39085973	266.1630496	-84.60668971	154.8789314	188.8629089
	-20	3.196447×10^{-4}	125.0694524	4066.112122	-99.91321589	3294.531815	4312.892689
	-40	1.839141×10^{-12}	1648836.106	60477477.18	-100.0000000	44751248.25	65635231.46
	-60	$3.135432201 \times 10^{13}$	3.9933400×10^{-7}	$1.797602013 \times 10^{-5}$	$8.512755102 \times 10^{15}$	-99.99998916	-99.99998049
c	+60	0.443099752	3.359187524	84.0076783	20.30238362	-8.827705692	-8.827681519
	+40	0.486002876	3.207492005	80.21402915	31.95065934	-12.94490022	-12.94487408
	+20	0.533060095	3.0626468	76.59169217	44.72677936	-16.87616892	-16.87614403
	-20	0.64128464	2.792283975	69.8303682	74.10993895	-24.21413376	-24.21411116
	-40	0.703377012	2.666188906	66.6769408	90.96813019	-27.63650202	-27.63648032
	-60	771481476	2.545788093	63.66591719	109.4586153	-30.90432148	-30.9043007
Δ_1	+60	0.368304942	3.684768192	92.14991087	-0.004542496	0.008935987	0.00896575
	+40	0.368296762	3.684931869	92.15400442	-0.006763381	0.013378373	0.01340842
	+20	0.368288582	3.685095556	92.15809801	-0.008984266	0.01782103	0.017851134
	-20	0.36872225	3.685422916	92.16628469	-0.013425221	0.026705965	0.02673602
	-40	0.368264047	3.685586594	92.17037803	-0.015645563	0.031148378	0.031178462
	-60	0.368255871	3.685750261	92.17447101	-0.017865361	0.035590493	0.035620514
Δ_2	+60	0.368321305	3.683949664	92.1318976	-0.000099912	-0.013279823	-0.010583791
	+40	0.368370411	3.683458537	92.11715886	0.013232455	-0.026609587	-0.026579532
	+20	0.368386786	3.683131088	92.10896988	0.017678297	-0.035496937	-0.035466914
	-20	0.368419543	3.682476153	92.0925911	0.026571881	-0.053272642	-0.046005872
	-40	0.368435927	3.682148658	92.084401	0.031020167	-0.062161241	-0.062131164
	-60	0.368452313	3.681821153	92.07621068	0.035468996	-0.071050111	0.07102000
Δ_3	+60	0.368304942	3.684277107	92.14991093	-0.004542496	-0.001192213	0.008965815
	+40	0.368296762	3.684195274	92.15400438	-0.006763381	-0.006613679	0.013408377
	+20	0.368288582	3.684113452	92.15809798	-0.008984266	-0.008834425	0.017851102
	-20	0.368272225	3.683949826	92.16628469	-0.013425221	-0.013275426	0.02673602
	-40	0.368264047	3.683868027	92.17037806	-0.015645563	-0.015495547	0.031178495
	-60	0.368255871	3.683786228	92.17447108	-0.017865361	-0.017715668	0.03562059
Δ_4	+60	0.368354039	3.684768248	92.12534754	0.008787427	0.008937507	-0.017692476
	+40	0.3683704	3.684932018	92.11715887	0.013232455	0.013382417	-0.026579521
	+20	0.368386786	3.685095816	92.10896986	0.017678297	0.017828087	-0.035466936
	-20	0.368419543	3.685423508	92.09259115	0.026571881	0.026722033	-0.053242513
	-40	0.368435927	3.685587392	92.08440094	0.031020167	0.031170037	-0.032131299
	-60	0.368452313	3.68575131	92.08440094	0.035468996	0.035618964	-0.071020032

Table 3.3. Sensitivity Analysis for H=15

Parameter	% Change	λ_*	q_*	M_{TC_*}	$\bar{\lambda}_*\%$	$\bar{q}_*\%$	$\bar{M}_{TC_*}\%$
a	+60	0.228900461	3.816279371	143.1422788	-28.86213416	18.56311042	18.56311067
	+40	0.174683745	4.368549356	163.8570054	-45.71164793	35.72088134	35.72088138
	+20	0.110043864	5.504027285	206.4468898	-65.80048114	70.99759512	70.99759451
	-20	0.110043864	5.504027285	123.8681337	-65.80048114	70.99759512	2.598556953
	-40	0.174633745	4.368549356	98.31420288	-45.71164793	35.72088134	-18.56747144
	-60	0.228900461	3.816279371	85.88536705	-28.86213416	18.56311042	-28.86213383
b	+60	0.173928832	4.37801961	172.4228301	-45.94626039	36.01510057	42.81585599
	+40	0.10165524	5.7266232	233.2071849	-68.40750432	77.91314335	93.16284122
	+20	0.042443803	8.862502044	376.7400381	-86.80928142	175.3377587	212.0494602
	-20	$1.62653150 \times 10^{-4}$	143.1634825	6980.770708	-99.94945053	4347.763419	5682.092453
	-40	2.423260×10^{-13}	3706555.668	204043484.6	-100.0000000	115154144.8	169006782.2
	-60	2.379646×10^{14}	$1.18360643 \times 10^{-7}$	$7.991118602 \times 10^{-6}$	$7.39548261 \times 10^{16}$	-99.99999632	-99.99999338
c	+60	0.387097619	2.934629259	110.0730525	20.30250339	-8.827751047	-8.827750862
	+40	0.424578339	2.802106107	105.1031544	31.95079112	-12.94494363	-12.94426056
	+20	0.465688127	2.798451114	100.4370351	44.72692346	-13.05849594	-16.80915375
	-20	0.560234476	2.439375052	91.49689258	74.11011239	-24.21417157	-24.21417144
	-40	0.614479168	2.329216786	87.36503957	90.96832056	-27.63653806	-27.63653804
	-60	0.673976101	2.224033094	83.4197745	109.4588243	-30.90435587	-30.90435587
Δ_1	+60	0.321755916	3.219060688	120.7416013	-0.004442611	0.008885803	0.008885962
	+40	0.321748769	3.219203692	120.7469652	-0.006663761	0.013328612	0.013328819
	+20	0.321741623	3.219346691	120.7523288	-0.008884601	0.017771265	0.017771427
	-20	0.321727333	3.219632678	120.7630557	-0.013325658	0.02665623	0.026656396
	-40	0.32172019	3.219775661	120.7684188	-0.015545565	0.031098386	0.03109859
	-60	0.321713046	3.21991865	120.7737821	-0.017765783	0.035540729	0.03554095
Δ_2	+60	0.321798808	3.218202616	120.7094164	0.008887398	-0.017772539	-0.017772421
	+40	0.32181311	3.217916569	120.6986873	0.013332185	-0.026659368	-0.026659212
	+20	0.321827415	3.217630507	120.6879576	0.017777904	-0.035546663	-0.0355465
	-20	0.321856033	3.217201375	120.6664968	0.026671828	-0.048878817	-0.053322235
	-40	0.321870346	3.21677224	120.6557654	0.031120034	-0.062211064	-0.06221093
	-60	0.321884661	3.216486128	120.6450339	0.035568861	-0.071099913	-0.071099709
Δ_3	+60	0.321760679	3.218679335	120.7380263	-0.002962362	-0.002961965	0.00592483
	+40	0.321755914	3.218631666	120.7416026	-0.004443232	-0.004442932	0.008887039
	+20	0.321751149	3.218584001	120.7451788	-0.005924103	-0.005923775	0.011849164
	-20	0.32174162	3.218488679	120.7523312	-0.008885533	-0.008885213	0.017773415
	-40	0.321736856	3.218417195	120.7559072	-0.010366093	-0.011106058	0.020735375
	-60	0.321732092	3.21839337	120.7594833	-0.011846652	-0.011846247	0.023697418
Δ_4	+60	0.321789276	3.218965402	120.7165468	0.005925035	0.005925484	-0.011866393
	+40	0.321798811	3.219060778	120.7094141	0.00888833	0.008888599	-0.017774326
	+20	0.321808347	3.219156167	120.7022605	0.01151936	0.011852118	-0.023699571
	-20	0.321827422	3.219346979	120.6879527	0.017780079	0.017780213	-0.035550558
	-40	0.321836961	3.219442403	120.6807985	0.020744617	0.020744816	-0.0414763
	-60	0.321846501	3.219537839	120.6736442	0.023709466	0.023709798	-0.047402125

Table 3.4. Sensitivity Analysis for H=20

Parameter	% Change	λ_*	q_*	$M_{\tilde{TC}_*}$	$\bar{\lambda}_*\%$	$\bar{q}_*\%$	$\bar{M}_{\tilde{TC}_*}\%$
a	+60	0.207973722	3.467384111	173.3981002	-28.86213389	180.56311025	18.56311008
	+40	0.158713654	3.969163978	158.7930202	-45.7116478	35.720881	8.576704772
	+20	0.099983337	5.000833246	250.0833357	-65.80048108	70.99759487	70.9975947
	-20	0.099983337	5.000833246	150.0500014	-65.80048108	70.99759487	2.598556804
	-40	0.158713654	3.969163978	119.094765	-45.7116478	35.720881	-18.56747152
	-60	0.207973722	3.467384111	104.0388601	-28.86213389	18.56311025	-28.86213397
b	+60	0.152613239	4.047716449	212.5405312	-47.79830808	38.4068901	45.32723465
	+40	0.086642856	5.372047042	291.6740152	-70.36232176	83.69081281	99.43573967
	+20	0.034561259	8.505726406	482.0714975	-88.17824583	190.8432828	229.6223889
	-20	1.007097 3 x 10^{-4}	157.5688882	10243.6848	-99.96555202	5287.882295	6904.247031
	-40	5.7521076 x 10^{-14}	6593145.89	483577948.4	-100.000000	225444747.8	330652343.5
	-60	1.0025027 x 10^{15}	4.994171025 x 10^{-8}	4.495503022 x 10^{-6}	3.429082394 x 10^{-17}	-99.99999829	-99.99999693
c	+60	0.35170804	2.666336992	133.339069	20.3025037	-8.827751234	-8.827751301
	+40	0.385762163	2.545929494	127.3176908	31.95079089	-12.94494362	-12.94494362
	+20	0.423113577	2.430959397	121.568228	44.72692368	-16.87621049	-16.8762104
	-20	0.509016226	2.216360353	110.8364874	74.1101125	-24.2141717	-24.21417193
	-40	0.558301712	2.116273075	105.8312892	90.9683207	-27.63653813	-27.63653839
	-60	0.612359264	2.020705578	101.052118	109.4588245	-30.90435598	-30.90435607
Δ_1	+60	0.292340063	2.924764884	146.2626172	-0.004442573	0.008885742	0.008885698
	+40	0.292333569	2.924894816	146.269115	-0.00666386	0.13328614	0.013328649
	+20	0.292327077	2.925024738	146.2756121	-0.008884463	0.017771144	0.017771122
	-20	0.292314094	2.925284577	146.2938042	-0.013325326	0.026656032	0.030210197
	-40	0.292307603	2.925414494	146.2951031	-0.015545587	0.03109839	0.031098336
	-60	0.292301112	2.92554441	146.3016001	-0.017765848	0.035540715	0.035540741
Δ_2	+60	0.292379033	2.923985263	146.2236298	0.008887199	-0.017772477	-0.017772422
	+40	0.292392028	2.923725365	146.2106327	0.013332168	-0.026659383	-0.026659351
	+20	0.292405026	2.923465451	146.1976347	0.017778162	-0.035546835	-0.035546895
	-20	0.292431027	2.922945599	146.1716378	0.026671861	-0.053322561	-0.053322599
	-40	0.292444031	2.922685655	146.1586385	0.031119907	-0.06221104	-0.062211032
	-60	0.292457038	2.922425696	146.1456383	0.03556898	-0.071100031	-0.071100081
Δ_3	+60	0.292346555	2.924440048	146.2561209	-0.00222197	-0.00222164	0.004443772
	+40	0.292343308	2.92440756	146.2593701	-0.003332614	-0.003332529	0.006665453
	+20	0.29234006	2.924375079	146.2626197	-0.004443599	-0.004443179	0.008887407
	-20	0.292333566	2.924310115	146.2691182	-0.006664886	-0.006664546	0.013330837
	-40	0.292330319	2.924277636	146.2723674	-0.00777553	-0.007775127	0.015552518
	-60	0.292327073	2.924245154	146.2756163	-0.008885831	-0.008885811	0.017773994
Δ_4	+60	0.292366043	2.92463499	146.2366239	0.004443941	0.00444417	-0.008887544
	+40	0.29237254	2.924699983	146.2301247	0.006666255	0.006666529	-0.013331453
	+20	0.292379038	2.92476498	146.2236252	0.00888891	0.008889025	-0.017775567
	-20	0.292392035	2.924894995	146.210626	0.013334562	0.013334735	-0.026663932
	-40	0.292398534	2.924960012	146.2041263	0.015557559	0.015557914	-0.031108182
	-60	0.292405034	2.925025034	146.1976263	0.017780898	0.017781265	-0.035552638

Table 3.5. Sensitivity Analysis for H=25

Parameter	% Change	λ_*	q_*	M_{TC_*}	$\bar{\lambda}_*\%$	$\bar{q}_*\%$	$\bar{M}_{TC_*}\%$
a	+60	0.193067846	3.218870033	201.2062008	-28.8622124	18.5631102	18.56311002
	+40	0.147338342	3.68468637	230.4236371	-45.71170759	35.72088083	35.77982651
	+20	0.092817339	4.642413927	290.1895605	-65.80051891	70.99759493	70.99759675
	-20	0.092817339	4.642413927	174.1137341	-65.80051891	70.99759493	2.598556755
	-40	0.147338342	3.68468637	138.1941622	-45.71170759	35.72088083	-18.56747162
	-60	0.193067846	3.218870033	120.7237204	-28.8622124	18.5631102	-28.86213403
b	+60	0.137895084	3.808763664	249.983442	-49.19117087	40.29111502	47.3056706
	+40	0.07654237	5.112195627	346.9452425	-71.79719476	88.30142481	104.441547
	+20	0.029469886	8.238899195	583.6665077	-89.14152442	203.4696969	245.110848
	-20	6.9435072×10^{-5}	169.7340493	13792.73029	-99.97441595	6151.944498	8027.527841
	-40	$1.8851647 \times 10^{-14}$	10301103.9	944393761.2	-100.000000	379428365.4	556494983.0
	-60	3.0588859×10^{15}	2.5572711×10^{-8}	$2.87731355 \times 10^{-6}$	$1.127077242 \times 10^{18}$	-99.99999906	-99.9999983
c	+60	0.326500449	2.475235503	154.7228459	20.30237077	-8.827751134	-8.827751191
	+40	0.358113848	2.363457841	147.7358105	31.95064525	-12.94494354	-12.94494362
	+20	0.392788214	2.256727874	141.0642982	44.72676377	-16.87621032	-16.87621037
	-20	0.472534056	2.057509553	128.6114929	74.10992046	-24.21417163	-24.21417172
	-40	0.518287156	1.964595721	122.8036041	90.96810983	-27.63653811	-27.63653821
	-60	0.5684703	1.875877729	117.2579903	109.4585934	-30.90435599	-30.90435608
Δ_1	+60	0.271387489	2.715141371	169.7189619	-0.00455306	0.008885814	0.008885771
	+40	0.271381461	2.715261988	169.7265015	-0.006774137	0.013328593	0.013328569
	+20	0.271375434	2.715382599	169.7340407	-0.008994846	0.01777115	0.017771131
	-20	0.271363381	2.715623817	169.7491188	-0.013435895	0.026656118	0.026656078
	-40	0.271357356	2.715744419	169.7566573	-0.015655867	0.031098344	0.031098227
	-60	0.27135133	2.715865025	169.7641964	-0.017876207	0.035540718	0.03554073
Δ_2	+60	0.271423667	2.714417622	169.6737215	0.008777086	-0.017772587	-0.017772663
	+40	0.27143573	2.714176353	169.6586403	0.01322182	-0.026659433	-0.026659437
	+20	0.271447796	2.71393507	169.6435581	0.017667659	-0.035546795	-0.0355468
	-20	0.271471934	2.713452474	169.6133917	0.026561547	-0.053322624	-0.053322704
	-40	0.271484006	2.713211161	169.5983076	0.031009597	-0.062211091	-0.062211187
	-60	0.27149608	2.712969838	169.583223	0.035458384	-0.071099926	-0.071099964
Δ_3	+60	0.271394722	2.714851875	169.7099155	-0.001887989	-0.001777413	0.003555074
	+40	0.27139231	2.714827749	169.712932	-0.002776714	-0.002666064	0.005332582
	+20	0.271389898	2.714803623	169.7159486	-0.00366544	-0.003554716	0.007110149
	-20	0.271385075	2.714755371	169.7219812	-0.005442523	-0.005332019	0.0102664929
	-40	0.271382663	2.71473125	169.7249978	-0.006331248	-0.006220486	0.012442496
	-60	0.271380252	2.714707125	169.7439201	-0.007219606	-0.007109101	0.023592683
Δ_4	+60	0.271409195	2.714996657	169.6918162	0.003444372	0.003555453	-0.007110149
	+40	0.27141402	2.715044924	169.6857828	0.005222552	0.005333308	-0.010665401
	+20	0.271418846	2.715093192	169.6797491	0.00700074	0.0071112	-0.014220829
	-20	0.271428498	2.715189743	169.6676817	0.0028652	0.010667537	-0.021331686
	-40	0.271433324	2.715238026	169.6616481	0.012335305	0.012445982	-0.024887055
	-60	0.271438151	2.715286309	169.655614	0.014113862	0.014224427	-0.028442719

Table 4.

Conditions	$M_{\tilde{TC}}$	$M^*_{\tilde{TC}}$ for H=5	$M^*_{\tilde{TC}}$ for H=10	$M^*_{\tilde{TC}}$ for H=15	$M^*_{\tilde{TC}}$ for H=20	$M^*_{\tilde{TC}}$ for H=25
$\Delta_1 < \Delta_2$	Min $\left\{M_{\tilde{TC}_1}, M_{\tilde{TC}_2}\right\}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$
$\Delta_1 > \Delta_2$	Min $\left\{M_{\tilde{TC}_1}, M_{\tilde{TC}_2}\right\}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$
$\Delta_1 = \Delta_2$	Min $\left\{M_{\tilde{TC}_1}, M_{\tilde{TC}_2}\right\}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$	$M_{\tilde{TC}_2}$
$\Delta_3 < \Delta_4$	Min $\left\{M_{\tilde{TC}_3}, M_{\tilde{TC}_4}\right\}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$
$\Delta_3 > \Delta_4$	Min $\left\{M_{\tilde{TC}_3}, M_{\tilde{TC}_4}\right\}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$
$\Delta_3 = \Delta_4$	Min $\left\{M_{\tilde{TC}_3}, M_{\tilde{TC}_4}\right\}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$	$M_{\tilde{TC}_4}$

Table 5.1. Values of the Best Order Quantity q^* (H=5)

Δ'_4 \ Δ'_2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.03	4.641589525	4.642620244	4.643650849	4.644681338	4.645711717	4.646741981	4.647772129	4.648802163
0.04	4.632860263	4.641075536	4.642620758	4.643651020	4.644681668	4.645711200	4.646741122	4.647770929
0.05	4.639531168	4.640561430	4.641591581	4.642621613	4.643651534	4.644681340	4.645711030	4.646740606
0.06	4.638503357	4.639533392	4.640563312	4.641593117	4.642622812	4.643652387	4.644681851	4.645711200
0.07	4.637476460	4.638506264	4.639535958	4.640565537	4.641595001	4.642624350	4.643653583	4.644682706
0.08	4.636450471	4.637480049	4.638509513	4.639538861	4.640568099	4.641597221	4.642626229	4.643655121
0.09	4.635425388	4.636454736	4.637483973	4.638513095	4.639542103	4.640570999	4.641599781	4.642628447
0.10	4.634401209	4.635430330	4.636459342	4.637488239	4.638517021	4.639545687	4.640574238	4.641602679

Table 5.1.a. Values of the Ratio q'^* (H=5)

Δ'_4 \ Δ'_2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.03	1.000000150	1.000222202	1.000444249	1.000666261	1.000888249	1.001110213	1.001332151	1.001554065
0.04	0.998119487	0.999889414	1.000222322	1.000444285	1.000666224	1.000888138	1.001110028	1.001331893
0.05	0.999556690	0.999778653	1.000000593	1.000222506	1.000444396	1.000666261	1.000888101	1.001109916
0.06	0.999335255	0.999557169	0.999779058	1.000000924	1.000222765	1.00044458	1.000666371	1.000888138
0.07	0.999114016	0.999335881	0.999557722	0.999779538	1.000001330	1.000223096	1.000444838	1.000666555
0.08	0.998892973	0.999114790	0.999336581	0.999558347	0.999780090	1.000001808	1.000223501	1.000445169
0.09	0.998672126	0.998893893	0.999115635	0.999337353	0.999559046	0.999780715	1.000002359	1.000223979
0.10	0.998451474	0.998673191	0.998894885	0.999116554	0.999338198	0.999559818	0.999781412	1.000002984

Table 5.2. Values of the Best Order Quantity q^* (H=10)

Δ_4' \ Δ_2'	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.03	3.683213193	3.682394654	3.681576025	3.680757307	3.679938498	3.679119594	3.678300601	3.677481518
0.04	3.682804309	3.681985864	3.681167323	3.680144409	3.679529977	3.678711166	3.677892266	3.677073271
0.05	3.682395608	3.681577250	3.680758808	3.679940270	3.679121638	3.678302920	3.677484108	3.676665206
0.06	3.681987088	3.681168824	3.680350469	3.679532020	3.678713481	3.677894852	3.677076132	3.676257324
0.07	3.681578751	3.680760574	3.679942308	3.679123957	3.678305505	3.677486969	3.676668338	3.675849617
0.08	3.68117059	3.680352507	3.679534333	3.678716070	3.677897712	3.677079263	3.676260725	3.675442098
0.09	3.680762615	3.679944625	3.679126540	3.678308364	3.677490099	3.676671744	3.675853294	3.684232319
0.10	3.680354822	3.679536920	3.678718927	3.677900840	3.677082668	3.676264400	3.675446043	3.674627597

Table 5.2.a. Values of the Ratio q'^* (H=10)

Δ_4' \ Δ_2'	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.03	0.999667314	0.999445153	0.999222968	0.999000758	0.998778524	0.998556264	0.998333979	0.998111671
0.04	0.999556339	0.999334203	0.999112041	0.998834410	0.998667646	0.998445411	0.998223152	0.998000867
0.05	0.999445412	0.999223300	0.999001166	0.998779005	0.998556818	0.998334609	0.998112373	0.997890114
0.06	0.999334535	0.999112449	0.998890338	0.998668201	0.998446040	0.998223854	0.998001644	0.997779410
0.07	0.999223708	0.999001645	0.998779558	0.998557448	0.998335310	0.998113150	0.997890964	0.997668753
0.08	0.999112928	0.998890891	0.998668829	0.998446742	0.998224630	0.998002494	0.997780333	0.997558148
0.09	0.999002199	0.998780187	0.998558149	0.998336086	0.998114000	0.997891888	0.997669751	0.999943917
0.10	0.998891519	0.998669531	0.998447518	0.998225479	0.998003418	0.997781330	0.997559219	0.997337083

Table 5.3. Values of the Best Order Quantity q^* (H=15)

Δ_3' \ Δ_2'	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.02	3.219728149	3.220443127	3.221158025	3.221872847	3.222587585	3.223302247	3.224016830	3.224731332
0.03	3.219966680	3.220681713	3.221396665	3.222111538	3.222826330	3.223541047	3.224255679	3.224970236
0.04	3.220205287	3.220920370	3.221635377	3.222350299	3.223065146	3.223779913	3.224494605	3.225209211
0.05	3.220443961	3.221159098	3.221874155	3.222589137	3.223304034	3.224018855	3.224733597	3.225448258
0.06	3.220682705	3.221397897	3.222113009	3.222828040	3.223542992	3.224257868	3.224972660	3.225687376
0.07	3.220921520	3.221636767	3.222351929	3.223067015	3.223782022	3.224496948	3.225211794	3.225926565
0.08	3.221160412	3.221875709	3.222590925	3.223306062	3.224021123	3.224736099	3.225451000	3.226165820
0.09	3.221399370	3.222114717	3.222829988	3.223545179	3.224260290	3.224975326	3.225690277	3.226405152

Table 5.3.a. Values of the Ratio q'^* (H=15)

Δ_3' \ Δ_2'	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.02	1.000296223	1.000518350	1.000740453	1.000962532	1.001184585	1.001406614	1.001628619	1.001850598
0.03	1.003703290	1.000592474	1.000814593	1.001036688	1.001258757	1.001480804	1.001702823	1.00192482
0.04	1.000444459	1.000666619	1.000888755	1.001110865	1.001332952	1.001555014	1.001777052	1.001999064
0.05	1.000518609	1.000740786	1.000962938	1.001185067	1.001407169	1.001629248	1.001851302	1.002073331
0.06	1.000592782	1.000814976	1.001037145	1.001259289	1.001481408	1.001703504	1.001925573	1.002147619
0.07	1.000666976	1.000891870	1.001111372	1.001333533	1.001555669	1.001777780	1.001999867	1.002221930
0.08	1.000741194	1.000963421	1.001185622	1.001407799	1.001629952	1.001852079	1.002074182	1.002296261
0.09	1.000815433	1.001037675	1.001259894	1.001482087	1.001926401	1.001926401	0.986819404	1.002370616

Table 5.4. Values of the Best Order Quantity q^* (H=20)

Δ_2' \ Δ_1'	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.02	2.924017742	2.923367921	2.922718035	2.922068069	2.921418037	2.920767926	2.920117750	2.91946750
0.03	2.924180203	2.923530349	2.922880426	2.922230428	2.921580354	2.920930211	2.920279998	2.91962971
0.04	2.924342702	2.923692811	2.923042851	2.922392816	2.921742710	2.921092530	2.920442280	2.919791955
0.05	2.924505241	2.923855313	2.923205315	2.922555243	2.921905101	2.921254883	2.920604596	2.919954239
0.06	2.924667813	2.924017849	2.923611621	2.92271771	2.922067530	2.921417276	2.920766951	2.920116557
0.07	2.924830420	2.924180419	2.923530352	2.922880205	2.922229994	2.921579702	2.920929346	2.920278910
0.08	2.924993062	2.924343028	2.923692924	2.923042745	2.922392492	2.921742168	2.921091769	2.920441301
0.09	2.925155742	2.924505671	2.923855530	2.923205314	2.922555029	2.921904668	2.921254237	2.920603732

Table 5.4.a. Values of the Ratio q'^* (H=20)

Δ_3' \ Δ_1'	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.02	0.999833381	0.999611182	0.999388961	0.999166713	0.998944442	0.998722144	0.998499824	0.998277479
0.03	0.999888932	0.999666722	0.999444489	0.999222230	0.998999945	0.998777636	0.998555303	0.998332945
0.04	0.999944497	0.999722274	0.999500028	0.999277756	0.999277756	0.999055460	0.998610794	0.998388423
0.05	1.000000076	0.99977784	0.999555581	0.999333296	0.999110988	0.998888654	0.998666296	0.998443914
0.06	1.000055665	0.999833417	0.999694512	0.999388850	0.999166529	0.998944182	0.998721811	0.998499416
0.07	1.000111267	0.999889006	0.999666723	0.999444413	0.999222081	0.998999722	0.998777340	0.998554931
0.08	1.000166880	0.999944608	0.999722313	0.999499992	0.999277646	0.999055275	0.998832879	0.998610459
0.09	1.000222507	1.000000223	0.999777914	0.999555580	0.999333223	0.999110840	0.998884333	0.998666000

Table 5.5. Values of the Best Order Quantity q^* (H=5)

Δ_3' \ Δ_1', Δ_2'	**0.2,1.0**	**0.3,0.9**	**0.4,0.8**	**0.5, 0.7**	**0.6, 0.6**	**0.7,0.5**	**0.8,0.4**	**0.9,0.3**
0.02	4.649836887	4.647775558	4.645713778	4.643651537	4.641588835	4.639525679	4.637462063	4.634307484
0.03	4.650870642	4.648808859	4.646746615	4.644683915	4.64262076	4.642027402	4.638493065	4.636428530
0.04	4.651905321	4.649843074	4.647780375	4.645717216	4.643653601	4.641589522	4.639524988	4.637459994
0.05	4.652940916	4.650878214	4.648815056	4.646751437	4.644687363	4.642622824	4.640557830	4.638492377
0.06	4.653977439	4.651914276	4.649850658	4.647786580	4.645722041	4.643657047	4.641591593	4.639525680
0.07	4.655014881	4.652951259	4.650887181	4.648822642	4.646757648	4.644692193	4.642626280	4.640559901
0.08	4.656053251	4.653989173	4.651924630	4.649859635	4.647794176	4.645728261	4.643661886	4.641595048
0.09	4.657092551	4.655028007	4.652963007	4.650897547	4.648831632	4.646765251	4.644698416	4.642631121

Table 5.5.a. Values of the Ratio q'^* (H=5)

Δ_3' \ Δ_1', Δ_2'	**0.2,1.0**	**0.3,0.9**	**0.4,0.8**	**0.5, 0.7**	**0.6, 0.6**	**0.7,0.5**	**0.8,0.4**	**0.9,0.3**
0.02	1.001776990	1.001332890	1.000888693	1.000444397	1.000000001	0.999555507	0.9991109150	0.9984321281
0.03	1.001999706	1.001555508	1.001111211	1.000666816	1.000222323	1.000094487	0.9993330230	0.9988882470
0.04	1.002222621	1.001778323	1.001333928	1.000889434	1.000444841	1.000000149	0.9995553580	0.9991104690
0.05	1.002445733	1.002001337	1.001556843	1.001112250	1.000667559	1.000222767	0.9997778770	0.9993328890
0.06	1.002669045	1.002224550	1.001779957	1.001335265	1.000890258	1.000445584	1.0000005950	0.9995555070
0.07	1.002892555	1.002447961	1.002003269	1.001558478	1.001113588	1.000668599	1.0002235120	0.9997783240
0.08	1.003116265	1.002671573	1.002226781	1.001781891	1.001336901	1.000891813	1.0004466260	1.0000013400
0.09	1.003340175	1.002895383	1.002450492	1.002005502	1.001560414	1.001115226	1.0006699400	1.0002245550

Table 5.6. Values of the Best Order Quantity q^* (H=10)

Δ_4' \ Δ_1', Δ_2'	0.5,1.1	0.6,1.0	0.7,0.9	0.8,0.8	0.9,0.7	1.0,0.6	1.1,0.5	1.2,0.4
0.10	3.685260326	3.683625522	3.681990352	3.680354822	3.678718927	3.677082668	3.675446043	3.673809055
0.09	3.685668667	3.684033677	3.682398326	3.680762615	3.679126540	3.677490099	3.675853294	3.674216125
0.08	3.686077184	3.684442018	3.682806487	3.681170590	3.679534333	3.677897712	3.676260725	3.674623375
0.07	3.686485889	3.684850536	3.683214824	3.681578751	3.679942308	3.678305505	3.676668338	3.675030807
0.06	3.686894769	3.685259236	3.683623342	3.681987088	3.680350469	3.678713481	3.677076132	3.675438420
0.05	3.687303832	3.685668123	3.684032048	3.682395608	3.680758808	3.679121638	3.677484108	3.675846215
0.04	3.687713077	3.686077187	3.684440931	3.682804309	3.681167323	3.679529977	3.677892266	3.676254186
0.03	3.688122505	3.686486433	3.684849991	3.683213193	3.681576025	3.679938498	3.678300601	3.676662345

Table 5.6.a. Values of the Ratio q'^* (H=10)

Δ_4' \ Δ_1', Δ_2'	0.5,1.1	0.6,1.0	0.7,0.9	0.8,0.8	0.9,0.7	1.0,0.6	1.1,0.5	1.2,0.4
0.10	1.000222931	0.999779225	0.999335421	0.998891519	0.998447518	0.998003418	0.997559219	0.997114921
0.09	1.000333759	0.999890003	0.999446150	0.999273611	0.998558963	0.998114000	0.997669751	0.997225405
0.08	1.000444636	1.000000832	0.999556930	0.999112928	0.998668829	0.998224630	0.997780333	0.997335937
0.07	1.000555630	1.000111709	0.999667757	0.999223708	0.998779558	0.998335310	0.997890964	0.997446519
0.06	1.000666538	1.000222635	0.999778634	0.999334535	0.998890338	0.998446040	0.998001644	0.997557150
0.05	1.000777562	1.000333612	0.999889561	0.999445412	0.999001166	0.998556818	0.998112373	0.997667830
0.04	1.000888636	1.000444636	1.000000537	0.999556339	0.999112041	0.998667646	0.998223152	0.997778558
0.03	1.000999760	1.000555711	1.000555711	1.000111561	0.999667314	0.998778524	0.998333979	0.997889337

Table 5.7. Values of the Best Order Quantity q^* (H=15)

Δ_3', Δ_4' \ Δ_2'	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
0.04,0.3	3.218544557	3.217831424	3.217118210	3.216404922	3.215691549	3.214978100	3.214264576	3.213550967
0.05,0.2	3.221156239	3.220442526	3.219728739	3.219014871	3.218300923	3.217586895	3.216872787	3.216158599
0.06,0.1	3.223776425	3.223062131	3.222347758	3.221633309	3.220918781	3.220204172	3.219489483	3.218774719
0.07,0.09	3.224253736	3.223539339	3.222824861	3.222110308	3.221395670	3.220680956	3.219966163	3.219251290
0.08,0.08	3.224731332	3.224016830	3.223302247	3.222587585	3.221872847	3.219012731	3.220443127	3.219728149
0.09,0.07	3.225209211	3.224494605	3.223779913	3.223065146	3.222350299	3.220920012	3.220920370	3.220205287
0.1,0.06	3.225687376	3.224972660	3.224257868	3.223542992	3.222828040	3.222113009	3.221397897	3.220682705
0.2,0.05	3.228322353	3.227503373	3.226891674	3.226176214	3.225460680	3.224745064	3.224029369	3.223313594

Table 5.7.a. Values of the Ratio q'^* (H=15)

Δ_3', Δ_4' \ Δ_2'	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
0.04,0.3	0.999928507	0.999706953	0.999485374	0.999263772	0.999042143	0.998820490	0.998598815	0.998377113
0.05,0.2	1.000739898	1.000518164	1.000296406	1.000074624	0.999852816	0.999630984	0.999409127	0.999187245
0.06,0.1	1.001553930	1.001332015	1.001110076	1.000888113	1.000666125	1.000444112	1.000222075	1.000000014
0.07,0.09	1.001702220	1.001480273	1.001258301	1.001036306	1.000814284	1.000592238	1.000370169	1.000148074
0.08,0.08	1.001850598	1.001628619	1.001406614	1.001184585	1.000962532	1.000073959	1.000518350	1.000296223
0.09,0.07	1.001999064	1.001777052	1.001555014	1.001332952	1.001110865	1.000666508	1.000666619	1.000444459
0.1,0.06	1.002147619	1.001925573	1.001703504	1.001481408	1.001259289	1.001037145	1.000814976	1.000592782
0.2,0.05	1.002966246	1.002711808	1.002521767	1.002299490	1.002077190	1.001854864	1.001632514	1.001410139

Table 5.8. Values of the Best Order Quantity q^* (H=20)

Δ_3', Δ_4' \ Δ_1'	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**
0.3,0.10	2.922718577	2.923369555	2.924020459	2.924671287	2.925322046	2.925972730	2.926623344	2.927273888
0.4,0.09	2.924512860	2.925164233	2.925815536	2.926466764	2.927117923	2.927769011	2.928420025	2.929070963
0.5,0.08	2.926311550	2.926963328	2.927615032	2.928266665	2.928918224	2.929569707	2.930221121	2.93087246
0.6,0.07	2.928114680	2.928766859	2.929418964	2.930070993	2.930722953	2.931374842	2.932026657	2.932678402
0.7,0.06	2.929922259	2.930574840	2.931227346	2.931879782	2.932532144	2.933184435	2.933836651	2.934488798
0.8,0.05	2.931734311	2.932387294	2.933040208	2.933693046	2.934345810	2.934998504	2.935651123	2.936303672
0.9,0.04	2.933550854	2.934204241	2.934857558	2.935510800	2.936163972	2.936817069	2.937470096	2.938123048
1.0,0.03	2.935371909	2.936025705	2.936679426	2.937330710	2.937986647	2.938640154	2.939293580	2.939946941

Table 5.8.a. Values of the Ratio q'^* (H=20)

Δ_3', Δ_4' \ Δ_1'	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**
0.3,0.10	0.999389146	0.999611741	0.999834310	1.000056853	1.002793720	1.000501866	1.000724336	1.000946782
0.4,0.09	1.000002681	1.000225410	1.000448115	1.000670795	1.000893451	1.001116083	1.001338690	1.00156127
0.5,0.08	1.000617722	1.000830315	1.001063421	1.00128625	1.001509043	1.001731810	1.001954553	1.002177271
0.6,0.07	1.003715805	1.001457286	1.001680265	1.001903219	1.002126149	1.002349055	1.002571935	1.002794792
0.7,0.06	1.001852361	1.002075503	1.002298620	1.002521713	1.002744780	1.002967824	1.003190841	1.003413835
0.8,0.05	1.002471971	1.002695051	1.002918507	1.003141737	1.003364942	1.003588123	1.003811278	1.004034410
0.9,0.04	1.003093116	1.003316534	1.003539928	1.003763297	1.003986641	1.004209960	1.004433255	1.004656524
1.0,0.03	1.003715805	1.003939362	1.004162886	1.004385593	1.004609883	1.004833342	1.005056774	1.005280183

Table 5.9. Values of the Best Order Quantity q^* (H=25)

Δ_3', Δ_4' \ Δ_1', Δ_2' .	**0.2, 0.25**	**0.3, 0.3**	**0.4, 0.35**	**0.5, 0.4**	**0.6,0.45**	**0.7, 0.5**	**0.8, 0.55**	**0.9, 0.6**
0.02, 0.025	2.714658890	2.714357303	2.714055697	2.713754075	2.713452439	2.713150788	2.712849116	2.712547430
0.03, 0.030	2.714719211	2.714417620	2.714116008	2.713814382	2.713512736	2.713211079	2.712909398	2.712607706
0.04, 0.035	2.714779541	2.714477940	2.714176323	2.713874692	2.713573040	2.713271374	2.712969687	2.712667991
0.05, 0.040	2.714839879	2.714538272	2.714236646	2.713935004	2.713633348	2.713331676	2.713029985	2.712728278
0.06, 0.045	2.714900219	2.714598603	2.714296971	2.713995325	2.711860384	2.713512589	2.713090285	2.712788569
0.07, 0.05	2.714960563	2.714658942	2.714357305	2.714055653	2.716109593	2.713452295	2.713150589	2.712848867
0.08, 0.055	2.715020915	2.714719288	2.714417642	2.714115985	2.713814304	2.713512612	2.713210900	2.712909174
0.09, 0.06	2.715081270	2.714779638	2.714477987	2.714176320	2.713874633	2.713572937	2.713271215	2.712969484

Table 5.9.a. Values of the Ratio q'^* (H=25)

Δ_3', Δ_4' \ Δ_1', Δ_2' .	**0.2, 0.25**	**0.3, 0.3**	**0.4, 0.35**	**0.5, 0.4**	**0.6,0.45**	**0.7, 0.5**	**0.8, 0.55**	**0.9, 0.6**
0.02, 0.025	0.999911142	0.999800056	0.999688963	0.999577864	0.999466760	0.999355651	0.999244534	0.999133412
0.03, 0.030	0.999933360	0.999822273	0.999711178	0.999600078	0.999488970	0.999400067	0.999266738	0.999155613
0.04, 0.035	0.999955582	0.999844491	0.999733394	0.999622292	0.999511182	0.999400067	0.999288945	0.999177819
0.05, 0.040	0.999977807	0.999866714	0.999755613	0.999644507	0.999533396	0.999422279	0.999311155	0.999200025
0.06, 0.045	1.000000033	0.999888936	0.999777833	0.999666726	0.998880347	0.999488916	0.999333365	0.999222232
0.07, 0.05	1.000022260	0.999911161	0.999800057	0.999688947	1.000445491	0.999466707	0.999355578	0.999244442
0.08, 0.055	1.000044401	0.999933389	0.999822281	0.999711169	0.999600049	0.999488924	0.999377793	0.999266655
0.09, 0.06	1.000066721	0.999955618	0.999844508	0.999733393	0.999622270	0.999511144	0.999400009	0.99928887

Table 6.1. Simulated Sampling Distribution

H	Sample Size n	Mean $\bar{x}$	Standard Deviation σ	Skewness	Kurtosis
5	10	4.365199	0.162544	0.016865	1.851675
	25	4.427767	0.134044	0.290297	2.139292
	50	4.372178	0.181885	0.177548	2.252878
	75	4.389462	0.220237	0.378736	4.141036
	100	4.356456	0.157625	0.018133	2.633181
	150	4.352150	0.169417	0.068435	2.043601
	200	4.370565	0.169089	0.111821	2.576298
	250	4.365365	0.172009	0.037441	2.470689
	300	4.358217	0.168921	0.045316	2.293105
10	10	3.577240	0.073720	0.030391	1.901382
	25	3.610458	0.061027	0.486862	2.446707
	50	3.582542	0.088321	0.197897	2.345901
	75	3.587596	0.097364	0.090387	3.284802
	100	3.574557	0.073196	0.099806	2.771479
	150	3.571897	0.079265	0.130015	2.182581
	200	3.580865	0.078057	0.244178	2.665714
	250	3.578076	0.079543	0.099518	2.484007
	300	3.575679	0.079128	0.093955	2.290149
15	10	3.161705	0.045506	0.059349	1.972329
	25	3.184784	0.038528	0.576960	2.689910
	50	3.166326	0.057486	0.155727	2.352791
	75	3.168448	0.059877	0.051294	3.132471
	100	3.160770	0.046183	0.136328	2.904075
	150	3.158833	0.050046	0.148548	2.270863
	200	3.164713	0.048818	0.294278	2.782167
	250	3.162767	0.049854	0.108460	2.551390
	300	3.161759	0.050093	0.088725	2.318178
20	10	2.890062	0.032384	0.088654	2.057577
	25	2.908086	0.028322	0.577664	2.830716
	50	2.894277	0.042709	0.105012	2.331451
	75	2.895203	0.042548	0.044281	3.097650
	100	2.889886	0.033550	0.131523	3.002758
	150	2.888345	0.036281	0.142627	2.334399
	200	2.892733	0.035116	0.297848	2.874302
	250	2.891209	0.035925	0.091979	2.616260
	300	2.890813	0.036414	0.063366	2.346792
25	10	2.692876	0.025068	0.109245	2.137106
	25	2.707886	0.022749	0.520967	2.862945
	50	2.696806	0.034220	0.062101	2.300172
	75	2.697121	0.032866	0.047985	3.097250
	100	2.693099	0.026466	0.101543	3.070260
	150	2.691804	0.028515	0.122223	2.379015
	200	2.695325	0.027425	0.271612	2.936914
	250	2.694053	0.028093	0.065316	2.670536
	300	2.693983	0.028692	0.035267	2.373990

or $\Delta_1^{'}$, the best order size is an increasing function of $\Delta_3^{'}$ and $\Delta_4^{'}$, while it is a decreasing or an increasing function of $\Delta_2^{'}$ or $\Delta_1^{'}$ respectively. Table **5.9** shows that for a fixed value of $\Delta_1^{'}$ and $\Delta_2^{'}$, the best order size is an increasing function of $\Delta_3^{'}$ and $\Delta_4^{'}$, while it is a decreasing function of $\Delta_1^{'}$ and $\Delta_2^{'}$ for fixed $\Delta_3^{'}$ and $\Delta_4^{'}$.

Entries in Tables **5.1(a)** to **5.9(a)** supplemented by calculations of similar values of the ratio q'^* point to possible robustness of the optimal order quantity when the decision parameters Δ_1, Δ_2, Δ_3 and Δ_4 are under, over or equally estimated.

CONCLUSION

An inventory model under flexibility and reliability criteria is considered in a non-random optimization environment. For highly demanded quality products normally, planning horizon is taken as finite and of crisp in nature. But in reality due to rapid change of environment, demand for the items fluctuates every year. So it is worthwhile to consider the planning horizon as uncertain in nature. Here stochastic nature of the uncertain planning horizon is considered. Complexities of the models increase when some other parameters of the model become imprecise in non-stochastic sense that is fuzzy in nature. Due to constraints of the system, the average level of investment necessary to operate effectively is assumed to be fixed in managing the long term investment in inventory management. Clamping by the backdrop of many factors, the system cost seems to be considered fixed. The methodology presented here shows that the total variable cost in the same industry with the crisp and fuzzy are almost the same. The optimal solution will be a good approximation with fuzzy strategy than that of crisp, when flexibility and reliability criteria are considered.

For giving managerial insights into the non-random optimization approach, a sensitivity analysis is performed to determine the sceneries which would result in vibration of the setup cost and inventory holding cost. It is examined that the total cost corresponding to our fuzzy solution in presence of reliability criteria is 0.0256 % more consistent than the cost obtained by the crisp. The proposed approach continues to work dynamically as the market evolves; it makes sense from our experience to conclude that this model can be an effective tool for dynamic inventory models.

Simulated sampling distribution converges around skewness of zero and kurtosis of three. In Table **6.1** the results obtained from the data admitting normality are what would be expected. The conclusion drawn from this analysis is that managers making financial aid decisions are better equipped to make informed decisions using vibrated values of decision parameters during implementations of the model.

In order to provide an assessment of the effectiveness of the present paper, further research should be included to a comprehensive numerical investigation of real life situations and identify the nature of system cost and empirically test more extricable for these effect of the values of the firm. With vibration in the system cost, the risk of the total cash flow can be reduced and hence the risk implication of the altering system cost may now be ignored.

There have been many methods to be proposed for the modelling in a non-random optimization environment with a disposal mechanism under flexibility and reliability criteria. However they cannot meet the real choice process with performance evaluation because most of the methods are the mathematical models. This is real critical subject that can result in choosing a disposal mechanism with various performances. The most important part of this, it gives a concrete result by recording the expert's previous experiences and process this with fuzzy logic arithmetic. As a future study it is possible to develop this method by using numerical performances criteria (DEA) to use objective and subjective evaluations together.

Also further research should include a comprehensive investigation of real life system and identify the nature of the system cost and empirically test most extricable for their effect. Further this model can be extended to consider the time value of money. Future study will incorporate more realistic propositions.

APPENDIX-A

$$A = \frac{q^*}{r^{*3}}\left(H+\phi_2\right) + a\lambda^{*-(b+1)}\left\{-2c(1-r^*)^{c-1} + r^*c(c-1)(1-r^*)^{c-2}\right\} \quad \text{(A1)}$$

$$B = \frac{2r^*}{q^*\lambda^{*3}}(S+\phi_1) + a(b+1)(b+2)r^*(1-r^*)^c\lambda^{*-(b+3)} \quad \text{(A2)}$$

$$C = 2\lambda^* q^*\left(H+\phi_2\right) \quad \text{(A3)}$$

$$D = -\frac{1}{\lambda^{*2}q^*}\left(S+\phi_1\right) - a[(1-r^*)^c - r^*c(1-r^*)^{(c-1)}]\times(b+1)\lambda^{*-(b+2)} \quad \text{(A4)}$$

$$E = -2\lambda^* q^*\left(H+\phi_2\right) + 2r^{*2}\lambda^{*-b}a[(1-r^*)^c - r^*c(1-r^*)^{c-1}] \quad \text{(A5)}$$

$$F = \frac{r}{\lambda^{*2}q^{*2}}\left(S+\phi_1\right) \quad \text{(A6)}$$

$$G = \left(S+\phi_1\right) + a(b+1)\lambda^{*-b}q^*\times[(1-r^*)^c - r^*c(1-r^*)^{(c-1)}] \quad \text{(A7)}$$

$$I = \frac{-1}{\lambda^* q^{*2}}\left(S+\phi_1\right) - \frac{1}{2r^{*2}}\left(H+\phi_2\right) \quad \text{(A8)}$$

$$J = q^{*2}\left(H+\phi_2\right) \quad \text{(A9)}$$

$$\text{Relative Error (R.E.)} = \left((M^* - Z_1)/Z_1\right)\times 100 \quad \text{(A10)}$$

$$q'^* = \frac{q'}{q^*} \quad \text{(A11)}$$

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