Two-Machine Flow-Shop Scheduling with Job Delivery Coordination

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Abstract: This study considers a class of two-machine flow-shop scheduling problems with job delivery coordination. Both vehicle capacity and transportation times are also investigated. The objective is to minimize the mean arrival time of jobs. Two integer programming models are developed to optimally solve this problem. These two integer programming models are Models 1 and 2. Model 1 adopts the concept of assignment problem to formulate the proposed problems, while Model 2 bases on the dichotomous constraints models. Model 2 is theoretically better than Model 1 in size complexity analysis.

Keywords: Scheduling, Two-machine flow-shop, Integer programming.

INTRODUCTION

An important and active topic in manufacturing research over the last ten years has been supply chain management [1]. A supply chain, also termed a value chain, is a network of interlinked suppliers, manufacturers, distributors, and customers. Supply chain management represents a new focus on how to link organizational units to best serve customer needs and to improve the competitiveness of a supply chain [2].

Lee and Chen [3] investigated machine scheduling models that impose constraints on both transportation capacity and transportation times. They categorized this class of scheduling problems based on two different types of transportation situations. The first type, Type-1, was the intermediate transportation in a flow-shop where jobs were transported from one machine to another for further processing. The second type, Type-2, was the delivery of finished jobs to customers. Jobs were delivered in batches by one or more vehicles with finite or infinite capacity. They assumed that the sizes of all jobs were of consistence. Chang and Lee [4] extended their work to the situation on which each job requires different physical space for delivery, whereas Li *et al*., [5] considered a problem involving job deliveries to multiple customers at different locations. Lee and Chen [3] and Soukhal *et al*., [6] studied the class of flow-shop problems by analyzing their complexity.

Chang and Lee [4] studied, for the first time, problems in which each job might occupy differing amounts of physical space in a transport vehicle. Based on machine scheduling and finished product delivery as discussed by Chang and Lee [4], this study deals with the situation in which jobs are processed on a two-machine flow-shop and delivered by a single vehicle to a single customer area. The objective is to determine the job processing sequence on the machine together with the delivery schedule to minimize the mean arrival time. Two integer programming models are developed to optimally solve this problem. These two integer programming models are Models 1 and 2. The rest of the paper is organized as follows. Section 2 defines the studied problems as well as the required notations. In section 3 two integer programming models are proposed and finally, conclusions drawn in section 4.

PROBLEM DESCRIPTION AND NOTATION DEFINITION

This study investigates a class of the two-stage scheduling problem where the first stage is job production and the second is job delivery. The investigative focus is on integrating production scheduling with delivery of finished products to a single customer. In this problem, jobs are first processed in a two-machine flow-shop then delivered in batches by a vehicle to a single customer. Jobs require varying physical space while being loaded into a vehicle and delivered to a single customer. The vehicle is associated with a capacity constraint, measured by the total physical space of the jobs it can deliver in one trip and has a specific transportation time for each delivery. Job completion time denotes the time when a job arrives at the customer. All jobs delivered in one shipment to a single customer have the same completion times. The cost function measures the customer service level, taking production and job delivery as one system. In particular, this study aimed to minimize all jobs delivery times for the customer.

The proposed problem was described as follows. There is a set of n independent jobs, $N = \{J_1, J_2, \ldots, J_n\}$, to be processed without preemption at a manufacturing system consisting of two machines and then delivered to a single customer. These two machines, ordered as M_1 and M_2, are continuously available from time zero onwards. Every job comprises two operations associated with respective processing times and release times on both machines. Before

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the first operation has been completed on M_1, the second one can not be started for processing on M_2. Let s_i be the size of J_i, representing the physical space J_i occupied when loaded in the vehicle. One vehicle was available for delivery, with a capacity of z and was first located at the manufacturing facility. Vehicle capacity was measured by the total physical space the vehicle provides for one delivery. Assuming that while the total physical space of jobs loaded did not exceed z, they could be arranged to fit in the physical space provided by the vehicle. A delivery batch denotes all jobs delivered together in one shipment and a transportation time is associated with each delivery batch. Furthermore, we define a single customer area as a location. In this study, the situation of deliveries made to just one customer area was considered. Let t_{01} be the one-way travel time between the manufacturing area (called area 0) and the single customer area (called area 1). The problem was to find a schedule for processing and delivering finished jobs to the customer in the minimal time required for all jobs to be processed and delivered. For convenient analysis, the arrival time of J_i, denoted as A_i, is defined in this study as the time when the vehicle finishes delivering J_i to the customer site.

The three-field notation scheme, $\alpha|\beta|\gamma$, introduced by Graham *et al.*, [7], was applied to represent the problems being studied. In the three-field scheme, α was the scheduling environment, β described job characteristics or restrictive requirements, and γ defined the objective function to be minimized. This study consider the following problem: $F_2 \to D, k = 1|v = 1, c = z|(1/n)\sum_{i=1}^{n} A_i$. In the α field, "$F_2 \to D, k = 1$" represents the problem in which jobs are first processed on a two-machine flow-shop environment and then are delivered to one customer area. In the β field, "$v = 1, c = z$" means that only one vehicle with a capacity of z is considered in our problems. The sum of sizes of the jobs loaded in the vehicle cannot exceed z in one shipment. In the γ field, "$(1/n)\sum_{i=1}^{n} A_i$" denotes the mean arrival time of the set of jobs.

The following notation was used throughout the study:

H = a very large positive number;

n = number of jobs for processing at time zero;

J_i = job number i;

M_k = machine number k;

B_j = the jth delivery batch;

e_i = the physical space J_i occupies when loaded in the vehicle;

t_{01} = the one-way travel time between the manufacturing area (called area 0) and the single customer area (called area 1);

z = the vehicle capacity, that is, the total physical space provided by the vehicle for one delivery;

l_i = the release time of J_i;

p_{ik} = the processing time of J_i on M_k;

A_i = the arrival time of J_i, that is, the time when the vehicle finished delivering J_i to the customer site;

r_j = the ready time of B_j, representing the latest completion time on the machine of the jobs assigned to B_j. Note $u_j \geq r_j$ in any feasible solution;

u_j = the departure time from the machine for the vehicle to deliver B_j;

h_{kq} = the earliest start time of the job in the sequence position q on machine M_k;

s_{ik} = the starting time of J_i on machine M_k;

X_{ikq} = 1 if J_i was scheduled at position q on machine M_k; 0 otherwise;

Y_{ij} = 1 if J_i was scheduled at batch B_j; 0 otherwise;

$Z_{ii'k}$ = 1 if J_i precedes $J_{i'}$ (not necessarily immediately) on machine M_k; 0 otherwise.

THE OPTIMIZATION MODEL

The optimization model employed integer programming technique to find the optimal solution for the problem $F_2 \to D, k = 1|v = 1, c = z|(1/n)\sum_{i=1}^{n} A_i$. Two integer programming formulations were presented. Model 1 adopts the concept of assignment problem to formulate the proposed problems, while Model 2 bases on the dichotomous constraints models.

Model 1

$$\text{Minimize } \frac{1}{n}\sum_{i=1}^{n} A_i \tag{1}$$

Subject to

$$\sum_{j=1}^{n} Y_{ij} = 1 \quad i = 1, 2, \ldots, n \tag{2}$$

$$\sum_{i=1}^{n} Y_{i,j+1} \leq H\sum_{i=1}^{n} Y_{ij} \quad j = 1, 2, \ldots, n-1 \tag{3}$$

$$\sum_{i=1}^{n} e_i Y_{ij} \leq z \quad j = 1, 2, \ldots, n \tag{4}$$

$$\sum_{q=1}^{n} X_{ikq} = 1 \quad i = 1, 2, \ldots, n;\ k = 1, 2 \tag{5}$$

$$\sum_{i=1}^{n} X_{ikq} = 1 \quad k = 1, 2;\ q = 1, 2, \ldots, n \tag{6}$$

$$h_{kq} + \sum_{i=1}^{n} p_{ik} X_{ikq} \leq h_{k,q+1} \quad k = 1, 2;\ q = 1, 2, \ldots, n-1 \tag{7}$$

$$h_{1q} + \sum_{i=1}^{n} p_{i1} X_{i1q} \leq h_{2q} \quad q = 1, 2, \ldots, n \tag{8}$$

$$h_{1q} \geq l_i - H(1 - X_{i1q}) \quad i = 1, 2, \ldots, n;\ q = 1, 2, \ldots, n \tag{9}$$

$r_j \geq h_{2q} + p_{i2} - H(2 - Y_{ij} - X_{i2q})$

$i, j, q = 1, 2, \ldots, n$ (10)

$u_1 = r_1$ (11)

$u_j \geq r_j \; j = 2, 3, \ldots, n$ (12)

$u_j \geq u_{j-1} + t_{01} + t_{10} \; j = 2, 3, \ldots, n$ (13)

$A_i \geq u_j + t_{01} - H(1 - Y_{ij}) \; i, j = 1, 2, \ldots, n$ (14)

$A_i \geq 0 \; i = 1, 2, \ldots, n;$

$r_j \geq 0, u_j \geq 0 \; j = 1, 2, \ldots, n;$

$h_{kq} \geq 0 \; k = 1, 2; q = 1, 2, \ldots, n$

Y_{ij} is binary $i, j = 1, 2, \ldots, n$

X_{ikq} is binary $i, q = 1, 2, \ldots, n; k = 1, 2$ (15)

The objective was found in Eq. (1). The objective was to minimize the mean arrival time of the set of jobs during the time horizon. Constraint set (2) ensured that each job could be processed on only one batch. Constraint set (3) ensured that if $\sum_{i=1}^{n} Y_{ij} = 0$ then $\sum_{i=1}^{n} Y_{i,j+1}$ could not equal 1 for $j = 1, 2, \ldots, n - 1$. That is, if no jobs placed at batch number j then also no jobs placed at batch number $j + 1$. Constraint set (4) restricted the total batch size to vehicle capacity. Constraint set (5) ensures that job J_i is uniquely placed on M_k. Constraint set (6) satisfies the requirement that each position of M_k has a unique job. Constraint sets (7) and (8) enforce the technological requirements of each job. Constraint set (9) defines the earliest start time of the job in the sequence position j on machine M_1. Furthermore, constraint set (10) defines the ready time r_j. Constraint sets (11) to (13) state the departure time u_j. Constraint set (14) defines the arrival time A_i. Finally, constraint set (15) specifies the non-negativity of A_i, u_j, r_j, and h_{kq}, and establishes the binary restrictions for Y_{ij} and X_{ikq}.

Model 2

According to the concept of dichotomous constraints, constraint sets (5)-(10) can be rewritten as constraint sets (16)-(20).

$s_{i1} + p_{i1} \leq s_{i2} \; i = 1, 2, \ldots, n$ (16)

$s_{ik} + p_{ik} \leq s_{i'k} + H(1 - Z_{ii'k}) \; 1 \leq i < i' \leq n;$

$k = 1, 2$ (17)

$s_{i'k} + p_{i'k} \leq s_{ik} + H Z_{ii'k} \; 1 \leq i < i' \leq n; k = 1, 2$ (18)

$s_{i1} \geq l_i \; i = 1, 2, \ldots, n$ (19)

$r_j \geq s_{i2} + p_{i2} - H(1 - Y_{ij}) \; i, j = 1, 2, \ldots, n$ (20)

Constraint set (16) restricts the starting time of J_i on machine M_2 to follow the finish time of J_i on machine M_1. Constraint sets (17) and (18) meet the requirement that only one job can be processed at any time, that is, either $s_{ik} + p_{ik} \leq s_{i'k}$ or $s_{i'k} + p_{i'k} \leq s_{ik}$ will hold. Incorporating binary variable $Z_{ii'k}$ and a very large positive number H, equations (17) and (18) together ensure that one of these two constraints holds while the other is eliminated. Constraint sets (19) and (20) define the starting time s_i and the ready time r_j, respectively.

The non-negativity of A_i, r_j, u_j, and s_{ik}, and the binary restrictions of Y_{ij} and $Z_{ii'k}$ are specified in (21).

$A_i \geq 0 \; i = 1, 2, \ldots, n;$

$r_j \geq 0, u_j \geq 0 \; j = 1, 2, \ldots, n;$

$s_{ik} \geq 0 \; i = 1, 2, \ldots, n; k = 1, 2$

Y_{ij} is binary $i, j = 1, 2, \ldots, n$

$Z_{ii'k}$ is binary $1 \leq i < i' \leq n; k = 1, 2$ (21)

Hence, Model 2 is comprised of (1)-(4), (11)-(14), and (16)-(21).

Comparisons of the Above Two Models

French [8] stated that the speed with which integer programming problems can be solved depends upon the number of variables and constraints in the problem, and that the dominant factor is the number of binary variables. Wilson [9] and Liao and You [10] showed that, if two models have the same number of binary variables, then the number of constraints is the next most influential element. Accordingly, Table **1** summarizes the sizes of these two models. Model 2 has the same number of continuous variables as that of Model 1, but has $n^3 - 2n^2 + 7n - 2$ fewer constraints and $n^2 + n$ fewer binary variables. Thus, Model 2 is theoretically better than Model 1.

Table 1. The Size of Integer Programming Model

Model	Number of Binary Variables	Number of Constraints	Number of Continuous Variables
1	$3n^2$	$n^3 + 2n^2 + 12n - 3$	$5n$
2	$2n^2 - n$	$4n^2 + 5n - 1$	$5n$

CONCLUSION AND FUTURE RESEARCH

This study considers a two-machine flow-shop scheduling for jobs delivered to a single customer area. The objective was to minimize the mean arrival time. Two integer programming formulations were proposed. These two integer programming models are Models 1 and 2. Model 1 adopts the concept of assignment problem to formulate the proposed problems, while Model 2 bases on the dichotomous constraints models. Model 2 is theoretically better than Model 1 in size complexity analysis.

Future research should address problems with multiple customer areas or different shop environments, including flow-shop and job-shop. Problems with other performance measures, including minimum makespan, mean tardiness, and multi-criteria measures, should also be studied. Meta-heuristics could be used to achieve solutions.

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