PV Generator Driven First Order Circuit - Transient Analysis using LambertW Function

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Abstract: The study of dynamic behaviour of circuits containing one or more energy-storing circuit elements powered by photovoltaic (PV) generator has been an intriguing aspect to the PV module design engineer. A PV module constitutes a special power source that has a transcendental current-voltage relationship. As can be expected this makes the transient response analysis for such circuits far more complicated than the conventional power source. Methods and models have been suggested to study the transient behaviour of such circuits. Earlier works on first order circuits have assumed that there are no current leakage losses in the form of Shunt resistance for a PV generator. In absence of an exact closed form solution for the transcendental current-voltage relations of a PV generator, these methods approximately describe the transient behaviour both quantitatively and qualitatively. Here we present an approach that uses exact closed form solutions based on Lambert W function, having incorporated finite value of shunt resistance for the PV generator.

Keywords: First order circuit analysis, Solar cell, Transient analysis, LambertW function, Photovoltaic generator.

1. INTRODUCTION

For higher efficiency of Photovoltaic (PV) systems it is very important for the designer to be able to have as accurate a value as possible for a) various design parameters, b) the load analysis of the PV generator for the given load as well as c) the transient analysis of the entire system. Specific case studies and analysis [1-6] have been formulated earlier by others. A generic study of the first order circuits with PV generator has been performed by Zacharias *et al.* [7].

Zacharias *et al.* [7] established that of the available methods incremental method is the best approach to solve the first-order non-linear differential equation. According to this method, the current i(t) will be calculated only for some discrete values of t. In particular, with the value of current i_N known at t=NT, an equation i_{N+1} relating to i_N. Such an equation is a special case of a difference equation, a technique for converting the governing non-linear differential equation into a linear differential equation in each time interval T and then into a difference equation.

However the PV generator (PVG) model considered by them does not account for current leakage losses in the form of Shunt resistance for the PVG, and their belief of non-existence of an exact closed form solution to the transcendental current-voltage relation of a PVG. Exact closed form solution to solve the transcendental current-voltage relation of a PVG has been proposed by Jain *et al.* [8-11].

In this study to have a greater degree of accuracy we have based our solution on LambertW function incorporating

a finite shunt resistance thus overcoming the above inabilities. Incremental model has been used to analyze the transient behaviour of first order circuit.

2. PV GENERATOR

For an equivalent circuit representing a PVG appropriately it requires to take into account the presence of parasitic series resistance and shunt conductance's [12-14]. No model would be complete without the consideration of these losses. We here use the single diode exponential model that takes care of these losses in the form of a series and a shunt resistance connected to the diode and a source as shown in Fig. (**1**):
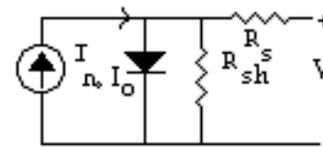
Fig. (1). Single diode exponential model for a Solar Cell.

This model mathematically can be expressed as

$$i = I_0\left(\exp^{\left(\frac{v-iR_s}{nV_{th}}\right)} - 1\right) + \frac{v - iR_s}{R_{sh}} - I_{ph} \qquad (1)$$

The above relation is a transcendental current-voltage relation. An exact closed form solution, based on LambertW function, to this equation is

$$v = i\left(R_s + R_{sh}\right) + R_{sh}\left(I_0 + I_{ph}\right) - nV_{th}LambertW\left(\frac{I_0 R_{sh}}{nV_{th}}\exp^{\left(\frac{R_{sh}\left(i + I_0 + I_{ph}\right)}{nV_{th}}\right)}\right) \qquad (2)$$

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$$i = \frac{v - R_{sh}(I_0 + I_{ph})}{R_s + R_{sh}} + nV_{th} LambertW\left(\frac{I_0 R_t R_s}{nV_{th}} \exp^{\left(\frac{R_t(v + R_s(I_0 + I_{ph}))}{nV_{th}}\right)}\right) \quad (3)$$

where $R_t = R_{sh}/(R_s + R_{sh})$ and other symbols have their usual meaning. The above expressions are valid for a single cell. In case of an array we need to have an expression that takes into account the current and voltages due to all the cells. PVG is a collection of interconnected solar cells and other components. It is assumed that the entire array is dealt as a system constituting N strings in parallel having M cells connected in series per string and all the cells are identical.

For a series array consisting of M identical cells where the current through the array is equal in each cell and the voltage be given as

$$V_a(i_a) = \sum_{n=1}^{M} V_n(i_a) \quad (4)$$

Considering these series strings in parallel to each other thus keeping the voltage across the array constant and the current through the array is equal to sum of currents through parallel strings.

$$i_a(V) = \sum_{n=1}^{N} i_n(V) \quad (5)$$

here i_n is the total current passing through the string of M cells connected in series as above. Now replacing various parameters as considered by Applebaum *et al.* [15] R_s = 0.9Ω, I_0 = 0.0081 A, V_{th} = 0.0422 V^{-1} and I_{ph} = 13.615 A at an insolation of 1000 Wm^{-2} in expressions as suggested by us earlier (Jain *et al.* [11]). Also the value of R_{sh} is taken to be as 1000 Ω. The Array constitute has 324 cells connected in a series to form a string and 18 strings in parallel.

2.1. PVG Connected to a Series RL Load

The circuit equation using Kirchoffs Voltage Law for the circuit shown in Fig. (**2a**)

(a)   (b)
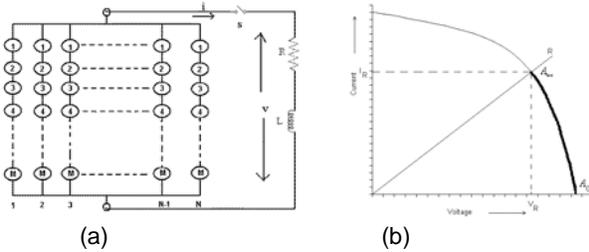
Fig. (2). PVG connected to a series RL Load (**a**) circuit diagram (**b**) dynamic route (heavy line) & the linear model.

$$L\frac{di}{dt} + Ri = v(i) \quad (6a)$$

therefore replacing v(i) from above we have

$$L\frac{di}{dt} + Ri = \sum_{q=1}^{M} V_q(i_a) \quad (6b)$$

Applying the linear operator Δ to both sides of equation (6a) one gets

$$L\frac{d(\Delta i)}{dt} + R\Delta i = \Delta v(i) \quad (7)$$

where $\Delta i = i_{N+1} - i_N$, for all $i = i_{N+1}$ for $t_N < t < t_N + T$ (8)

and $i = i_N$ for $t = t_N$

However by Taylor's expansion we get

$$\Delta v(i) = \frac{dv(\Delta i)}{di} + \delta\Delta i \quad \text{implying} \quad \Delta v(i) = \frac{dv(\Delta i)}{di} \quad (9)$$

defining $R_{so} = |dv/di|$ as the dynamic or incremental resistance of the PV element, then from eqn we get

$$R_{so} = \frac{R_s}{1 - \dfrac{R_t}{1 + LambertW\left(\dfrac{I_0 R_s R_t}{nV_{th}} e^{\left(\frac{R_t(v + R_s(I_0 + I_{ph}))}{nV_{th}}\right)}\right)}} \quad (10a)$$

for our array

$$R_{sa} = R_{so} * M / N \quad (10b)$$

Since dv/di < 0 it follows that

$$\frac{dv}{di}\bigg|_{i=i_n} = -R_{sa_n}$$

therefore equation (9) can be written as

$$\Delta v = -R_{sa_n}(i_{N+1} - i_N) \quad (11)$$

Substituting equation (8) & (11) equation (7) becomes

$$L\frac{di_{N+1}}{dt} + \left(R + R_{sa_n}\right)i_{N+1} = v_N + R_{sa_n}i_N \quad (12)$$

This equation is valid for the interval $t_N < t < t_N + T$. The solution of this equation yields the current i_{N+1} at $t = t_N + T$ as

$$i_{N+1} = \frac{v_N + R_{sa_N}}{R + R_{sa_N}} + \left(i_N - \frac{v_N + R_{sa_N}}{R + R_{sa_N}}\right)e^{-B_{sa_n}T} \quad (13)$$

where v_N is calculated for $i = i_N$ from

$$v = i(R_s - R_{sh}) + k + V_{oc}LambertW\left(\frac{V_{oc} - R_{sh}I_{ph}}{V_{oc}\left(e^{\frac{V_{oc}}{nV_{th}}} - 1\right)}e^{\left(\frac{k}{V_{oc}} - iR_{sh}\right)}\right) \quad (14)$$

where $k = \dfrac{V_{oc} - I_{ph}R_{sh}e^{\frac{V_{oc}}{nV_{th}}}}{e^{\frac{V_{oc}}{nV_{th}}} - 1}$ and $B_{sa_n} = 1/\tau_N$

implying $B_{sa_n} = \left(R + R_{sa_N}\right)/L \quad (15)$

equation (13) is the desired difference equation and is recursive in nature.

(a)　　　　　　　　(b)
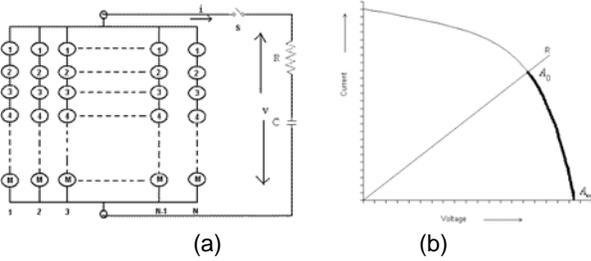
Fig. (3). PVG connected to a series RC Load (**a**) circuit diagram (**b**) dynamic route (heavy line) & the linear model.

(a)　　　　　　　　(b)
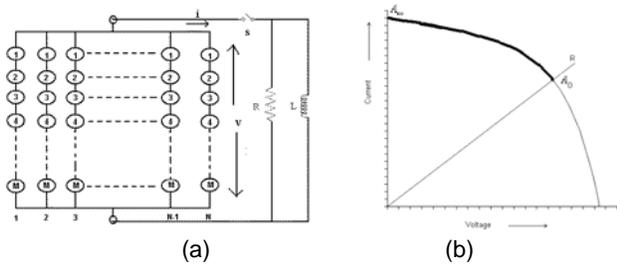
Fig. (4). PVG connected to a parallel RL Load (**a**) circuit diagram (**b**) dynamic route (heavy line) & the linear model

(a)　　　　　　　　(b)
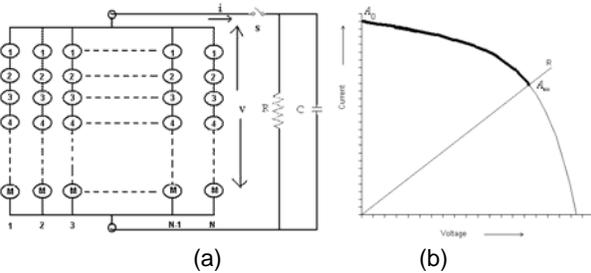
Fig. (5). PVG connected to a parallel RC Load (**a**) circuit diagram (**b**) dynamic route (heavy line) & the linear model.
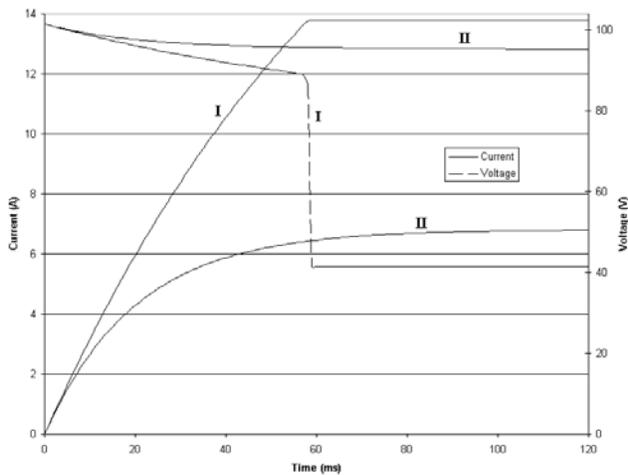
Fig. (6). PVG current & voltage vs. time in a PV RL series circuit for (I) R = 3Ω & L=0.3 H (II) R = 14Ω & L=0.3 H.
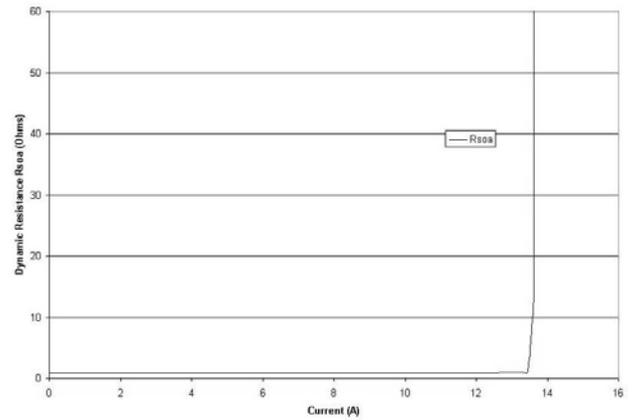
Fig. (7). Dynamic Resistance vs PV Generator current in a PV RL series circuit for R = 3Ω & L=0.3 H.

2.2. PVG Connected to a Series RC Load

The circuit equation using Kirchoffs Voltage Law for the circuit shown in Fig. (**3**):

$$\frac{C\left(R + R_{sa_N}\right)d\left(\Delta v_{CN+1}\right)}{dt} + v_{CN+1} = v_N + R_{sa_N} i_N \quad (16)$$
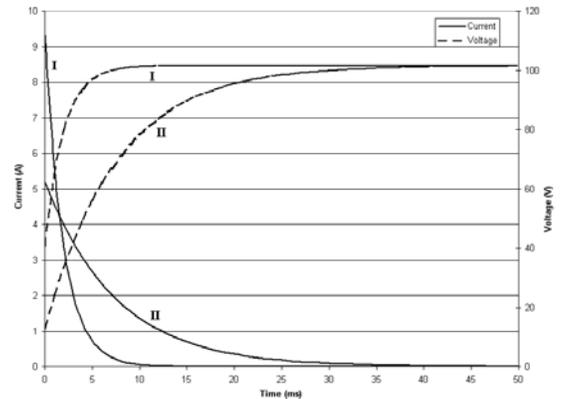
Fig. (8). PVG current & voltage vs. time in a PV RC series circuit for (I) R = 3Ω & C = 500μf (II) R = 14Ω & C = 500μf.

The solution for the capacitor voltage at t = t_N+T is

$$v_{CN+1} = v_N + R_{sa_N} i_N + \left(v_{CN} - v_N - R_{sa_N} i_N\right)e^{-B_{sa_n}T} \quad (17)$$

where $B_{sa_n} = \dfrac{1}{C\left(R + R_{sa_N}\right)}$ \quad (18)

the PVG current is given by

$$i_{N+1} = \frac{v_N + R_{sa_N} i_N - v_{CN+1}}{R + R_{sa_N}} \quad (19)$$

2.3. PVG Connected to a Parallel RL Load

The circuit equation using Kirchoffs Voltage Law for the circuit shown in Fig. (**4a**)

$$L\left(\frac{1}{R} + \frac{1}{R_{sa_N}}\right)\frac{di_{LN+1}}{dt} + i_{LN+1} = \frac{v_N + R_{sa_N}i_N}{R_{sa_N}} \qquad (20)$$

leads to the solution for the inductor current i_{LN+1} at $t = t_N+T$ is

$$i_{LN+1} = \frac{v_N + R_{sa_N}i_N}{R_{sa_N}} + \left(i_{LN} - v_N - R_{sa_N}i_N\right)e^{-B_{sa_n}T} \qquad (21)$$

where $\quad B_{sa_n} = \dfrac{RR_{sa_N}}{L\left(R + R_{sa_N}\right)} \qquad (22)$

the PVG current is given by
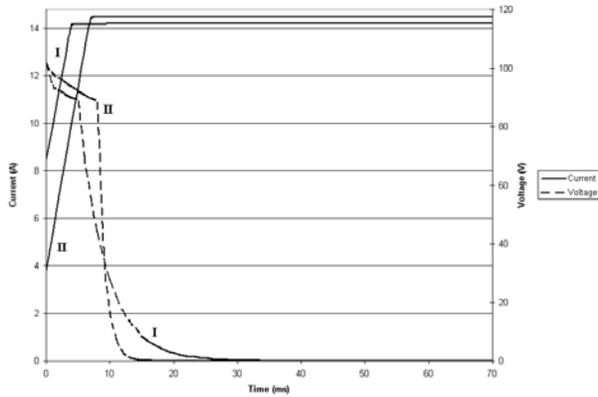
Fig. (9). PVG current & voltage vs. time in a PV RL parallel circuit for (I) R = 14Ω & L= 0.3 H (II) R = 50Ω & L= 0.3 H.

$$i_{N+1} = \frac{v_N + R_{sa_N}\left(i_N + i_{LN+1}\right)}{R + R_{sa_N}} \qquad (23)$$

2.4. PVG Connected to a Parallel RC Load

The circuit equation using Kirchoffs Voltage Law for the circuit shown in Fig. (**5**).
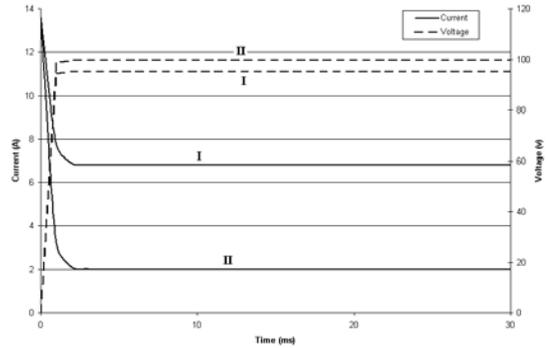
Fig. (10). PVG current & voltage vs. time in a PV RC parallel circuit for (I) R = 14Ω & C = 500 μf (II) R = 50Ω & C = 500 μf.

$$\frac{Cdv_{CN+1}}{dt} + v_{CN+1}\left(\frac{1}{R} + \frac{1}{R_{sa_N}}\right) = \frac{v_N + R_{sa_N}i_N}{R_{sa_N}} \qquad (24)$$

The solution for the capacitor voltage at $t = t_N+T$ is

$$v_{CN+1} = \frac{R\left(v_N + R_{sa_N}i_N\right)}{R + R_{sa_N}} + \left(v_{CN} - \frac{R\left(v_N + R_{sa_N}i_N\right)}{R + R_{sa_N}}\right)e^{-B_{sa_n}T} \qquad (25)$$

where $\quad B_{sa_n} = \dfrac{R + R_{sa_N}}{CRR_{sa_N}} \qquad (26)$

the PVG current is given by

$$i_{N+1} = \frac{v_N + R_{sa_N}i_N - v_{CN+1}}{R_{sa_N}} \qquad (27)$$

Table 1. Comparison of Results Between Zacharias *et al.* [7] (ZIM) and Using LambertW Function (LIM) for Solutions in the PV – R-L Series Circuit for Different Time Steps (R=3Ω, L= 0.3H)

Time (ms)	Step Size = 0.1 ms		Step Size = 1.0 ms		Step Size = 2.5 ms		Step Size = 5.0 ms	
	Z IM	LIM	Z IM	LIM	Z IM	LIM	Z IM	LIM
5	2.7966	1.637843	2.7967	1.637843	2.7974	1.637844	2.7997	1.637845
10	5.3226	3.172524	5.3229	3.172524	5.3244	3.172527	5.3295	3.172531
15	7.5787	4.610524	7.5792	4.610528	7.5818	4.610535	7.5905	4.610545
20	9.5572	5.957922	9.5581	5.957931	9.5622	5.957944	9.5760	5.957964
25	11.2357	7.22041	11.2370	7.220424	11.2437	7.220446	11.2658	7.220479
30	12.5579	8.403316	12.5604	8.403337	12.5723	8.40337	12.6103	8.403421
35	13.3757	9.511628	13.3802	9.511659	13.4041	9.511708	13.4806	9.511782
40	13.5388	10.55001	13.5392	10.55006	13.5407	10.55013	13.5559	10.55023
45	13.5409	11.52282	13.5409	11.52289	13.5409	11.52299	13.5425	11.52314
50	13.5409	12.4341	13.5409	12.43419	13.5409	12.43434	13.5409	12.43456
55	13.5409	13.28747	13.5409	13.28763	13.5409	13.28787	13.5409	13.28822
60	13.5409	13.63772	13.5409	13.77709	13.5409	13.70251	13.5409	14.08708
65	13.5409	13.63772	13.5409	13.77709	13.5409	13.70251	13.5409	14.09541

3. CONCLUSION

In this paper a comprehensive qualitative and quantitative analytical study of first order circuits powered by PVG module using incremental method incorporating LambertW function has been completed. Various graphs have been plotted to describe the circuit behavior for a PVG current & voltage on a time line for: a RL series load circuit for (I) R = 3Ω & L=0.3 H (II) R = 14Ω & L=0.3 H (Fig. **6**). Dynamic Resistance vs PV Generator current in a RL series circuit for R = 3Ω & L=0.3 H (Fig. **7**). RC series circuit for (I) R = 3Ω & C = 500 μf (II) R = 14Ω & C = 500 μf (Fig. **8**). RL parallel load circuit for (I) R = 14Ω & L= 0.3 H (II) R = 50Ω & L= 0.3 H (Fig. **9**). RC parallel circuit for (I) R = 14Ω & C = 500 μf (II) R = 50Ω & C = 500 μf (Fig. **10**). Results obtained by using incremental model without incorporating shunt Resistances as reported by Zacharias *et al.* [7] (Z I M) and incremental model using LambertW function (L I M) incorporating shunt resistance for solutions in the PV – R-L series circuit for different time steps (R=3Ω, L= 0.3H) are tabulated in Table **1**.

Thus from the above study it may affirm that W function technique incorporating a finite shunt and series resistance overcomes the known inabilities due to the absence of Shunt resistance in the PVG, and hence provides good correspondence to existing experimental results.

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